Coverage and Achievable Rate Analysis for Indoor Terahertz Wireless Networks

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***Abstract*—With the emergence of numerous novel data-intensive applications, the demand on fast wireless access is experiencing an unprecedented growth. Following this trend, the "Terabit era" is expected to become a reality in the near future. Terahertz technology is promising as an enabler due to its feature of an extremely high bandwidth. However, due to the high carrier frequency along with a high molecular absorption, the transmission distance is limited to a few meters only. In this work, the coverage problem and the achievable data rate performance of indoor THz wireless networks (THz-WNs) are investigated. In order to overcome the severe propagation loss and to improve the transmission range, a single frequency network (SFN) is advocated. The minimum individual user rate for different resource allocation schemes is analyzed, taking into account the effects of inter-symbol interference due to channel dispersion in the THz band and as a consequence of the SFN transmit protocol. Results demonstrate that the proposed SFN scheme is able to provide a high minimum achievable user data rate. The coverage probability increases from 25% when only a single access point (AP) is employed up to 95% when 20 APs are considered, for an output power of 1 W.**

I. Introduction

Over the last few years, the global demand for mobile data services has experienced an explosive growth because of the rapid proliferation of smart mobile devices which need to support high data rates and low latency applications and services. Moreover, recently, both augmented and virtual reality applications have experienced a lot of media attention and are anticipated to evolve rapidly.

In an era in which consumers progressively demand high quality bandwidth-intensive applications, Terahertz (THz) networks are becoming a reality, since the THz band (0.1 - 10 THz) offers a huge bandwidth which is essential for increasing the capacity of the current wireless systems. However, because of the high transmission frequency and the atmospheric attenuation due to molecular absorption caused by water vapor and oxygen molecules in the atmosphere, the THz signals experience a severe path loss which limits the communication distance to a few meters. Therefore, THz wireless systems will be most likely limited to short-link indoor applications [1], [2]. Moreover, the indoor performance is also affected by a very high reflection loss depending on the shape, material and roughness of the reflecting surface [3].

In order to tackle the problem of the severe propagation loss and improve the transmission range, various solutions have been already proposed. Some of them are based on the implementation of THz systems with multiple antenna subarrays using an adaptive hybrid beamforming scheme [4], [5], whereas others employ distance-aware bandwidth-adaptive modulation schemes which take advantage of the distance-and-frequency-dependent channel peculiarities [6]. However, none of these approaches takes into account the effects of inter-symbol interference due to channel dispersion in the THz band.

In this work, we investigate the coverage and the achievable data rate performance for indoor THz wireless networks (THz-WNs). We propose to adopt a single frequency network (SFN), where a certain number of access points (APs) simultaneously send the same signal over the channel in the same frequency band. It is well known that SFNs can be an attractive choice for extending the coverage area of a network with the simple insertion of additional low-power transmitters [7]. This is beneficial for THz communications, since the potential use of current THz systems is limited by the low output power of THz sources [8]. To the best of our knowledge, there is no previous work on a similar approach in the THz band.

Moreover, we address the performance of the multiuser THz communication network in terms of the signal-to-noise ratio (SNR) per transmitted data symbol at the output of an equalizer. We suggest guidelines for the AP placement and AP density in order to improve the coverage probability for a target minimum achievable data rate. Besides, we evaluate the effects of the AP density, user density and total power budget on the average minimum theoretically achievable data rate of all users. We propose a frequency division scheduling (FDS) resource allocation scheme, in which each sub-channel is used by exactly one user only in order to maximize the minimum individual data rate. Furthermore, we compare the proposed scheme with an optimal time division scheduling (TDS) resource allocation scheme in which each frequency sub-channel is time shared by several users.

This paper is organized as follows. In Section II the system model and some preliminaries regarding the considered THz-WN are introduced. In Section III, we formulate a max-min optimization problem for the FDS and TDS resource allocation policies. The proposed resource allocation algorithms as well as the corresponding rate outage probabilities are discussed in Section IV. Conclusions are drawn in Section V.

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II. System Model and Preliminaries

The physical mechanisms governing a wireless transmission in the THz band are different from those which affect schemes operating in the lower frequency bands where the propagation is mainly influenced by the spreading loss. The strong absorption properties of atmospheric gases related to molecular resonances primarily of water vapor and oxygen [9] result in a very high and frequency-selective path loss for line-of-sight (LOS) links. For non-LOS (NLOS) propagation, in addition to the previously mentioned peculiarities, a very high reflection loss depending on the shape, material and roughness of the reflecting surface governs the THz wave propagation [3].

