Throughput-Optimal LIFO Policy for Bounded Delay in the Presence of Heavy-Tailed Traffic

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***Abstract*—Scheduling is one of the most important resource allocation for networked systems. Conventional scheduling policies are primarily developed under light-tailed (LT) traffic assumptions. However, recent empirical studies show that heavy-tailed (HT) traffic flows have emerged in a variety of networked systems, such as cellular networks, the Internet, and data centers. The highly bursty nature of HT traffic fundamentally challenges the applicability of the conventional scheduling policies. This paper aims to develop novel throughput-optimal scheduling algorithms under hybrid HT and LT traffic flows, where classic optimal policies (e.g., maximum-weight/backpressure schemes), developed under LT assumption, are not throughput-optimal anymore. To counter this problem, a delay-based maximum-weight scheduling policy with the last-in first-out (LIFO) service discipline, namely LIFO-DMWS, is proposed with the proved throughput optimality under hybrid HT and LT traffic. The throughput optimality of LIFO-DMWS gives that a networked system can support the largest set of incoming traffic flows, while guaranteeing bounded queueing delay to each queue, no matter the queue has HT or LT traffic arrival. Specifically, by exploiting asymptotic queueing analysis, LIFO-DMWS is proved to achieve throughout optimality without requiring any knowledge of traffic statistic information (e.g., the tailness or burstiness of traffic flows). Simulation results validate the derived theories and confirm that LIFO-DMWS achieves bounded delay for all flows under challenging HT environments.**

I. Introduction

Because of the emerging multimedia, data center, and the Internet applications over mobile devices, network traffic in both radio access networks and wired core networks has tremendously grown in past few years while the network capacity is rather limited. To address such a challenge, throughput-optimal scheduling is highly demanding, which determines the optimal transmission time for traffic flows, supports the largest set of traffic rates, and maintains the desired network stability. As being an important class of throughput-optimal scheduling, the maximum-weight scheduling (MWS) policy and many of its variants [1] are of great interests. On one hand, MWS policy can achieve throughput optimality without requiring any knowledge of the statistic information of arrival traffic flows and time-varying channel conditions. On the other hand, MWS policy has shown its throughput optimality in a variety of network settings, such as the Internet, cellular networks, WiFi networks, satellite networks, ad-hoc and sensor networks, and high-performance computing clusters.

Generally developed under the assumption of light-tailed (LT) traffic flows (e.g., Markovian or Poisson traffic), the celebrated MWS policies achieve strong stability for LT flows, which guarantees that each flow has bounded average queueing delay [2] whenever the incoming traffic rates are within the network stability region. However, this LT assumption has large discrepancy from recent large-scale empirical studies, which show the emergence of heavy-tailed (HT) traffic in a variety of networked systems, such as WLAN [3], mobile ad-hoc networks [4], cellular networks [5], and data center networks [6]. Such HT traffic is mainly caused by the inherent heavy-tailed distribution in traffic sources (e.g., the file size on the Internet servers, the message size on cellular base stations, the flow length of data centers, and the frame length of VBR video streams [7]).

Different from conventional LT traffic, HT traffic exhibits high burstiness or dependence over a long range of time scale. Such a highly bursty nature can induce significant degradation in network stability [8], thus having a destructive impact on the throughput optimality of scheduling policies. In particular, it is proved that the celebrated MWS policies are not throughput-optimal anymore in the presence of HT traffic flows because MWS can lead to unbounded average queueing delay even if the arrival traffic rates are within network capacity region [8]. Such surprising phenomenon attributes to the fact that the queues with HT traffic arrivals (HT queues) inherently experience heavy-tail distributed queueing delay, which implies that HT queues have much higher chance to experience very large queueing delay, compared with the queues with LT arrivals (LT queues). As a result, based on MWS policies [8], [9], HT queues will receive much more service opportunities, while LT queues are starved for scheduling service and their queueing delay can be of unbounded mean. To counter such challenges, maximum power weight scheduling policies (MPWS) are investigated recently [10], [11], which make scheduling decisions based on queue backlog raised up to the α-th power, where α is determined by the burstiness or heavy tailness of traffic flows. Intuitively, by properly selecting α to allocate more service opportunities to LT queues, MPWS can guarantee that all LT queues experience bounded average queueing delay, completely shielding those LT queues from the destructive impact of HT traffic. Despite such promising feature, MPWS cannot ensure the delay boundness of the HT queues and thus is still not a throughput-optimal scheduling policy. Moreover,

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MPWS policy requires the statistical information (i.e., tailness or burstiness of arrival flows), which is difficult to estimate.

