Performance Analysis of Computer Communication Networks with Local and Global Window Flow Control

I. F. Akyildiz

School of Information and Computer Science Georgia Institute of Technology Atlanta, Georgia 30332 U. S. A.

ABSTRACT

A message switched network with local and global window flow controls is analyzed. Messages arriving to find the global window full or the first destination's local window full, are assumed to be lost. Further, if a message from a source system is sent to a destination system of which the local window is full, the message blocks the source system until it can be received from the destination. In that time no other message can be processed by the source system. An open tandem queueing network model with blocking is developed where the service times are assumed to be general and the scheduling disciplines First Come First Served. An approximation algorithm for the effective computation of performance measures, such as the throughput of the system, mean number of messages in the global window and the mean delay time of the system, is given. The approximation has been validated by executing of several examples and comparing the analytical results with simulation.

1. Introduction

Flow control is a system of algorithms, which are used in a communication network to prevent a single user or a user group from monopolizing the network resources to the detritment of other users. One flow control, *Global Control*, places a limit on the total number of messages in transit in the network. The number of messages in the entire network is constrained to be no more than some positive integer, W, called the *global window size*. Another flow control, the *Local Control*, places limits on the number of messages at the buffers of individual switching nodes of the network. In this case, the number of messages in a node is constrained to be no more than some positive integer, B, called the *local window size*. A third flow control is the *End-to-End control*, where a limit on the number of messages is placed in transit in the network, belonging to each source-destination pair. In this case, the number of messages between any source-destination pair is constrained to be no more than some positive integer, T, called the end-to-end window size.

In recent years considerable work [8,11,12,13,15]. has been done on the performance analysis of flow control procedures. Pujolle [11] presented models of store-andforward tandem links with congestion control and gave ergodicity conditions.

Reiser [12] analyzed the virtual route using a tandem queueing model. He assumed *losses* in the system, i.e., arriving messages facing a full window are discarded. Varghese, Chou and Nilsson [18] assumed that such messages are not discarded; instead they wait for their turn to enter the network in a global window queue. Reiser [13] has analyzed a similar model (which he refers to as a *wait system* to distinguish it from the previous *loss systems* using a

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standard decomposition procedure [15]. Such a decomposition procedure requires the assumption of an exponentially distributed sojourn time for each message dispatched on the virtual route. Reiser [14] gave a survey about the studies of window flow control problem.

In this work we consider a packet switched network with local and global flow-control. We develop an open tandem queueing model of the network which considers losses of messages due to the global window and blocking of messages due to the local windows. The network is analyzed in a pair-node fashion, i.e., each two-node starting with the final node, is analyzed and replaced by a flowequivalent node which is then analyzed with its predecessor node and so forth.

2. Model Description

We consider a message-switching network having N switching nodes where each node is considered as composed of multiple servers $(m_i \ge 1)$ and a buffer pool shown in Figure 1.

due to the transition probabilities $p_{ij} = 1$ for i = 1, 2, ..., Nand $j = (i+1) \mod N$.

We consider two levels of flow control in the system. The first level is the *Global Flow Control*, (shown in the Figure 1 by dotted lines) whereby we assume a global window size W. The second level is *Local Flow Control*, whereby we assume that each node has a finite capacity B_i for i = 1,...,N. When the capacity $B_{(i+1)}$ of the node (i+1)is reached then the message seeking admission to it, is blocked in the predecessor node *i*. The blocked message will remain in the server of the *i*-th node, until node (i+1)is free to accept the blocked message. Loss of incoming messages occurs if the sum of the number of messages in all nodes becomes W, the global window size or the first node is full of B_1 messages.

Note that if $\sum_{i=1}^{N} B_i \leq W$, this model is a traditional

blocking network with finite node capacities, studied in several papers for which a bibliography is given by Perros [10]. We want to analyze the case where the condition



Figure 1.

Messages arrive to the network from a single source in a Poisson fashion. All messages are assumed to have the same type. Message lengths are assumed to have a general distribution with mean value $1/\mu_i$ and coefficient of variation $c_i^2 \ge 0.5$ having rational Laplace transform in the *i*th node (for i = 1,...,N). The queueing discipline at each node is FCFS. The messages proceed through the network

 $W < \sum_{i=1}^{N} B_{i}$, i. e., the global window size is smaller than the total capacity of the network.