A. Channel and Noise Modeling

The overall channel transfer function (CTF) of an indoor THz communication link was derived in [10]. A certain amount of the incident power at a reflection point is scattered diffusely [11]. However, no relevant impact of the diffusely scattered paths on the channel characteristics has been observed in channel measurements [12]. Therefore, in our current analysis, we focus on the LOS and the four specular[1]-reflected rays corresponding to the walls of a simplified indoor environment. For the sake of completeness, we provide the CTF for indoor THz communications according to the stated assumptions [10],

$$H(f,r,\boldsymbol{\zeta}) = H^{\mathrm{LOS}}(f,r)\,\mathrm{e}^{-j2\pi f\tau_{\mathrm{LOS}}} + \sum_{i=1}^{4} H_i^{\mathrm{NLOS}}(f,\boldsymbol{\zeta}_i)\,\mathrm{e}^{-j2\pi f\tau_{\mathrm{NLOS}_i}}, \tag{1}$$

where $H^{\mathrm{LOS}}(f,r) = H_{\mathrm{spread}}(f,r)\cdot H_{\mathrm{abs}}(f,r)$ is the magnitude frequency response of the LOS path and accounts for the molecular absorption and the spreading loss, $\tau_{\mathrm{LOS}} = r/c$ and $\tau_{\mathrm{NLOS}_i} = \frac{r_{i1}+r_{i2}}{c}$ are the propagation delays of the LOS path and i^{th} NLOS path, respectively. Here, r denotes the distance between the transmitter (TX) and receiver (RX), r_{i1} is the distance between the TX and the i^{th} scattering point and r_{i2} is the distance between the i^{th} scattering point and the RX, and c is the speed of light. The vector $\boldsymbol{\zeta} = [\boldsymbol{\zeta}_1,\ldots,\boldsymbol{\zeta}_4]$ is related to the geometry of scattering, where $\boldsymbol{\zeta}_i = [r_{i1}, r_{i2}, \theta_{i1}, \theta_{i2}]$ corresponds to the i^{th} scattering point.

The i^{th} NLOS path contribution including the effects of the molecular absorption, the spreading and the reflection loss is defined as [10]

$$H_i^{\mathrm{NLOS}}(f,\boldsymbol{\zeta}_i) = H_{\mathrm{refl},i}(f,r_{i2},\theta_{i1},\theta_{i2}) \times H_{\mathrm{spread},i}(f,r_{i1},r_{i2})\cdot H_{\mathrm{abs},i}(f,r_{i1},r_{i2}). \tag{2}$$

Detailed information regarding the channel model and the definitions of the transfer functions in (2) can be found in [10].

The total noise power spectral density in the THz band is obtained as $\Phi_{\mathrm{N}}(f,r) = \Phi_{\mathrm{mol}}(f,r) + \Phi_{\mathrm{thermal}}(f)$ [13] where

$$\Phi_{\mathrm{mol}}(f,r) = k_B T_0 (1 - e^{-\alpha(f)r}) \tag{3}$$

[1]"Specular" means that the incident angle θ_1, i.e., the angle between the incident ray and the surface normal equals the reflected angle θ_2, i.e., the angle between the reflected ray and the normal of the surface.

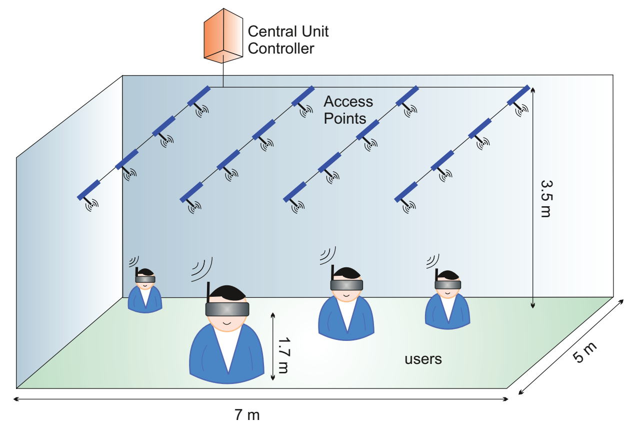

Figure 1: Considered scenario.

is the noise power spectral density (PSD) of the molecular absorption noise (k_B: Boltzmann's constant, T_0: 290 K, $\alpha(f)$: molecular absorption coefficient [9]), and $\Phi_{\mathrm{thermal}}(f)$ is the thermal noise PSD at high frequencies [14].

B. Considered Scenario and System Assumptions

In the considered THz communication network, M APs deliver a high bit-rate data stream to K users in the downlink. The APs and the users are equipped with a single antenna each as depicted in Fig. 1. Moreover, we consider an SFN, where the APs simultaneously send the same signal over the same frequency channel. The immediate benefit of such networks is that the transmitters can coordinate their signals to provide stronger and more robust coverage. This is beneficial especially for THz systems which will most likely be limited to short-range transmission. For transmitting the M signals to a user we assume optimal beamforming, i.e., maximum ratio transmission (MRT), which requires the feedback of both magnitude and phase information for its beamforming filter [15].