To counter above challenges, in this paper, we propose a delay-based maximum-weight scheduling policy with LIFO service discipline (LIFO-DMWS) and prove its throughput optimality in the presence of HT traffic. Specifically, rather than adopting queue backlog as link weight, we focus on delay-based scheduling, which exploits the *head-of-line (HoL) delay* metric in inter-queue scheduling decisions (i.e., determining the serving order for the packets from different queues). Moreover, instead of using the classic FIFO service discipline, we exploit LIFO service discipline for intra-queue scheduling (i.e., determining the serving order for the packets within each queue). Furthermore, by exploiting asymptotic queueing delay analysis along with moment theory, we prove that LIFO-DMWS is throughput-optimal with respect to strong stability in the presence of heavy tails. That is, we show that with LIFO-DMWS, no matter the incoming traffic flows are HT or LT, all queues will experience bounded average queueing delay as long as the incoming traffic rates are within the network capacity region. Such a throughput optimality feature is of great importance, since it prevents the QoS performance of LT traffic from being significantly degraded by the bursty HT traffic. Simulation results confirm the throughput optimality of LIFO-DMWS and show that LIFO-DMWS brings considerable delay reduction as compared to classic maximum-weight scheduling policies [12]. To the best of our knowledge, this work is the first throughput-optimal scheduling for bounded delay with emerging HT traffic.

The rest of the paper is organized as follows. Section II introduces the system model and preliminaries. Section III proposes the LIFO-DMWS policy for system stabilization with hybrid HT and LT traffic. Section IV presents the performance evaluation and Section V concludes the paper.

II. System Model and Preliminaries

A. System Model

We consider a multi-queue single-server queueing system, where F queues share a single server. Such a queueing system can be used to model downlink/uplink scheduling in cellular or WiFi networks, where multiple users compete a single wireless channel or a set of wireless channels.

We consider a time-slotted system. A *traffic flow* $f \in \{1,\ldots,F\}$ is a discrete-time stochastic arrival process $\{A_f(t); t \in \mathbb{Z}_+\}$, which represents the total number of packets that arrive at a queue at the beginning of the time slot t and is independent and identically distributed from time slot to time slot. The arrival processes or traffic flows are mutually independent. Let $\lambda_f = E[A_f(0)] > 0$ be the rate of traffic flow f and $\lambda = (\lambda_1,\ldots,\lambda_F)$ be the rate vector.

Let stochastic processes $\{Q_f(t); t \in \mathbb{Z}_+\}$ and $\{D_f(t); t \in \mathbb{Z}_+\}$ be the number of packets and the queueing delay, respectively, in queue f at the beginning of time slot t. $Q(t) = (Q_1(t),\ldots,Q_F(t))$ captures the queue backlog at time slot t, and its initial state $Q(0)$ can be an arbitrary element of $\mathbb{Z}_+^F$. Moreover, not all traffic flows can be served simultaneously due to wireless interference. Accordingly, a set of flows that can be served simultaneously is called a *feasible schedule*. In our network model, each feasible schedule only contains one flow, because only one queue can be served during each time slot. Let S denote the set of all feasible schedules. Then, for each feasible schedule $\pi \in S$, let $\pi_f(t)$ denote the number of packets transmitted from queue f under schedule π at time t. For simplicity, we assume that the service rate is one packet per time slot. Thus, the average service rate of queue f under schedule π is $E[\pi_f(t)] = \pi_f \leq 1$. Accordingly, the time-varying scheduling vector $S(t) = (\pi_1(t),\ldots,\pi_F(t)), \pi \in S$ is determined by the proposed scheduling policy. Hence, the set of processes $\{Q(t), D(t), S(t)\}$ with $Q(0)$ completely captures the dynamic of the entire stochastic queueing system.

