Gateway performance analysis in interconnected networks

Ian F Akyildiz and Jörg Liebeherr demonstrate the effect of finite buffers of gateways and NAUs on network performance

The performance of interconnected networks is highly dependent on the performance of the gateways. Since the finite storage capability of gateways affects the throughput it has to be considered for the analysis of interconnected networks. Different configurations of networks are studied: (i) gateways and the channels of local area networks have no buffer capacity constraints; (ii) only gateways have buffer capacity constraints; (iii) only the channels of local area networks have buffer capacity constraints; (iv) both gateways and the channels of the local area networks have buffer capacity constraints, (iv) both gateways and the channels of the local area networks have buffer capacity constraints. An approximation method is introduced which allows computation of the throughput for the above network configurations. Examples are given to demonstrate the impact of gateway buffer capacity on the performance of the network. Approximate results are compared and validated by simulation.

Keywords: communication networks, performance evaluation, gateways, finite buffers, throughput

The interconnection of heterogeneous local area networks (LAN) is accomplished by dedicated processors (i.e. the gateways) attached to each network. The gateways are the interface between the local and long haul network. They perform necessary protocol conversion, implement flow control algorithms and route packets over the long

College of Computing, Georgia Institute of Technology, Atlanta, GA 30332, USA

Paper received: 25 August 1989. Revised paper received: 18 April 1990

haul network. Additionally, gateways act as a buffer between networks with different transmission rates. Recent technological advances changed the paradigm of slow, long distance communication and (relatively) fast communication in a LAN. The task of a gateway is more difficult when communication has to be maintained between LANs with different transmission capabilities. e.g. 'classical' Ethernet-type networks exchanging data with high speed LANs. On the other hand, the imbalance of fast local transmission rates and slow long distance transmission rates may be reversed, e.g. a metropolitan area network (MAN) backbone with a capacity by far exceeding the transmission rates of the LANs connected to the backbone. In the near future, several communication systems from different generations will coexist. Internetwork design has to consider the implications of different transmission speeds of the network components. Otherwise, the system will suffer from throughput decrease due to link congestion, and packet loss caused by overflow of the gateway buffers.

Few performance studies of gateways in interconnected networks have been done so far. Exley and Merakos^{1, 2} study two interconnected broadcast networks by simulation, and obtain stability conditions for the network load. They compute values for packet delays under different network access strategies. Lazar and Robertazzi³ investigate flow control issues using a queueing model of two interconnected networks. Ben-Michael and Rom⁴ study two Aloha networks connected via a gateway. By assuming unbounded buffer capacities of gateways they derive analytical formulas for throughput and queueing delay. Varakulsiripunth *et al.*⁵ analyse a special flow control policy which constrains the amount of traffic accepted by

0140-3664/91/001015-12 © 1991 Butterworth-Heinemann Ltd

a LAN. They consider the finite buffer space of the gateways and obtain blocking probabilities at the gateways. Heath⁶ simulates high speed LANs, and demonstrates the importance of performance decrease due to finite buffer capacity stations. Cheng and Robertazzi⁷ give an overview of recent studies on performance analysis of interconnected networks.

In this paper we present analytical solutions for different network configurations to demonstrate the effect of finite buffers of gateways and network access units on the performance of the network. A classification of different network configurations is introduced. For these configurations we develop queueing models, and present analytical solutions for these queueing models. Numerical examples are given, and we demonstrate how the performance of the network is affected if certain parameters are varied. Conclusions are then given, and the algorithm we use to solve load dependent queueing networks with finite buffer capacities is explained in detail in the Appendix.

SYSTEM DESCRIPTION

We consider interconnected packet switching networks with several LANs connected by a long haul network. The LANs are connected to a long haul network by gateways, as shown in Figure 1.

In the network there are two different types of communication:

- communication between hosts of each LAN (intranetwork traffic);
- communication between hosts at different LANs (internetwork traffic).

Hosts in the same LAN communicate with each other using a shared broadcast channel. The channel is accessed by the hosts via an interface, the so-called network access unit (NAU). Based on communication



Figure 1. Interconnected network: LANs connected to a long haul network using gateways



Figure 2. Local area network structure

protocols considered in this study, only one packet is allowed to be sent on the channel at a time. If a host wants to transmit a packet to another host in the same LAN it forwards it to its NAU. The access protocol of the LAN decides which packet will be transmitted on the channel next. All packets in the NAU of the hosts can be seen as waiting in a global queue to access the channel. Though physically still residing at the hosts (in the buffers of the NAU), these packets logically belong to the broadcast channel. Once a packet obtains access to the channel it is immediately transmitted to the destination host, if source and destination hosts belong to the same LAN. If they do not belong to the same LAN, the packet is put into the NAU of the source LAN and the channel sends the packet to the gateway of the source LAN. The gateway then transmits the packet to the gateway of the destination LAN, which forwards the packet to the according host through its broadcast channel. The packet has to obtain access to the broadcast channel competing with intranetwork packets.

We investigate four different network configurations based on buffer capacity constraints of channels and gateway buffers.