The proposed approximation method is based on solving of several two node networks. In other words, the total network has been solved in a pair-wise fashion starting with the nodes (N - 1) and N and replacing them by a

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flow-equivalent server and so forth. Similar pair-wise analysis techniques have also been utilized by Suri/Diehl [17], Goto/Takahashi/Hasegawa [5] and Goto [6]. Suri/Diehl analyze open networks with the assumption that the source is treated as an additional node and the total number of messages must be less than the total number of capacity of the network, i.e., $K < \sum_{i=1}^{N} B_i$. That is, the traditional case of blocking networks. Their technique is different from our approach in the sense that they compute the marginal probabilities for each node-pair and weight them with the throughputs and obtain so-called a view capacity seen by the messages coming from the predecessor node. The capacity of the composite node is obtained as probabilities. Additionally they consider only Markovian networks.

Our work is motivated by Goto/Takahashi/Hasegawa [5] and Goto [6] study where they analyze Markovian networks with the blocking type where the message is blocked at the head of the queue if the destination node is full to its capacity.

In section 3 we introduce a new approximation technique for blocking networks consisting of two nodes and having general service time distributions, multiple servers and FCFS scheduling discipline. The proposed technique is a direct application of the extended version [1] of the Method of Marie [MARI80] to blocking networks. In section 4 we discuss the pair-wise analysis of the nodes. Section 5 contains the evaluation of the proposed technique. The simulation is executed developing a RESQ model. Several examples are given and the approximate results are compared with simulation.

3. Two Node Closed Networks with Blocking

We have assumed that the service times follow an arbitrary distribution with a rational Laplace transform. In the modeling of non-exponential service time distributions we have an estimation of the first and second moments of the service distribution, i.e., the expected value and the variance. Marie [MARI80] has shown that for any mean μ and any squared coefficient of variation $e^2 \geq 0.5$, such that it is possible to represent a node's server by a Coxmodel with 2 phases.

Each time a message completes a phase it may either depart from the node or it may proceed to the next phase. The total time a message spends in a phase is exponentially distributed. In general, each phase has a different mean service rate. Let $\mu_{ij}(k)$ denote the load-dependent service rate at the j-th phase (j = 1,2) of the *i*-th node (i = 1,2). Also, let a_j be the probability that a message upon completion of its service at the *j*-th phase will proceed to the (j + 1)-th phase and b_j denotes the probability that a message upon completion of its service at phase *j* will depart from the node.

The parameters $\mu_{i1}(k)$, $\mu_{i2}(k)$, a_i and b_i are determined as follows [9]:

$$\mu_{i1}(k) = 2 \mu_i(k)$$
 (1a)

$$\mu_{i2}(k) = \frac{\mu_i(k)}{c_i^2}$$
(1b)

$$a_i = \frac{1}{2 c_i^2}$$
 (1c)

$$b_i = 1 - a_i \tag{1d}$$

where the $\mu_i(k)$ is the load-dependent service rate which is computed by:

$$\mu_i(k) = \begin{cases} k \cdot \mu_i & \text{if } k \leq m_i \\ m_i \cdot \mu_i & \text{if } k \geq m_i \end{cases}$$

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A state of a node is denoted by a pair:

$$\underline{k} = (k, j)$$

where

k is the number of messages, k = 0, 1, 2, ..., K

j is the number of phases, j = 1, 2.

A transition from one state to another takes place either when a new message arrives or when a message leaves the node through either phase. We will denote the arrival rates by $\lambda_i(k)$, where the arrival rates are dependent on the number of messages in the node. Since we are dealing with two-node networks, it is clear that the arrival rate to the *i*-th node is:

$$\lambda_i(k) := \mu_j(K - k) \tag{2}$$

for i, j = 1, 2; $i \neq j$; k = 0, 1, ..., (K-1). Note that $\lambda_i(K) = 0$, since no message is in the *j*-th node and consequently the throughput of the *j*-th node will be zero. A message leaving the node after phase one is denoted by $b_i \mu_{i1}$. μ_{i2} is the departure rate of a message leaving the node after phase two at a rate $a_i \mu_{i1}$ and may do so only after receiving service in phase one. The state (0) denotes that no message is in the *i*-th node.