B. Mathematical Preliminaries

Definition 1 (Heavy Tail) *A random variable (r.v.) X is heavy-tailed (HT), if for all $\theta > 0$, $\lim_{x\to\infty} e^{\theta x}\Pr(X > x) = \infty$, or equivalently, $E[e^{zX}] = \infty$, $\forall z > 0$. A r.v. is light-tailed (LT), if it is not HT, or equivalently, if there exists $z > 0$ so that $E[e^{zX}] < \infty$.*

A HT r.v. has tail distribution decreases slower than exponentially (e.g., Pareto and log-normal); a LT r.v. has tail distribution decreases exponentially or even faster (e.g., exponential and Gamma). From the existence of the moments, we define the tail index of a nonnegative r.v. X as

$$\kappa(X) := \sup\{k \geq 0 : E[X^k] \leq \infty\}, \tag{1}$$

which defines the maximum order of finite moments that X can have. Moreover, to show the sufficient condition for finite tail indexes [13], we have the following: A nonnegative r.v. X has $\kappa(X)$ if and only if the tail distribution of X satisfies

$$\lim_{t\to\infty} \frac{\log \Pr(X > t)}{\log t} = -\kappa(X). \tag{2}$$

In the following, we define an important class of HT distributions (i.e., regularly varying distributions).

Definition 2 (Regularly Varying) *A r.v. X is called regularly varying with tail index $\beta > 0$, denoted by $X \in \mathcal{RV}(\beta)$, if $\Pr(X > x) \sim x^{-\beta}\mathcal{L}(x)$, where for any two real functions $a(t)$ and $b(t)$, $a(t) \sim b(t)$ denote $\lim_{t\to\infty} a(t)/b(t) = 1$ and $\mathcal{L}(x)$ is a slowly varying function.*

Regularly varying distributions are a generalization of Pareto/Zipf/power-law distributions and can effectively, stochastically characterize a wide range of network attributes, including the frame length of VBR traffic, the session duration of network users in WLANs, the files sizes at the Internet severs, and the message sizes at cellular base stations. The smaller values of β imply heavier tail. In particular, if the arrival process $A_f(t)$ follows regular varying distribution, $A_f(t) \in \mathcal{RV}(\beta)$, then $\kappa(A_f(t)) = \beta$. This indicates that if $0 < \beta < 1$, X has infinite mean and variance. If $1 < \beta < 2$, X has finite mean and infinite variance.

C. System Stability and Throughput Optimality

Definition 3 (Steady-state Stability) *Given the queueing system described in Section II-A, if there exists a scheduling policy under which the Markov chain of queue lengths is positive Harris recurrent (i.e., $\{Q(t); t \in \mathbb{Z}_+\}$ converges in distribution), then the queueing network is steady-state stable.*

Steady-state stability only guarantees the convergence of the steady-state distribution for queue backlog. To characterize the delay performance of queueing systems, strong stability needs to be adopted as follows.

Definition 4 (Strong Stability) *A queueing system is strongly stable, if all traffic flows experience bounded average queueing delay (i.e., $E[W_f] < \infty, \forall f \in F$).*

The throughput-optimality of scheduling algorithms characterizes their capability of achieving strong stability in the network capacity region, defined as follows.