Configuration 1: no capacity constraints

Gateways and channels are able to store an infinite number of packets. Even though the infinite buffer capacity assumption is unrealistic, we take this configuration into consideration for the sake of comparison with other configurations.

Configuration 2: gateways with capacity constraints

If a gateway has a buffer capacity for storing only a limited number of packets for transmission on the long haul network, it is possible that the storage capacity will be exhausted. In this case no more packets are allowed to be forwarded to the gateway because of the buffer overflow problem. All hosts wanting to transmit internetwork packets using this gateway have to wait until a space becomes available in the buffer of the gateway. This type

of situation occurs if the transmission rate of the LANs is much higher than the transmission rate of the internetwork. High-speed local networks (HSLN) are a representative example where the high speed of the LAN may cause buffer overflows at the gateways. A low transmission rate for internetwork traffic such as in packet switched satellite networks has the same effect.

Configuration 3: channels with capacity constraints

The finite buffer capacity of the channel may result in each channel being congested. A gateway, for instance, cannot deliver a packet to a host because the access to the channel is not possible due to its congestion. The congestion of the channel can also affect the hosts within each LAN transmitting packets to each other. Heavily loaded LANs are a representative example for this type of configuration.

Configuration 4: channels and gateways with capacity constraints

This configuration is a mixture of configurations 2 and 3. In addition to the performance decrease as in the two above cases, a backlog of untransmitted messages can be observed in a network with a high rate of internetwork transmissions. Suppose a LAN is congested, and packets received by a gateway cannot be forwarded locally. After some time the buffer capacity of the gateway will be exhausted. As an effect, internetwork packets cannot be transmitted to that particular LAN. They have to remain in their LAN until a space becomes available in the congested destination LAN. In this way, the congestion may propagate from one LAN to another, and may affect the entire internetwork communication. Consequently, the performance of each individual LAN decreases. In the worst case, a deadlock situation may occur where no packets can be transmitted at all.

MODELLING

We develop queueing models for different configurations described above. The model of the interconnected network consists of M LANs denoted by LAN_m for m = 1, ..., M. Each LAN_m contains a station for broadcast channel CH_m and a fixed number R_m of host stations H_{mr} for $r = 1, ..., R_m$. Each LAN_m has a gateway station which is composed of an input queue $GW_{m,in}$ and an output queue $GW_{m,out}$, as shown in Figure 3.

The detailed model of a LAN is given in Figure 4.

All host stations H_{m1} , H_{m2} , ..., H_{mR_m} put their packets to the buffer of the according channel CH_m which represents the broadcast channel of LAN_m . From CH_m the packets are routed to either one of the host stations



Figure 3. Queueing model of a gateway



Figure 4. Queueing model of a LAN

 $H_{m1}, H_{m2}, \ldots, H_{mR_m}$ or to the according gateway $GW_{m,out}$. A station of type $GW_{m,in}$ transmits all its output to CH_m . Internetwork connections are established by routing from a station $GW_{m,out}$ to one or more stations $GW_{i,in}$ ($j \neq m$).

The complete model of the interconnected network has *N* total number of stations, where *N* is composed of:

$$N = \left\{ \sum_{m=1}^{M} \text{number of hosts in } LAN_m \right\}$$
$$+ \left\{ \text{number of channels} \right\}$$
$$+ \left\{ \text{number of gateway queues} \right\}$$
$$= \sum_{m=1}^{M} R_m + M + 2 \cdot M$$

The load of the interconnected network is determined by the fixed number of packets traversing the network at a time, and is denoted by K. In our model packets are routed with fixed probabilities. The service time of all stations is exponentially distributed. The scheduling discipline of all stations is FCFS. All stations CH_m , $GW_{m,in}$ and $GW_{m,out}$ may have a finite buffer size denoted by B_{CH_m} , $B_{GW_{m,in}}$ and $B_{GW_{m,out}}$, respectively. The host stations are assumed to have no buffer constraints ($B_{H_{mr}} = \infty$; m = 1, 2, ..., M; $r = 1, 2, ..., R_m$). Buffer overflows are handled as follows:

A packet in any station is not allowed to leave if the destination station is full, i.e. the number of packets in the destination station is equal to its buffer capacity. In this case, the packet is blocked in the current station until a packet in the destination station is transmitted and a buffer becomes available.

The complete queueing model of the interconnected network has the structure as given in Figure 5. For the sake of simplicity we give a model with only M = 3 local area networks.

PERFORMANCE ANALYSIS

In this section we present analytical solutions providing throughput values for interconnected networks. Since we



Figure 5. Queueing model of the interconnected network (M = 3)

are primarily interested in studying the performance of interconnected LANs due to gateway buffer constraints, we introduce an approach which allows a separate analysis of intranetwork and internetwork traffic. The analysis of the suggested network configurations is carried out in two steps: first, we analyse the performance of each LAN independently; second, we obtain the overall performance of the interconnected network by analysing the internetwork communication. This approach does not only have the advantage of separating the analysis for different types of traffic (i.e. intra- and internetwork traffic), but also reduces the computational complexity of the analysis and allows us to analyse internetwork traffic under different workload without re-doing the computations for the entire network.