Since the nodes have capacity limits, only a subset of states are feasible. The feasible state for the blocking network are obtained by considering the fact that the number k_i of messages in the *i*-th node may not exceed the capacity of the node B_i . On the other hand, at least $(K - B_j)$ messages can be accomodated by the *i*-th node due to the capacity restriction of the *j*-th node. Let us define g_i as the number of servers that can be blocked at the *i*-th node in a given state:

$$g_i = \begin{cases} m_i & \text{if } K > B_j + m_i \\ K - B_j & \text{if } K \le B_j + m_i \end{cases}$$
(3)
for $i = 1, 2 \quad i \ne j.$

Note that the first case of equation (3) states that all the servers of the *i*-th node can be blocked while the second case states that some servers of the *i*-th node can be empty. The g_i neighbors of a feasible state represent the blocking states in the network, i.e., whenever a transition occurs from one state to another state where the connective limit of a node would be violated, we assume that the transition causes a blocking of a server of that particular node. As many servers the node possesses as many neighbors of the feasible states are included in the state space of the blocking network. All other states violating the node capacities are non-feasible and are cancelled. The bounds for the number of messages in the *i*-th node are extended as follows:

$$(K - B_j - g_i) \leq k_i \leq (B_i + g_j) \qquad (4)$$

As in case of Markovian networks [AKYL87], also here we can find an equivalent network without limitation of node capacities with appropriate total number of messages K' which provides exactly the same state space structure as the blocking network. The equivalent non-blocking network has the same number of nodes; the same generally distributed service times; the same transition probabilities. The only difference is that the node capacities are unlimited, hence no blocking can occur and the total number of messages in both systems is not equal, $K \neq K'$, where K'is computed by [2]:

$$K' = \min \{K, B_1 + m_2\} + \min \{K, B_2 + m_1\} - K(5)$$

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The non-blocking network with K' messages is analyzed using the algorithm given in [1]. In the following we outline the application of the algorithm to two-node blocking networks.

1. Determine the parameters for the Cox distribution using equation (1).

2. Iterative Part

- i) Assign the service rates $\mu_j(k)$ with k messages to the arrival rates of the *i*-th node with (K' - k) messages, equation (2). Initially, $\mu_j(k) := \mu_j$ for all k = 1, ..., K'
- ii) Compute the conditional throughputs $\nu_i(k)$ using [1]:

If the test is not successful, then adjust the service rates:

 $\mu_i(k) := \nu_i(k)$ for k = 1, ..., K'. (11) Informally equation (11) states that the conditional throughputs, equations (7 and 8) are assumed to be the new service rates and the next iteration is carried out with these new service rates. Iterations continue until acceptable tolerances are obtained. The performance measures for the two node networks with blocking are then computed by the following formulas.

The marginal probability that there are k messages in the *i*-th node of the blocking network, is obtained from the marginal probabilities of the equivalent non-blocking net-

$$v_{i}(k) = \frac{\left[\mu_{i2}(k-1) - b_{i} \ \mu_{i1}(k-1)\right] \left[\mu_{i1}(k) \ \mu_{i2}(k) + b_{i} \ \mu_{i1}(k) \ \lambda_{i}(k)\right]}{\left\{\left[\lambda_{i}(k) + \mu_{i2}(k) + a_{i} \ \mu_{i1}(k)\right] \left[\mu_{i2}(k-1) - b_{i} \ \mu_{i1}(k-1)\right]\right\} - \left\{\left[\nu_{i}(k-1) - b_{i} \ \mu_{i1}(k-1)\right] \left[\mu_{i2}(k) - b_{i} \ \mu_{i1}(k)\right]\right\}}$$

work:

with

$$\nu_i(1) = \frac{b_i \lambda_i(1) \mu_{i1}(1) + \mu_{i1}(1) \mu_{i2}(1)}{\lambda_i(1) + \mu_{i2}(1) + a_i \mu_{i1}(1)}$$
(7)

iii) Compute the equilibrium state probabilities for nodes

i = 1, 2, using

$$p'_i(k) = p_i(0) \prod_{n=0}^{k'-1} \frac{\lambda_i(n)}{\nu_i(n+1)}$$
 (8)

for k = 1, \cdots , K'. where

$$p'_{i}(0) = \left[1 + \sum_{k=1}^{K'} \prod_{n=0}^{k-1} \frac{\lambda_{i}(n)}{\nu_{i}(n+1)}\right]^{-1}$$
(9)

iv) Check the termination condition to see if the sum of the mean number of messages is equal to the total number of messages in the given network within a tolerance level ϵ :

$$\frac{\left|K' - \sum_{i=1}^{N} \sum_{k=1}^{K'} k \cdot p'_i(k)\right|}{K'} < \epsilon \qquad (10)$$

where ϵ is a tolerance level. Usual value is $\epsilon = 10^{-4}$.