In an SFN, the signal arriving at a user from different APs experiences multi-path propagation. In order to overcome inter-symbol interference (ISI), caused by both the peculiarities of the THz band and the SFN transmit protocol, we employ an equalization scheme for the signal detection. It is well known that a decision-feedback equalization (DFE) scheme based on an unbiased minimum mean-squared error (MMSE) criterion is a canonical receiver structure for ISI channels, i.e., ideal MMSE-DFE (with perfect decision feedback) is a lossless equalization scheme if used in conjunction with coding [16].

Moreover, in order to get rid of the distance dependence of the PSD of the molecular absorption noise, we assume the maximum value of the PSD, $\Phi_{\mathrm{mol}}(f) = k_B T_0$, which will give a lower bound on the achievable performance.

Our considered scenario is based on a large rectangular room (length $\times$ width $\times$ height: $7\ \mathrm{m}\times 5\ \mathrm{m}\times 3.5\ \mathrm{m}$) and corresponds to a virtual reality application, e.g., a museum scenario, where each visitor (user) equipped with a head mounted display streams a high-definition 3D video corresponding to different artworks (cf. Fig. 1). Thus, each user should be served at least with a certain target minimum data rate R_{min} in order to ensure data integrity during high-rate data transfer. Due to the high signal strength loss in the THz band and when transmitting from only one AP, it might happen that the users experience a poor data rate below R_{min} when moving only a few steps within the room. Installing additional APs will improve the coverage, decreasing the outage probability. The APs are placed at the ceiling of the room equidistantly along the length and width and no APs are placed at the perimeter of the room as depicted in Fig. 1. A uniform placement of small power APs is beneficial for an SFN, since such placement creates a quite uniform signal strength, which is important when users move within the room [7]. Also, by adopting an SFN, we avoid frequency planning and thus the processing of handoffs.

Additionally, we assume that there is a central unit which takes care of the synchronization between all connected APs and collects the channel state information (CSI) of all wireless links for computation of the resource allocation. Typically, the CSI for each downlink channel can be acquired by estimating it at the corresponding user terminal and sending it to the AP via a feedback channel.

III. Resource allocation problem formulation

In the following, we formulate the optimization problem for maximization of the minimum achievable user data rate.

We consider the downlink of the multiuser THz communication network. The total channel bandwidth B from 0.1 THz to 1 THz is divided into K orthogonal sub-bands, each of equal width $B_n = \frac{B}{K}$, which are shared among the K users. Note that in our analysis the number of sub-bands equals the number of users K. We refer to n and k as the index of the sub-band and the index of the user, respectively. Moreover, if a whitened matched filter, matched to the cascade representing the equivalent overall channel (square-root raised cosine (SRRC) transmit filter, channel including MRT, continuous-time noise whitening filter) is applied at the receiver side for each sub-band link, for a fixed symbol interval T_n, we may sample without any information loss the continuous-time signal at the rate $1/T_n$ to obtain a discrete-time model with white noise for each sub-band. In our system, $T_n = \frac{1+\alpha}{B_n}$, where α is the roll-off factor of the SRRC filter. Since the total discrete-time transmission channel is frequency-selective in each sub-band n, we employ an MMSE-DFE scheme at each user terminal. The unbiased SNR at the output of the MMSE-DFE receiver of user k on sub-channel n is expressed as

$$\mathrm{SNR}_{\text{MMSE-DFE-unbiased},k,n} = \exp\left(T_n \int_{-1/2T_n}^{1/2T_n} \ln\left(\frac{P_{k,n}|H_{k,n}(\mathrm{e}^{j2\pi f T_n})|^2}{\Phi_{\text{noise},k,n}(\mathrm{e}^{j2\pi f T_n})} + 1\right) \mathrm{d}f \right) - 1 \tag{4}$$

where T_n is the symbol interval of the sub-band, and $H_{k,n}(\mathrm{e}^{j2\pi f T_n})$ and $\Phi_{\text{noise},k,n}(\mathrm{e}^{j2\pi f T_n})$ are the discrete-time overall CTF and the discrete-time noise PSD, respectively. For simplicity of notation, we denote $H_{k,n}(\mathrm{e}^{j2\pi f T_n}) = H_{k,n}(f)$ and $\Phi_{\text{noise},k,n}(\mathrm{e}^{j2\pi f T_n}) = \Phi_{\text{noise},k,n}(f)$.[2] The central unit controller allocates the sub-bands to the users and determines the amount of power to be transmitted on each sub-band by performing an optimal power allocation.

A. Frequency Division Scheduling Resource Allocation Scheme

Let $R_{k,n}$ denote the achievable data rate of user k on sub-band n,

$$\begin{aligned} R_{k,n} &= B_n \cdot \log_2(1 + \mathrm{SNR}_{\text{MMSE-DFE-unbiased},k,n}) \\ &= \frac{B_n}{\ln 2} \cdot T_n \int_{-1/2T_n}^{1/2T_n} \ln\left(\frac{P_{k,n}|H_{k,n}(f)|^2}{\Phi_{\text{noise},k,n}(f)} + 1\right) \mathrm{d}f, \end{aligned} \tag{5}$$

where $P_{k,n}$ is the allocated power of user k on sub-channel n. For what follows, we define $\gamma_{k,n}(f) = |H_{k,n}(f)|^2/\Phi_{\text{noise},k,n}(f)$ and $b = B_n/\ln 2$.