The queueing models of the configurations described above are different due to buffer size restrictions of gateway and channel stations. The following restrictions apply:

- Configuration 1: $B_{CH_m} = B_{GW_{m,in}} = B_{GW_{m,out}} = \infty$
- Configuration 2: $B_{GW_{m,in}} < K$; $B_{GW_{m,out}} < K$; $B_{CH_m} = \infty$
- Configuration 3: $B_{CH_{m,}} < K$; $B_{GW_{m,in}} = B_{GW_{m,out}} = \infty$
- Configuration 4: $B_{CH_m} < K$; $B_{GW_{m,in}} < K$; $B_{GW_{m,out}} < K$

for
$$m = 1, 2, ..., M$$

In the following we discuss the solutions for each configuration.

Configuration 1 networks

Networks of Configuration 1 fulfill the requirements for a product form network⁸, i.e. all stations have exponential service times, scheduling is according to FCFS and all buffers are infinite. Hence, analytical methods like mean



Figure 6. Reduced queueing network

value analysis, in short form MVA^9 , can be applied. In the following we discuss our approach, which is based on that of Chandy et *al.*¹⁰.

For each LAN_m $(1 \le m \le M)$ we construct one flowequivalent composite station representing the stations $H_{m1}, H_{m2}, \ldots, H_{mR_m}$ and CH_m . We refer to the flowequivalent station for LAN_m as station Lan_m . The load dependent service rates $\mu_{Lan_m}(k)$ (for $k = 1, 2, \ldots, K$) of Lan_m are determined by analysing the m-th LAN separately, i.e. we set the service times of all stations not belonging to LAN_m equal to zero and compute the throughput values $\lambda_{LAN_m}(k)$, for $k = 1, 2, \ldots, K$ using MVA. Then, the values $\lambda_{LAN_m}(k)$ are assigned to the load dependent service rates of the flow-equivalent station $\mu_{Lan_m}(k)$. By this way a reduced network is constructed where the stations belonging to LAN_m are replaced by the flow-equivalent station Lan_m ($1 \le m \le M$). The reduced network has the structure shown in Figure 6.

The reduced network can be also analysed by MVA⁹, thus providing exact results. Since all actual systems have finite storage space the results for configuration 1 networks can be seen as best case estimations for the investigated networks. The remaining configurations (2-4) cannot be solved by MVA. Since finite buffer capacity networks do not satisfy the conditions for product form networks, we have to apply approximate techniques. However, we follow the major steps taken in the above procedure, i.e. aggregating the stations of each LAN to a flow-equivalent station and then solving the reduced network from Figure 6.

Configuration 2 networks

Here we assume that the buffer size of the gateways is limited ($B_{GW_{m,in}}$; $B_{GW_{m,out}} \leq K$, for m = 1, 2, ..., M). A gateway does not accept packets if its buffer is full. Hence, packets which are ready for transmission to a full station have to remain in the current station until a space becomes available in the full destination buffer, thus keeping the server of the current station idle. We refer to this phenomenon as a *blocking event*. Since the gateway has one queue for incoming traffic ($GW_{m,in}$), as well as for outgoing traffic ($GW_{m,out}$), a full buffer in the gateway may

cause blocking at one or more remote gateways $GW_{j,out}$ $(j \neq m)$ or at the local channel CH_m .

As for configuration 1 networks, we first aggregate the stations of each local area network LAN_m , i.e. H_{m1} , H_{m2}, \ldots, H_{mR_m} and CH_m (1 < m < M), to a flow-equivalent station Lan_m . The construction of the load dependent stations Lan_m and of the reduced network is carried out exactly as in configuration 1 because the stations of each LAN obey the product form assumptions. However, in the network given in Figure 6 blocking events may occur because of the finite capacity buffers of the gateways. Since blocking causes inter-dependencies between the stations, MVA cannot be applied for performance analysis. Therefore, we use the algorithm introduced in the Appendix.

Note that in configuration 2 networks only stations CH_m (which are included in the flow-equivalent sations Lan_m in the reduced network) and stations $GW_{m,out}$ may be blocked. Therefore, for configuration 2 networks we have to add blocking delay phases to all gateway servers $GW_{m,out}$ and all flow-equivalent stations Lan_m . A formal procedure for delay phase construction is given in the Appendix. We show the result of the phase construction for $GW_{1,in}$ in Figure 7 and for Lan_1 in Figure 8.

Applying the algorithm for load dependent queueing networks with finite buffer capacities given in the Appendix provides us with throughput values for the configuration 2 networks.

Configuration 3 networks

If the buffer capacity of the broadcast channel in the LAN is finite, throughput degradation occurs for both intranetwork and internetwork traffic, i.e. hosts cannot send packets to the local channel and packets from remote LANs cannot be delivered to the hosts of the LAN with the congested channel.