 $p_i(k + s) = p'_i(s)$ (12) for s = 0, 1, 2, ..., K', where k is defined in the range given by equation (4).

4. Analysis of the Tandem Network

As mentioned before the network is analyzed in a pair-wise fashion starting with the nodes N and (N-1) first and replacing them by a flow-equivalent (composite) node. This is explained in step 1 below. By repeating step 2, all the remaining nodes are reduced to one. Finally, total throughput and average delay time are determined in step 3.

STEP 1. The last two nodes in the network of Figure 2 are replaced with a composite (flow-equivalent) node for which the parameters such as the service rates $\mu_{e}(k)$, the

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coefficient of variation value $c_c(k)$ and the capacity B_c^* of the composite (flow-equivalent) node.









In order to obtain these values, the open network shown in Figure 2 is transformed into a closed network as shown in Figure 3.

Note that if K is smaller than W, then $\mu_{c}^{*}(k)$ is set

to
$$\mu_{\epsilon}(K)$$
 for $k = K+1$, ..., W .



We assume that the closed network has

 $K = \min \{W, B_{N-1} + B_N - 1\}$ (13) total number of messages. All input parameters for the closed network given in Figure 3 are known, so the algorithm given in section 3 is called and the marginal probabilities $p_i(k)$, equation (12), are obtained. Note that the closed network shown in Figure 3 is analyzed with the number of messages starting from from 1 to K. Therefore, the algorithm given in section 3 is called K number of times, and in each case, the corresponding marginal probabilities, $p_N(k)$, equation (12) are returned. The loaddependent service rates, $\mu_c(k)$ for k = 1,...,K, are computed then from:

$$\mu_{c}(k) = \sum_{n=max\{jmin,1\}}^{jmax} \mu_{N} \cdot p_{N}(n) \qquad (14)$$

The capacity of the composite (flow-equivalent) node

is determined by:

$$B_c^* = \min \{W, B_{N-1} + B_N\}$$
 (16)

The coefficient of variation value of the flowequivalent node is computed by [SEVC77]:

$$c_{c}^{2}(K) = 1 + \rho_{N}^{2}(K) [c_{N}^{2} - 1] + (17)$$
$$[1 - \rho_{N}(K)]^{2} [c_{N-1}^{2}(K) - 1]$$

where $\rho_N(K)$ denotes the utilization of the N-th node and is computed by:

$$_{N}(K) = \frac{\lambda_{N}(K)}{\mu_{N}} = \frac{\mu_{c}(K)}{\mu_{N}}$$
(18)

where $\mu_c(K)$ are obtained from equation (14).

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STEP 2.

Further nodes i = C, $(N-2), (N-3), \dots, 2, 1$ are analyzed as follows: Note that C denotes the composite

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(flow-equivalent) node representing the nodes (N-1) and N analyzed above.



Figure 4.

The open network of Figure 4 is transformed into a closed network, Figure 5, with the total number of messages K obtained by:

 $K = \min \{ W, B_{N-j} + B_{N-j+1} - 1 \}$ (19)



Figure 5.

The coefficient of variation values of the composite (flow-equivalent) node is computed by:

$$c_{c_{N-j+1}}^{2}(K) = 1 + \rho_{N-j+1}^{2}(K) [c_{N-j+1}^{2} - 1] + (20)$$

[1 - \rho_{N-j+1}(K)]^{2} [c_{c_{N-j}}^{2}(K) - 1]

where $\rho_{N-j+1}(K)$ denotes the utilization of the N-j+1-th node and is determined from equation (18). Note that the node (N-j) has now load-dependent service rates $\hat{\mu}_{N-j}(k)$ which are computed by:

$$\hat{\mu}_{N-j}(k) = \begin{cases} \mu_{N-j} & \text{for } 1 \le k \le B_{N-j+1} + m_{N-j+1} \\ TRU_{N-j}) & \text{for } (B_{N-j+1} + m_{N-j+1}) \le k \le w \end{cases}$$
(21)

 TRU_{N-j} is the throughput of the following network given in Figure 6. The reason why we have to adjust the



service rate of node (N-j) is that in the originally given network, node (N-j) can be blocked because node (N-j+1) has the finite capacity B_{N-j+1} .