The problem we consider here is to optimize the allocation of sub-bands and powers under a total transmit power budget, P_{max}, so as to maximize the minimum achievable date rate among all users in the system. The optimum resource allocation for this max-min fairness optimization problem can be obtained by solving problem $\mathsf{P1}$,

$$\begin{aligned} \mathsf{P1} : \underset{d_{k,n}, P_{k,n}}{\text{maximize}} \ \min_{k} \quad & \sum_{n=1}^{K} d_{k,n} \cdot b \cdot \\ & \cdot T_n \int_{-1/2T_n}^{1/2T_n} \ln\left(P_{k,n}\gamma_{k,n}(f) + 1\right) \mathrm{d}f \\ \text{subject to} \quad & \sum_{k=1}^{K}\sum_{n=1}^{K} P_{k,n} = P_{\text{max}}, \ P_{k,n} \geq 0, \\ & \sum_{k=1}^{K} d_{k,n} = 1, \forall\, n, \ \sum_{n=1}^{K} d_{k,n} = 1, \forall\, k \\ & d_{k,n} \in \{0, 1\}, \forall\, k, n, \end{aligned}$$

where, the last three constraints impose that each sub-channel n is used by only a single user k and each user gets a sub-band.

Selecting the optimization variables $d_{k,n}$ and $P_{k,n}$ for all k and n in an optimal way is a mixed integer programming problem. In a system with K users and K sub-bands, there are $K!$ possible sub-band assignments since each sub-channel can be used by one user only. For each sub-channel assignment, the total available power should be distributed among users to maximize the minimum achievable user data rate.

Since this problem is not easily tractable we propose the following sub-optimal solution, by breaking up the problem $\mathsf{P1}$ into two sub-problems: $\mathsf{P1}_{\text{A}}$ which addresses the optimal sub-channel allocation for a uniform power allocation over the sub-bands and $\mathsf{P1}_{\text{B}}$ which considers the optimal power allocation among the user/sub-band combinations selected in Problem $\mathsf{P1}_{\text{A}}$. It should be noted that alternating optimization between $\mathsf{P1}_{\text{A}}$ and $\mathsf{P1}_{\text{B}}$ was also studied, however the additional gains were limited. In what follows we describe the proposed solution

[2] From now on, we will refer to $H(f)$ as a discrete-time CTF, whereas in Eq. (1) $H(f)$ stands for a continuous-time CTF.

Algorithm 1 Optimal sub-channel selection for uniform power allocation.

1: Input: $\mathbf{R} = [R_{k,n}]$, $\mathbf{D} = [d_{k,n}]$, $\mathbf{D} = \mathbf{J}_K$
2: Output: $\widetilde{\mathbf{D}}^*$ (permutation matrix)
3: **while** $\sum_{k=1}^{K} \sum_{n=1}^{K} d_{k,n} \neq K$ **do**
4: $\quad (k^*, n^*) \leftarrow \arg\min_{k,n} R_{k,n}|_{>0}$ (find nonzero minimum rate)
5: $\quad d_{k^*,n^*} \leftarrow 0$, $R_{k^*,n^*} \leftarrow 0$
6: $\quad$ **if** $\text{per}(\mathbf{D}) = 0$ **then**
7: $\qquad R_{k^*,n^*} \leftarrow \infty$
8: $\qquad d_{k^*,n^*} \leftarrow 1$
9: $\quad$ **else**
10: $\qquad$ go to 5.
11: $\quad$ **end if**
12: **end while**
13: $\widetilde{\mathbf{D}}^* = D$

of $\mathsf{P1}_{\mathrm{A}}$. First, let $\mathbf{R} = [R_{k,n}]$ be the rate matrix, assuming that the central unit controller distributes the total available power uniformly over the sub-bands, i.e., $P_{k,n} = P_{\max}/K$. Correspondingly, we define an allocation matrix $\mathbf{D} = [d_{k,n}]$, which initially is set to a $K \times K$ all-ones matrix $\mathbf{J}_K$ showing that each user k might possibly occupy every sub-channel n. Here, $d_{k,n1} = 1$ implies that user k gets sub-channel n. From a matrix $\mathbf{D}$ a number of $K!$ permutation matrices[3] $\widetilde{\mathbf{D}}$ can be obtained by setting a certain number of 1's to zero. To each such allocation matrix $\widetilde{\mathbf{D}}$ a corresponding matrix $\widetilde{\mathbf{R}}$ exists, which comprises the rates of the users in the selected sub-bands.