Figure 7. Phases of GW_{1,out} in configuration 2. 2: GW_{1,out}: 6: GW_{2,in}; 9: GW_{3,in}



Figure 8. Phases of Lan_1 in configuration 2. 1: LAN_1 ; 2: $GW_{1,out}$; 6: $GW_{2,in}$; 9: $GW_{3,in}$



Figure 9. Phases of $GW_{1,in}$ in configuration 3. 1: LAN₁; 2: $GW_{1,out}$

As in previous cases, we construct the flow equivalent stations Lan_m by analysing the stations H_{m1} , H_{m2} , ..., H_{mR_m} and CH_m separately for each local area network LAN_m $(1 \le m \le M)$. However, the throughput analysis to obtain $\lambda_{Lan_m}(k)$ (for k = 1, 2, ..., K) cannot be done with standard product form algorithms since the buffer capacities of the stations CH_m are finite. We apply the throughput method for closed queueing networks with exponential service time distributions and finite buffer capacities as described by Akyildiz¹¹. With the throughput values we construct the flow-equivalent stations Lan_m with load dependent service rates $\mu_{Lan_m}(k)$ ($1 \le m \le M$).

The flow equivalent station Lan_m has a finite buffer capacity which is equal to the buffer capacity of the channel CH_m . The reduced network shown in Figure 6 is analysed using the algorithm given in the Appendix. However, since only stations Lan_m ($1 \le m \le M$) have finite buffer capacity, the construction of the delay phases can be simplified. Blocking delays occur only at gateway stations $GW_{m,in}$ waiting for buffer space at Lan_m . All other stations do not have blocking delays. The phase construction for station $GW_{1,in}$ is given in Figure 9.

 $GW_{m,in}$ has only one delay phase caused by blocking at CH_m . Note that $GW_{m,out}$, the successor station of CH_m , does not cause blocking at $GW_{m,in}$, since it has infinite buffer capacities. After constructing the delay phases we apply the algorithm given in the Appendix and obtain the throughput values.

Configuration 4 networks

As mentioned before, configuration 4 is a mixture of configuration 2 and configuration 3. However, deadlocks are possible in this configuration. A deadlock is a circular wait of stations, each having a packet ready for transmission and waiting for available buffer in the destination stations. The network is analytically tractable only if it is deadlock free. As proven by Kundu and Akyildiz¹², a network with finite buffer capacities is deadlock free if the sum of the buffer capacities in each cycle of the network exceeds the total number of packets.

The flow equivalent station for each LAN is constructed as described above. Note that in this case, possibly all stations in the network may cause blocking delays for a station. Applying rules R1 and R2 from the Appendix we obtain the phase server for station Lan_1 of Figure 6 shown in Figure 10.

After construction of the delay phases for each station we apply the algorithm given in the Appendix and compute the throughput values.



Figure 10. Phases of Lan₁ in configuration 4. 1: LAN₁; 2: $GW_{1,out}$; 3: $GW_{1,in}$; 4: LAN₂; 5: $GW_{2,out}$; 6: $GW_{2,in}$; 7: LAN₃; 8: $GW_{3,out}$; 9: $GW_{3,in}$

NUMERICAL EXAMPLES

Example 1

The example network is a simplified model of the network shown in Figure 5. Here, we consider an interconnection of two homogeneous LANs. Each LAN has only three hosts connected to it.

The service time of the channel is set to $1/\mu_{CH_m} = 1/3$ ms. Note that according to our discussion above the service time of the channel includes the time spent in the network access unit. Assuming a packet length of 1000 byte/packet the service time of the channel corresponds to a network with a maximum transmission rate of 3 Mbit/s. The buffer capacity of the channel stations is assumed to be $B_{CH_i} = 3$. The ratio of intranetwork traffic and intermetwork traffic is set to 3 : 7, and shows heavy internetwork activity. Intranetwork packets are distributed equally among the host stations. The service time of the gateway stations is assumed to be $1/\mu_{GW_{i,in}} = 1/\mu_{GW_{iout}} = 1$ (for m = 1, 2). The complete list of parameters is summarized in Tables 1 and 2.

We now present examples of different parameter configurations, and demonstrate how constraints on the buffer capacity of the stations may decrease the performance of the network.

(a) Configuration 1

According to the algorithm described above we obtain results given in Figure 12.

Figure 12 shows the values of the intertraffic through-



Figure 11. Interconnection of two local area networks

Table 1. Service times (m = 1, 2)

	LAN _m
$1/\mu_{H_{mr}}$ (r = 1, 2, 3)	3
_1/μ _{CHm}	1/3
$1/\mu_{GW_{m,in}}$	1
$1/\mu_{GW_{m,out}}$	1

Table 2. Transition probabilities

Pij	CH _m	$H_{m,r}$ (r = 1, 2, 3)	GW _{j,in} (j ≠ m)	GW _{m,out}	
$H_{m,r}$ (r = 1, 2, 3)	1	0	0	0	
CHm	0	0.1	0	0.7	
GW _{m.in}	1	0	0	0	
GW _{m,out}	0	0	1	0	

put, i.e. the throughput seen by each of the gateway stations $GW_{i,j}$ (for i = 1, 2 and j = in, out), under different network load. Note that these results are exact.