The closed network in Figure 6 is analyzed by the algorithm given in section 3.

Once all the parameters of the closed network given in Figure 5 are known, the algorithm in section 3 is called again and the desired throughput values are computed which are set equal to the according composite (flowequivalent) node's service rate.

STEP 3.

Note that the source is treated as an additional node with the service rate $\mu_o = \lambda$ and infinite capacity $B_o = \infty$. This merging process explained in step 2 is repeated until node 0 and node 1 are finally analyzed. The total throughput of the network is computed as follows:

$$\lambda_{NETWORK} = \begin{cases} [1 - p_1(B_{FIN}^*)] [1 - p_0(B_1)] \cdot \lambda & \text{for } B_1 \leq B_{FIN}^* \\ [1 - p_1(B_{FIN}^*)] \cdot \lambda & \text{for } B_1 > B_{FIN}^* \end{cases}$$

$$(22)$$

where

$$B_{FIN}^{\bullet} = \min\{W, \sum_{i=1}^{N} B_i - 1\} \qquad (23)$$

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The mean number of messages in the Global Window is computed by:

$$\overline{k} = \sum_{n=1}^{B_{FIN}^*} n \ p_i(n)$$
 (24)

The mean delay time of messages is obtained by (Little's Law):

$$\bar{t}_D = \frac{\bar{k}}{\lambda_{NETWORK}}$$
(25)

5. Evaluation

Our approximation technique is extensively evaluated in comparison with simulation results obtained using RESQ simulation package. In order to compare the estimations of the approximations with simulation results, there are a large number of the parameter combinations. We classify them from two points of view. One is whether the blocking in each node occurs or not and the loss of arrivals is only caused when the first node is full or also caused when the network is full of W messages. Another is the combination of service rates and arrival rates. In the examples 2-node, 3-node networks are chosen in order to demonstrate the performance measures such as the network throughput, mean number of messages in the global window and the mean delay time of the messages. The solid lines in the Figures show the approximate results where the dashed lines denote the simulation results.

In these examples, comparing with simulation results, in most cases, the performance measures are well estimated as it can easily be seen in the charts. In 4 and more node cases which are not given here the approximations are also close to the simulation. It can be stated that this approximation method provides accurate results for performance measures.

Example 1.

N = 2 nodes; Global Window Size W = 10; $B_1 = 4$, $B_2 = 10$;

$$\mu_1 = 0.5, \ \mu_2 = 2; \ m_1 = 3, \ m_2 = 5; \ c_1^2 = 7, \ c_2^2 = 0.707$$



Plot of mean number of messages as a function of arrival rates

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Example 2.

N = 3 nodes; Global Window Size W = 10; $B_1 = 5$, $B_2 = 5$, $B_3 = 5$; $\mu_1 = 2$, $\mu_2 = 7$; $\mu_3 = 5$; $m_1 = 1$; $m_2 = 4$, $m_3 = 2$; $c_1^2 = 4$, $c_2^2 = 0.707$, $c_3^2 = 1$

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Plot of mean number of messages as a function of arrival rates

Example 3.

N = 3 nodes; Global Window Size W = 10; $B_1 = 3$, $B_2 = 7$, $B_3 = 6$; $\mu_1 = 3$, $\mu_2 = 4$; $\mu_3 = 1.5$; $m_1 = 3$; $m_2 = 2$, $m_3 = 5$; $c_1^2 = 2.134$, $c_2^2 = 1$, $c_3^2 = 0.707$



Plot of mean_number_of_messages as a function of arrival rates

6. Conclusion

In this work, we suggest an efficient approximation method for $\lambda_{\{k\}} / G / m_i$ - FCFS type tandem queueing networks in which the number of messages in each node is restricted to a finite level and also the total number of messages in all nodes is limited. In this method the entire network is replaced with a composite (flow-equivalent) node with its rate depending on the number of messages in the network. Compared with simulation results, the estimations from this method (throughput, mean delay time, mean number of messages) show accurate results and it takes short computation time to obtain them. Using this technique we examined the characteristics of those queueing networks where we found that almost same throughput and shorter delay are obtained by limiting the total number of messages comparing with totally unlimited case and that in some 2-node cases exchanging the order of nodes has no effect on throughput, delay and the mean number of messages in the network.

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