Our goal is to retain that specific permutation matrix $\widetilde{\mathbf{D}}$, which is optimal in the sense that the corresponding rate matrix $\widetilde{\mathbf{R}}$ exhibits a maximum minimum non-zero element $\widetilde{R}_{k,n}$ among all rate matrices. For this purpose, the iterative Algorithm 1 is executed which receives as input parameters the matrix of the rates $\mathbf{R} = [R_{k,n}]$ and the allocation matrix $\mathbf{D} = [d_{k,n}]$, and outputs the optimal allocation matrix $\widetilde{\mathbf{D}}^*$, which finally is a permutation matrix. The idea of the algorithm is to disable those sub-channel/user pairs with an unfavorable SNR. More explicitly this means, that in each iteration, we find the nonzero minimum achievable data rate from the current matrix $\mathbf{R}$, corresponding to user k^* and sub-channel n^*. Accordingly, we set in the allocation matrix $\mathbf{D}$ the element d_{k^*,n^*} to zero, meaning that the user k^* will not occupy sub-channel n^* since on that specific sub-channel he would suffer from a minimum rate. In the course of this iterative procedure it might happen that we end up disabling a certain user k from all the available sub-channels, which leads to an empty solution set. In order to avoid this, each time we set $d_{k,n} = 0$ we need to make sure that by doing so, we can still find a feasible solution, i.e, a valid assignment for all users in the system under the constraint that each sub-channel is allowed to be occupied by one user only. Mathematically, this translates into checking the permanent[4], $\text{per}(\mathbf{D})$ of the matrix $\mathbf{D}$. In case the permanent is zero this means that disabling the respective user k^* on the corresponding sub-channel n^* leads to an empty solution set. Then, we decide not to deactivate sub-channel n^* for user k^*. This means we set again d_{k^*,n^*} to 1 and R_{k^*,n^*} to ∞ or a high enough value, in order to avoid considering this specific combination (k^*, n^*) again in the search. As a short illustration example, consider two 3×3 matrices,

$$\mathbf{D}_1 = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \quad \mathbf{D}_2 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$

where $\text{per}(\mathbf{D}_1) = 2$ means that two permutation matrices can be obtained from the matrix $\mathbf{D}_1$ by setting correspondingly some 1's to 0's, whereas $\text{per}(\mathbf{D}_2) = 0$ means that no permutation matrix can be obtained from $\mathbf{D}_2$, which can be also clearly seen by inspecting matrix $\mathbf{D}_2$. Several algorithms have been proposed for the evaluation of the permanent of a matrix [17], [18], [19]. The procedure of Algorithm 1 is iterated until each sub-channel is allocated exactly to one user, i.e., the sum of the 1's in the allocation matrix $\mathbf{D}$ equals the number of users, K. The proposed algorithm is optimal in terms of sub-channel assignment, since for the given power distribution it ensures that the minimum individual rate among users is maximized under the condition that all users in the system are transmitting on exactly one sub-band, and these sub-bands are disjoint.

Once the sub-channel assignment is known after completion of Algorithm 1, the power allocation among the selected sub-band/user pairs has to be determined. The aim is again to maximize the data rate of the user who has the lowest rate. The max-min fair power allocation can be obtained by solving the following optimization problem:

$$\begin{aligned} \mathsf{P1}_{\mathrm{B}} : \underset{P_k}{\text{maximize}} \;\; & \min_{k} \quad b \cdot T_n \int_{-1/2T_n}^{1/2T_n} \ln\left(P_k \gamma_k(f) + 1\right) \mathrm{d}f \\ \text{subject to} \;\; & \sum_{k=1}^{K} P_k = P_{\max},\; P_k \geq 0,\; \forall\, k. \end{aligned}$$

For solving problem $\mathsf{P1}_{\mathrm{B}}$ we introduce an auxiliary optimization variable t that denotes the minimum rate achieved among the individual users. Then problem $\mathsf{P1}_{\mathrm{B}}$ is transformed into the following equivalent convex optimization problem $\overline{\mathsf{P1}}_{\mathrm{B}}$,

$$\begin{aligned} \overline{\mathsf{P1}}_{\mathrm{B}} : \max_{P_k, t} \quad & t \\ \text{s. t.} \quad & b \cdot T_n \int_{-1/2T_n}^{1/2T_n} \ln\left(P_k \gamma_k(f) + 1\right) \mathrm{d}f \geq t, \forall\, k, \\ & \sum_{k=1}^{K} P_k = P_{\max},\; P_k \geq 0,\; \forall\, k. \end{aligned}$$

[3] A permutation matrix is a matrix for which every row and column contains precisely a single 1 with 0's everywhere else.

[4] The permanent of a square matrix is the sum of all additive terms in the determinant, without "-" signs.