(b) Configuration 2

We assume the buffer capacities of all gateway stations as:

 $B_{GW_{ij}} = 2$ for i = 1, 2 and j = in, out

The analytical method given above provides the results shown in Figure 13.

Our approximate results* are compared with the results of configuration 1. It can be observed that the difference between the results of configuration 1 and configuration 2 increases if the load of the network increases. This is explained by the increased occurrence of blocking events for networks under a heavy load.

^{*}Approximate results are compared with the confidence intervals of simulations. Simulations are done on an IBM 4381 using the RESQ simulation package¹³ with confidence intervals set to 95%. The confidence intervals in the Figures are denoted by 'l'.









Figure 13. Throughput for configuration 2

(c) Configuration 3

The buffer capacity constraints for the channel stations are now assumed:

$$B_{CH_1} = 2$$
 for $i = 1, 2$

The results are plotted in Figure 14.

The results for configuration 3 are almost identical to results for configuration 1. Thus, the decrease of



Figure 14. Throughput for configuration 3

performance due to blocking events occurring at the channel station is negligible. For the chosen set of parameters the finite buffer capacity of the channel station does not influence the global performance.

(d) Configuration 4

We choose the same buffer capacity constraints we had for the previous configurations:

$$B_{CH_i} = 2$$

$$B_{CH_i} = 2$$
for $i = 1, 2$ and $i = in, out$

The results are plotted in Figure 15.

Note that due to the deadlock freedom, property performance measures can only be computed up to a maximum load of K = 11. It is obvious that the given configuration is not sensitive to buffer constraints of the channel stations in LANs. However, if finite buffers of the gateway stations are considered as in configurations 2 and 4 we observe that the overall performance is significantly affected. The performance difference for configurations 2 and 4 and configurations 1 and 3 networks is illustrated in Figure 16.



Figure 15. Throughput for configuration 4



Figure 16. Summary for example 1

Table 3.	Buffer sizes	and	service	times	(m =	1,	2,	3)	
----------	--------------	-----	---------	-------	------	----	----	----	--

B _{CHm}	10
B _{GWm inf} B _{GWm out}	2
$1/\mu_{CH_m}$	1/3 ms
$1/\mu_{GW_{min}}, 1/\mu_{GW_{mont}}$	2
$1/\mu_{H_{m_r}}$ (r = 1,2,,10)	3 ms

Example 2

Basic model

Three LANs are connected to each other via gateways (see Figure 5). The number of hosts in each LAN is set to ten. Including the channel and the gateway stations, the entire queueing network has N = 39 stations. Our goal for this example is, starting from a set of parameters, to show how the performance of the network changes if certain parameters are changed. We vary the parameters of the stations of one particular LAN (LAN₁). The parameters for the network are given in Table 3.

Transition probabilities are given in Table 4. The network described above is analysed according to the algorithm given for configuration 4 networks. Throughput results are given in Figure 17.

Modification 1

Now assume that LAN_1 is improved in such a way that the maximum transmission rate is increased:

 $\mu_{CH_1} = 40 \, m s^{-1}$



Figure 17. Throughput of the basic model

Table 4. Transition probabilities



Figure 18. Throughput of modification 1 and the basic model

All other parameters remain unchanged. It can be observed in Figure 18 that the total throughput of the network also remains unchanged.

In the following we investigate which parameters of LAN_1 have to be changed to achieve a better performance.

Modification 2

In addition to the faster channel from modification 1 we increase the service rate of the gateway belonging to LAN_1 :

- Modification 2a: $\mu_{GW_{1,in}} = \mu_{GW_{1,out}} = 2 ms^{-1}$ Modification 2b: $\mu_{GW_{1,in}} = \mu_{GW_{1,out}} = 20 ms^{-1}$

Clearly, the throughput values can be improved as demonstrated in Figure 19.



Figure 19. Throughput of modification 2a/b and the basic model

P _{ij}	CH _m	$H_{m,r}$ (r = 1,2,,10)	^{Gw} j,in (j ≠ m)	GW _{m,out}
$H_{m,r}$ (r = 1,2,,10)	1	0	0	0
CH _m	0	0.07	0	0.3
GW _{m,in}	1	0	0	0
GW _{m,out}	0	0	0.5	0





Figure 20. Throughput of modification 2a/b and modification 3

Modification 3

Another way to improve the performance starting from modification 1 is to increase the buffer space of the gateway connected to LAN_1 . We assume:

$$B_{GW_{1,i}} = 5$$
 for $j = in$, out

Other parameters are as given in modification 1. Figure 20 plots the throughput values of modification 3 and compares them with those from modification 2.

It can be seen that an increase of the gateway buffer size, as in modification 3, does not improve the performance, as the reduced service time of the gateway (modification 2). This can be verified by a combination of modifications 2 and 3, as given in modification 4.

Modification 4

We assume the same parameters as in modification 1 except:

$$\mu_{GW_{1,in}} = \mu_{GW_{1,out}} = 20 \text{ ms}^{-1}$$

 $B_{GW_{1,i}} = 5$ for $j = in$, out

Figure 21 plots the throughput values for modification 2a as well as for modification 4.



Figure 21. Throughput of modification 2a and modification 4

Different curves cannot be distinguished. Thus, we conclude for Example 2: if the transmission rate of the channel of a LAN (assuming the parameters given above) is increased without changing other devices of the network, the internetwork throughput can be improved by increasing the service rate of the gateway connected to it. An increase of the gateway buffer size does not have a similar effect. However, in Figure 19 we see that the improvements achieved by speeding up the gateway are limited.

CONCLUSION

This study investigated the performance degradation of interconnected computer networks due to finite storage space of the involved stations. We suggested a classification of interconnection networks, developed queueing network models and proposed computational algorithms which give analytical solutions for each class of networks. Numerical examples demonstrated the performance differences of the different classes of networks. Due to the topology of interconnected LANs, the performance of internetwork traffic was shown to be sensitive to variations of the parameters of gateway stations. An improvement of the transmission rate in a LAN was ineffective for internetwork communication, unless the parameters of the gateway stations were improved at the same time.

ACKNOWLEDGEMENTS

This work was supported in part by the Air Force of the Scientific Research (AFOSR) under grant no. AFOSR-&&-0028.

APPENDIX: THROUGHPUT ANALYSIS OF LOAD DEPENDENT QUEUEING NETWORKS WITH FINITE BUFFER CAPACITIES

Model assumptions

We consider closed queueing networks with N stations and K total packets. The service time at station i is exponentially distributed with load dependent mean values $1/\mu_i(k)$ (for i = 1, ..., N and k = 1, ..., K). The scheduling discipline at each station is assumed to be FCFS. Each station has a fixed finite buffer capacity B_i where $B_i = (queue \ capacity + 1)$, (for i = 1, ..., N). Any station whose buffer capacity exceeds the total number of packets in the network can be considered to have infinite capacity $(B_i = \infty \text{ for some } i = 1, 2, ..., N)$. A packet which is serviced by the ith station proceeds to the j^{th} station with probability p_{ii} , (for i, j = 1, ..., N), if the j^{th} station is not full, i.e. if the number of packets in the ith station k_i , is less or equal to B_i . Otherwise, the packet is blocked in the *i*th station until a packet in the *j*th station has completed its servicing and a place becomes available.

Algorithm

The basic idea of the solution algorithm is to replace the finite capacity queueing network, from now on denoted as Φ , by an infinite capacity queueing network, denoted as Γ . In substituting Φ by Γ we have to take the blocking events into account which occur between the stations in Φ , i.e. a packet being served in a station cannot leave the server of this station because the destination station is full. To consider the blocking events we modify the service mechanism of a station such that all delays a packet might undergo due to blocking events in Φ can be represented. Therefore, corresponding delay phases caused by blocking events are appended to the service unit of each station. The frame of the algorithm consists of three steps:

- Construction of delay phases for each station.
- Computation of service times and branching probabilities for phases.
- Solution of the Network Γ .

Construction of delay phases for each station

Let *i* be a station of Φ . For each possible blocking delay caused by another station *j* in Φ (for *i*, *j* = 1, ..., N; *i* ≠ *j*) we add a service phase to station *i*. The connection between the added phases and the original server of station *i* is the same as the transitions between stations in Φ .

We have to consider that blocking delays may not only be caused by a station's immediate successors, but also by stations which occur in each cycle of the network where a particular station is represented. Let *i* be an arbitrary station in the network, $C_i(l)$ the l^{th} cycle of stations that starts and ends at station *i* is defined by:

$$C_i(I) = (i, j_{l_1}, j_{l_2}, \ldots, i)$$

where j_{l_a} is the q^{th} station in cycle *l* of station *i*. Now let us consider one of these cycles, $(i, j_{l_1}, j_{l_2}, \ldots, i)$. For instance, assume that there are k_i packets at station i and the number of packets in the network be such that station j_{l_1} through j_{l_s} can be full at the same time. In this case, a packet upon service completion at station i may find station j_l, full, blocking station i's server. Now the question is when this blocked packet will depart from station i. If upon service completion at station j_{l_1} a packet chooses to go to station j_{l_2} which is not full, then the packet at station j_{l_1} will depart, and at the same time another packet at station *i* will join station j_{l_1} unblocking server of the station *i*. However, if a packet at station j_{l_2} gets blocked because its destination is full, then the blocked packet at station i cannot depart. Hence, in the worst case a packet at station i will wait for service completions at stations j_{l_1},\ldots,j_{l_s} before leaving station *i*.

In constructing the phases the following rules must be obeyed¹⁴:

R1) If two or more cycles are identical up to a certain element, then the elements prior to that element are represented only once in the phase construction.