The first constraint specifies the supralevel set of a concave function which is convex. The other constraints are affine and convex, respectively. As a result, $\overline{\mathsf{P1}}_{\mathrm{B}}$ is a convex optimization problem. In order to find the optimal solution P_k^* of $\overline{\mathsf{P1}}_{\mathrm{B}}$ we set up the dual problem and the corresponding Karush-Kuhn-Tucker (KKT) conditions [20]. The Lagrangian of the above problem is given by

$$\mathcal{L} = -\,t + \sum_{k=1}^{K} \beta_k \left[t - b \cdot T_n \int_{-1/2T_n}^{1/2T_n} \ln\left(P_k \gamma_k(f) + 1\right) \mathrm{d}f \right] + \mu \left(\sum_{k=1}^{K} P_k - P_{\max} \right) - \sum_{k=1}^{K} \delta_k P_k, \tag{6}$$

where $\boldsymbol{\beta} = [\beta_1, \ldots, \beta_K]$, μ and $\boldsymbol{\delta} = [\delta_1, \ldots, \delta_K]$ are the Lagrange multipliers. Let t^* and P_k^* denote the optimal solution. Applying the KKT conditions, we obtain the following necessary and sufficient conditions for t^* and P_k^*, and all corresponding Lagrange multipliers,

$$\sum_{k=1}^{K} P_k^* = P_{\max},\ \beta_k^*, \delta_k^* \geq 0,\ \forall\, k, \tag{7}$$

$$\frac{\partial \mathcal{L}}{\partial t^*} = 0,\ \frac{\partial \mathcal{L}}{\partial P_k^*} = 0,\ -\delta_k^* P_k^* = 0,\ \forall\, k, \tag{8}$$

$$\beta_k^* \left[t^* - b \cdot T_n \int_{-1/2T_n}^{1/2T_n} \ln\left(P_k^* \gamma_k(f) + 1\right) \mathrm{d}f \right] = 0,\ \forall\, k. \tag{9}$$

Evaluating the above equations, we observe that $\sum_{k=1}^{K} \beta_k^* = 1$, $\delta_k^* = 0,\ \forall\, k$, $t^* = b \cdot T_n \int_{-1/2T}^{1/2T} \ln\left(P_k^* \gamma_k(f) + 1\right) \mathrm{d}f$, where the last equation ensures that all users achieve the same data rate for the optimum solution. Exploiting the fact that the partial derivative of the Lagrangian with respect to P_k^* vanishes we get

$$b \cdot T_n \int_{-1/2T_n}^{1/2T_n} \frac{\beta_k^* \gamma_k(f)}{P_k^* \gamma_k(f) + 1} \mathrm{d}f = \mu^*. \tag{10}$$

Here the additional parameter β_k^* ensures fairness among different users and accounts for the individual rate of user k. The solution of the given coupled nonlinear equations can be obtained via various algorithms, such as the subgradient method or the ellipsoid method [20].

B. Time Division Scheduling Resource Allocation Scheme

Another approach to problem $\mathsf{P1}$ is to relax the constraint that each sub-channel is used by one user only. Then, $d_{k,n}$ becomes a time-sharing factor among the users, indicating the portion of time that user k is occupying sub-channel n. In addition, $w_{k,n} = d_{k,n} P_{k,n}$ becomes now the actual amount of power allocated to user k on sub-channel n, whereas $P_{k,n}$ is the power as if sub-channel n is occupied by user k only. A similar approach has already been proposed in [21], however with the purpose of maximizing the sum-rate of the users. Similarly to $\overline{\mathsf{P1}}_{\mathrm{B}}$, we again introduce an auxiliary variable τ and transform the problem to its epigraph representation. Thus, the optimization problem becomes

$$\mathsf{P2}:\ \underset{d_{k,n}, w_{k,n}, \tau}{\text{maximize}} \quad \tau \tag{11}$$

$$\text{subject to} \quad \sum_{n=1}^{K} d_{k,n} \cdot b \cdot T_n \int_{-1/2T_n}^{1/2T_n} \ln\left(\frac{w_{k,n} \gamma_{k,n}(f)}{d_{k,n}} + 1 \right) \mathrm{d}f \geq \tau, \forall\, k, \tag{12}$$

$$\sum_{k=1}^{K} \sum_{n=1}^{K} w_{k,n} = P_{\max},\ w_{k,n} \geq 0, \forall\, k, n,$$
$$\sum_{k=1}^{K} d_{k,n} = 1, \forall\, n,\ 0 \leq d_{k,n} \leq 1, \forall\, k, n. \tag{13}$$

The objective function in (11) is a linear function, and therefore is concave. The constraint (12) is jointly convex with respect to optimization variables $d_{k,n}$, $w_{k,n}$ and τ. Besides, the constraints in (13) are all convex. Therefore, the proposed optimization problem is convex. Here, we do not provide a detailed solution to problem $\mathsf{P2}$, due to space constraints, instead, we highlight the main observations. Formulating the Lagrangian of the above problem and setting up the KKT conditions we obtain the necessary and sufficient conditions for $d_{k,n}^*$, $w_{k,n}^*$ and τ^*. The optimal power allocation follows again a similar structure as for Problem $\overline{\mathsf{P1}}_{\mathrm{B}}$, except that the allocated power to user k on a sub-channel n is now available only for a fraction of time $d_{k,n}$. Moreover, regarding the optimal sub-channel allocation, for each sub-channel, a user with better channel conditions is more likely to be assigned to that sub-channel.

IV. Numerical Results

In this section, we discuss numerical results for the rate outage probability and the average minimum individual achievable data rate after applying the resource allocation schemes discussed in Section III. For our simulations we assume $M \in \{1, \ldots, 25\}$ APs and $K \in \{30, 40, 50, 60, 120\}$ users which are uniformly distributed in the room. In each case, 100 random realizations of user locations are considered. In Fig. 2, we provide rate outage probability results for a high user density scenario (K = 120) and a random channel selection (RCS) scheme. We distinguish between two cases: maximum value of the molecular absorption noise (lower performance bound approach) and no molecular absorption noise in the system (upper performance bound approach). As mentioned before, in the considered virtual reality use case, each user should be served with a certain minimum data rate in order to

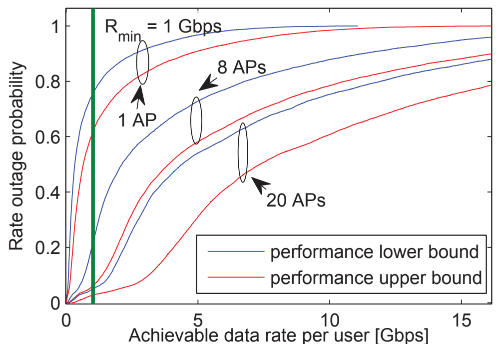

Figure 2: Rate outage probability vs. achievable data rate per user. RCS scheme, $P_{\max}$ = 1 W, K = 120 users.

ensure quality of service during transmission. Otherwise, the user is considered to be in outage. By setting a target minimum achievable data rate of 1 Gbps, $R_{\min}$ = 1 Gbps, and utilizing only a single AP, it can be observed that in 75% and 60% of the cases the users will be in outage when considering the lower bound performance and the upper bound performance, respectively. When adding further 7 APs we notice that the percentage of instances where the users are in outage decreases drastically reaching 20% and 6%, respectively. For a THz-WN with 20 APs, only for 5% and 2% of the instances the users will be in outage.

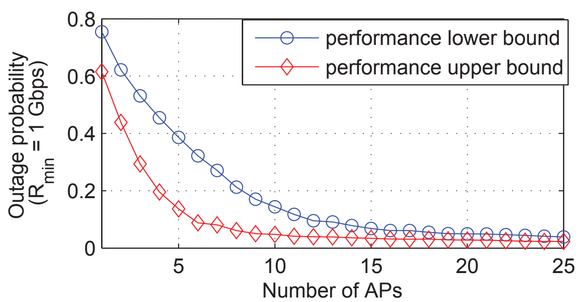

Figure 3: Rate outage probability vs. number of APs for $R_{\min}$ = 1 Gbps. RCS scheme, K = 120 users, $P_{\max} = 1$ W.

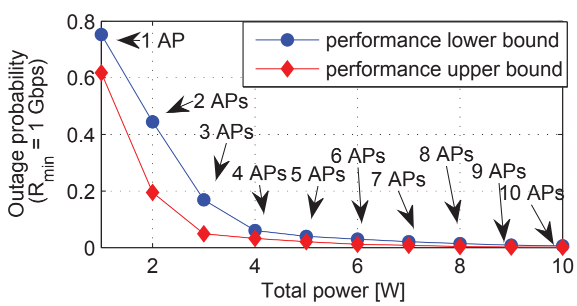

Figure 4: Rate outage probability vs. total power for $R_{\min}$ = 1 Gbps. RCS scheme, K = 120 users, $P_{\max} = M$ W.

In this context, it is of interest to analyze the outage probability corresponding to $R_{\min}$ when varying the number of APs in the room and the total power budget, cf. Figs. 3 and 4. It should be mentioned that when increasing $P_{\max}$ we also increase the number of APs M (i.e., $P_{\max} = M$ W). We observe from Fig. 3, that the outage probability tends to converge after installing more than 20 APs, when considering the lower bound performance, and more than 10 APs for the upper bound performance. From Figs. 3 and 4, we notice that considering the lower bound performance, a total power budget of 5 W, and 5 APs we achieve the same performance as when installing 20 APs with a total power of 1 W. The number of APs must be chosen such that it provides the best trade-off between the outage probability for the considered target minimum individual data rate and the implementation cost. Moreover, the difference in performance between the upper and lower bounds shrinks as the number of APs and the total power budget increases as observed from Figs. 3 and 4.