R2) If $\Omega_i(l) = (i, j_{l_1}, j_{l_2}, \dots, j_{l_q})$ is a path in cycle $C_i(l)$ starting from station *i* with:

$$\sum_{r=1}^{q-1} B_{j_k} < K \leq \sum_{r=1}^{q} B_{j_k}$$

then the stations $(j_{l_q'}, j_{l_{q+1}}, \ldots, i)$ of $C_i(l)$ are not considered in the phase construction for this cycle. In other words, if the sum of station buffer capacities in $\Omega_i(l)$ exceeds the total number of packets, then the last station of $\Omega_i(l)$ and all its successors in $C_i(l)$ are not taken into account in the phase construction for station *i* for that cycle.

Computation of service times and branching probabilities for phases

After the construction of phases we need to determine the parameters such as branching probabilities and service times of the phases.

Since a blocked packet cannot leave a station until space becomes available in the full destination station, the time a packet is blocked is equal to the mean remaining service time of the station which causes the blocking. Because of exponential service time distribution the mean remaining service time of a station is given by the mean service time¹⁵. Therefore, the blocking delay for a load dependent station *i* caused by station *j* is determined by $1/\mu_j(B_j)$, the mean service time of station *j* having a load of B_j packets.

 a_{ij} denotes the probability that a packet after a service or blocking delay in phase *i* enters the phase representing station *j*. The value of a_{ij} is computed by:

$$a_{ij} = \rho_{ij} \cdot P_i(k_j = B_i + 1)$$
 for $i, j = 1, ..., N$ (A1)

where p_{ij} is the transition probability of the original network, and $P_j(k_j = B_j + 1)$ are blocking probabilities computed by an iteration (discussed below).

For calculation of the probabilities $P_j(k_j = B_j + 1)$ we assume that each station behaves like an isolated station with exponential service time, Poisson arrivals and finite buffer, known as [M/M/1/N] station. In the considered blocking protocol a blocked packet has already been processed at the station at which it resides. Thus, the blocked packet belongs logically to the full destination station occupying the $(B_j + 1)$ th position in the destination station j which has a buffer capacity of B_j . Hence, we may approximate the blocking probability by the steady state probability $P_j(k_j = B_j + 1)$ of the finite capacity station having its buffer increased by one. The formula for the probability of a finite $[M/M/1/B_j + 1]$ station is computed by:

$$P_{j}(k_{j} = B_{j} + 1) = \begin{cases} \hat{\rho}_{j}^{B_{j} + 1} \cdot \frac{1 - \hat{\rho}_{j}}{1 - \hat{\rho}_{j}^{B_{j} + 2}} & \text{if } B_{j} < K \\ 0 & \text{if } B_{j} \ge K \end{cases}$$
(A2)

with:

$$\hat{\rho}_{j} = \frac{\hat{\lambda}_{j}}{\mu_{j}(k)}$$
 for $j = 1, ..., N; k = 1, ..., K$ (A3)

where λ_i is the arrival rate to station *j*. Since arrivals to station *j* are rejected once the buffer capacity of the station is exhausted ($k_j = B_j + 1$) we may express $\hat{\lambda}_j$ in terms of the effective arrival rate λ_j as follows:

$$\hat{\lambda}_j = \frac{\lambda_j}{1 - P_j(k_j = B_j + 1)} \quad \text{for } j = 1, \dots, N \quad (A4)$$

In other words, the effective arrival rate λ_j is the portion of all arrivals which are not rejected because of a full buffer. Equations (A2) and (A4) are used as fixpoint iteration to compute the values for $P_j(k_j = B_j + 1)$ for j = 1, 2, ..., N. With the $P_j(k_j = B_j + 1)$ values the probabilities a_{ij} of equation (A1) can be determined.

Solution of the network Γ

Now we are able to compute the throughput of Γ with the following iteration:

Initially we set all branching probabilities a_{ij} between service and delay phases of each station to zero, and hence eliminate all delay phases. The network Γ has then the same structure as the network Φ , except the stations' buffer capacities are now infinite. Since all stations have exponentially distributed service times with mean value $1/\mu_i(k)$ we obtain the initial throughput $\lambda^{(0)}$ by applying a product from algorithm such as mean value analysis⁹. The throughput λ_i of each station *j* is determined by:

$$\lambda_j = \lambda^{(n)} \cdot \mathbf{e}_j \quad \text{for } j = 1, \dots, N$$
 (A5)

Note that *n* is set to n = 0 in the initial iteration step. The value of e_j denotes the mean number of visits that a packet makes to node *j*, and is given by:

$$e_j = \sum_{i=1}^{N} e_i \cdot p_{ij}$$
 for $j = 1, ..., N$ (A6)

With the throughput values we compute the fixpoint iteration of equations (A2) and (A4) to obtain $P_j(k_j = B_j + 1)$ for j = 1, 2, ..., N. The values for $P_j(B_j(B_j + 1))$ are then used to determine the branching probabilities a_{ij} from equation (A1).

Now, for each multi-phase station j of network Γ we determine the total mean service times $\tilde{\mu}_j(k)$, the variance $\tilde{\sigma}_j^2(k)$ and the coefficient of variation $\tilde{c}_j(k)$. Since the time a packet spends in service phase and each blocking phase of a station are determined by independent random variables with exponential distribution functions, the total time a packet spends in the multi-phase server is itself given by an exponential random variable. The following equations show how to compute $\tilde{\mu}_i(k)$, $\tilde{\sigma}_i^2(k)$ and $\tilde{c}_i(k)$.

$$\frac{1}{\tilde{\mu}_j(k)} = \frac{1}{\mu_j(k)} + \sum_{\substack{\text{all blocking phases } I}} \frac{\text{prob}_l}{\mu_l(k)} \tag{A7}$$

$$\tilde{\sigma}_{j}^{2}(k) = \left(\frac{1}{\mu_{j}(k)}\right)^{2} + \sum_{\text{all blocking phases } l} \left(\frac{\text{prob}_{l}}{\mu_{l}(k)}\right)^{2}$$
(A8)

$$\tilde{c}_{j}(k) = \sqrt{\tilde{\sigma}_{j}^{2}(k) \cdot \tilde{\mu}_{j}(k)}$$

for $j = 1, \dots, N$ and $k = 1, \dots, K$ (A9)

with:

$$prob_{I} = \prod_{n=0}^{t} a_{s_{n}s_{n+1}}$$
(A10)

With $s_0 = j$, $s_t = l$ and $\langle j, s_1, s_2, \dots, s_{t-1}, l \rangle$ a path of phases in the multi-phase server of station *i*.

Having calculated the mean value and the coefficient of variation of the service times of all stations, we apply the algorithm for queueing networks with general service times and load dependent servers¹⁶ and obtain the throughput value $\lambda^{(n)}$. An iteration test is carried out:

$$|\lambda^{(n)} - \lambda^{(n-1)}| > \varepsilon \tag{A11}$$

for *n* as the number of iterations. If the difference between consecutive throughput values is greater than a threshold value (e.g. $\varepsilon = 10^{-4}$) we continue with the next iteration. Otherwise, the iteration terminates and the final throughput values are obtained.

REFERENCES

- Exley, G and Merakos, L 'Packet delay characteristics in interconnected random access networks' Proc. GLOBECOM '86 Houston, TX, USA (1986) pp 455-459
- 2 Exley, G and Merakos, L 'Throughput-delay performance of interconnected CSMA local area networks' *IEEE J. Selected Areas in Commun.* Vol 5 No 9 (December 1987) pp 1380–1390
- 3 Lazar, A A and Robertazzi, T G 'Optimal flow control of networks interconnected via gateways' Proc. 19th Ann. Conf. Inf. Sci. Syst. Baltimore, MD, USA (March 1985) pp 263-268
- 4 Ben-Michael, S and Rom, R 'Gatewaying two Aloha networks' Proc. INFOCOM '86 Miami, FL, USA (1986) pp 30-33
- 5 Varakulsiripunth, R, Shiratori, N and Noguchi, S 'Congestion control policy on internetwork gateway' Proc. ICCC '86 North-Holland, Netherlands (1986) pp 659–664
- 6 Heath, J R 'Analysis of gateway congestion in interconnected high-speed local networks' IEEE Trans. Comm. Vol 36 No 8 (August 1988) pp 986–989
- 7 Cheng, Y-C and Robertazzi, T G 'Annotated bibliography of local communication system interconnection' *IEEE J. Selected Areas in Comm.* Vol 5 No 9 (December 1987) pp 1492–1499
- 8 Baskett, F, Chandy, K M, Muntz, R R and Palacios, G 'Open, closed and mixed netework of queues with different classes of customers' J. ACM Vol 22 No 2 (April 1975) pp 248–260

- 9 Reiser, M and Lavenberg, S S 'Mean value analysis of closed multichain queueing networks' J. ACM Vol 27 No 2 (April 1980) pp 313-322
- 10 Chandy, K M, Herzog, U and Woo, L 'Parametric analysis of queueing network models' *IBM J. Res. & Dev.* Vol 19 No 1 (January 1975) pp 43–49
- 11 Akyildiz, I F 'Product form approximations for queueing networks with multiple servers and blocking' *IEEE Trans. Comput.* Vol 15 No 1 (January 1989) pp 99–114
- 12 Kundu, S and Akyildiz, I F 'Deadlock free buffer allocation in closed queueing networks' Queueing Syst. J. Vol 4 (January 1989) pp 47-56
- 13 Sauer, C H, MacNair, E A and Kurose, J F 'The

research queueing package version 2' IBM T J Watson Res. Center Yorktown Heights, New York, USA (19)

- 14 Akyildiz, I F and Liebeherr, J 'Application of Norton's theorem on queueing networks with finite capacities' *Proc. INFOCOM* '89 Ottawa, Canada (April 1989) pp 914–923
- 15 Akyildiz, I F 'Mean value analysis for blocking queueing networks' IEEE Trans. Softw. Eng. Vol 14 No 4 (April 1988) pp 418–429
- 16 Akyildiz, I F and Sieber, A 'Approximate analysis of load-dependent general queueing networks' IEEE Trans. Softw. Eng. Vol 14 No 11 (November 1988) pp 1537-1545