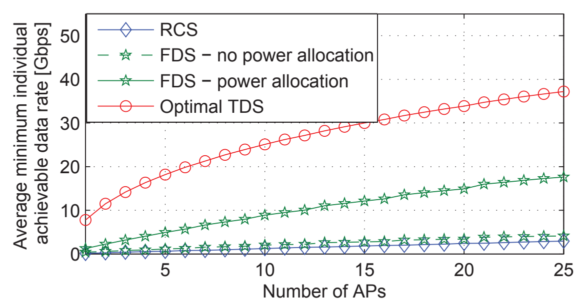

Figure 5: Average minimum individual achievable data rate vs. number of APs. $K = 30$ users, $P_{\max} = 1$ W.

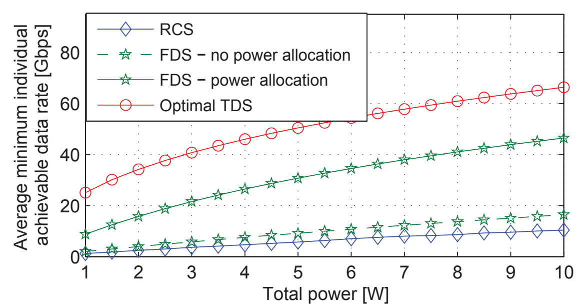

Figure 6: Average minimum individual achievable data rate vs. total power. $K = 30$ users, $M = 10$ APs

In what follows we continue the analysis by considering the lower bound performance, i.e., the performance corresponding to the maximum value of the molecular absorption noise.

Fig. 5 depicts the average minimum individual rate versus the number of APs when considering a total power budget of $P_{\max} = 1$ W, whereas Fig. 6 shows the average minimum individual rate when varying the total power, for $M = 10$ APs. Here, the number of users has been set to $K = 30$ and different channel selection policies are studied: RCS, FDS with power allocation and no power allocation (i.e., we consider the rates given by Algorithm 1), and the optimal TDS. As it can be observed for all considered schemes, the average minimum data rate is monotonically increasing as the number of APs or the power are increased. Also, it should be noticed

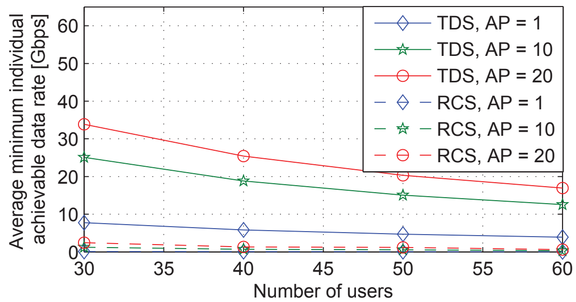

Figure 7: Average minimum individual achievable data rate vs. number of users. $P_{\max} = 1$ W.

that as we increase the number of APs or the power, the performance gap between the proposed schemes with optimal power allocation and the RCS scheme increases. For the schemes with no power allocation, the individual minimum data rate increases only slowly with respect to the number of APs or the total power budget. Therefore, the performance gain when considering optimal power allocation for the two proposed schemes FDS and TDS over the FDS-without power allocation and RCS is evident. Moreover, it can be observed that the TDS scheme offers the best performance, although at a higher implementation cost, due to the fact that it requires careful time synchronization, since users share the frequency bands in the time domain, whereas the transceivers for the FDS scheme are simpler.

In Fig. 7, we show the average minimum individual achievable rate versus the number of users K for a total power budget of 1 W and different numbers of APs, when TDS and RCS schemes are assumed. Increasing the number of users increases the cost of finding an optimal allocation of exactly one user per sub-channel, since computing the permanent of a large matrix is computationally expensive. Therefore, we don't consider the FDS scheme. We observe that as the number of users increases, the minimum individual data rate becomes smaller, since the resource allocation is required to ensure fairness for a larger number of users, despite their potentially poor channel conditions. Therefore, the average minimum data rate decreases with increasing number of users. Moreover, the high performance gain of the TDS over the RCS scheme is once again obvious.

V. Conclusions

In this paper, we have investigated the coverage problem and the achievable data rate performance for a multiuser THz single frequency network. We have addressed the performance in terms of the signal-to-noise ratio at the output of an equalizer. We have given recommendations for the AP placement and the AP density considering the outage probability for a target minimum achievable data rate. Besides, the results revealed a trade-off between the number of APs and the total power budget. Moreover, we have analyzed the minimum individual theoretically achievable data rate as a function of the AP density, user density and the power budget for different resource allocation schemes. In this context, the benefits of an SFN for tackling the high propagation loss and improving the transmission range become obvious. Simulation results have shown that the proposed resource allocation schemes achieve significant gains in the achievable data rate compared to the random channel selection, especially when considering optimal power allocation.

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