Definition 5 (Network Capacity Region [1]) *The network capacity region Φ of the queueing system is the set of all traffic rate vectors that are admissible by the system (i.e., λ can be covered by a convex combination of feasible schedules). Mathematically, $\Phi := \{\lambda \in \mathbb{R}_+^F | \lambda \leq \sigma$ componentwise, for some $\sigma \in Co(S)\}$, where $Co(S)$ denotes the convex hull of all feasible schedules.*

Definition 6 (Throughput Optimality) *A scheduling policy is throughput-optimal, if it can achieve strong stability for any admissible rate vector (i.e., any rates within the network capacity region).*

III. Delay-based MaxWeight Scheduling with LIFO Service Discipline (LIFO-DMWS)

MWS policies activate a feasible scheduler with the maximum weight at any given time slot. The classic MWS policies adopt FIFO service discipline for intra-queue scheduling and utilize queue-length as the weight to regulate inter-queue scheduling. That is, at each time slot, the queue f with the maximum queue length is served, i.e.,

$$Q_f^{FIFO}(t) = \max_{j \in F} Q_j^{FIFO}(t), \quad \forall f \in F. \tag{3}$$

The following theorem shows the instability of MWS algorithms in the presence of HT traffic flows.

Theorem 1 (Network Instability of MWS [10], [11]) *Under MWS, if one of queues has a HT traffic arrival with tail index smaller than 2 (i.e., $A_f \in \mathcal{RV}(\alpha)$ with $\alpha < 2$), all queues have bounded average queueing delay (i.e., $E[D_f] = \infty, \forall f \in F$).*

The above theorem indicates the classic MWS is not throughput-optimal anymore with HT traffic. Such surprising phenomenon can be attributed to the fact that HT queues inherently experience heavy-tail distributed queueing delay, which implies that compared with LT queues, HT queues have much higher chance to experience very large queueing delay, which can be of unbounded mean. As a result, based on MWS policies, HT queues will receive much more service opportunities, while LT queues are seriously starved and thus their average queueing delay can be unbounded.

To counter such a problem, we propose a LIFO-DMWS policy. Different from the classic MWS policies, the proposed LIFO-DMWS exploits LIFO service discipline for intra-queue scheduling and employs the HoL delay as the weight for inter-queue scheduling. In this case, the HoL packet delay $\{W_f^{LIFO}(t)\}$ is the queueing delay $\{D_f(t)\}$ experienced so far by the HoL packet in the queue. Specifically, at each time slot, the LIFO-DMWS policy serves queue f with the maximum HoL delay, i.e.,

$$W_f^{LIFO}(t) = \max_{j \in F} W_j^{LIFO}(t), \quad \forall f \in F \tag{4}$$

where ties are broken randomly. Despite its simplicity, LIFO-DMWS is the first throughput-optimal scheduling for bounded delay with heavy-tailed traffic.

Theorem 2 (Throughput Optimality of LIFO-DMWS) *The LIFO-DMWS policy is throughput optimal by ensuring that all queues have bounded average queueing delay $E[D_f] < \infty, \forall f \in F$ whenever (1) all arrival traffic flows have bounded mean $\lambda_f = E[A_f(t)] \in \infty, \forall f \in F$. That is, all arrival traffic flows have a tail index larger than 1, i.e., $\min_{f \in F} \kappa(A_f(t)) > 1$; (2) the incoming traffic rates are within the network capacity region.*

Intuitively speaking, the promising feature of LIFO-DMWS comes from the optimal asymptotic delay performance of the HT queues under LIFO service discipline, which can guarantees that HT queues experience sufficiently reduced delay, which are of bounded mean. Then, by adopting HoL delay as the weight metric, we can ensure that the delay of LT queues is at least of the same order as the delay of HT queues. This implies the queueing delay of LT queues is also of the bounded mean. Accordingly, we can show that LIFO-DMWS is throughput-optimal. More importantly, LIFO-DMWS does not require any knowledge of the statistical information of traffic arrivals.

Despite the intuitive advantage of LIFO-DMWS, proving its throughput optimality is very challenging. To prove the throughput optimality of delay-based maximum-weight scheduling (DMWS) with FIFO, fluid model-based schemes are generally adopted, which establish the linear relationship between queueing delay and queue length through fluid model solutions. This implies the throughput optimality of DMWS, since queue-length based policies are throughput-optimal. However, under HT environment, fluid model based schemes cannot be applied because Little's law does not hold for HT queues with LIFO discipline, which indicates that the linear relationship between queueing delay and queue length may not hold for HT queues in the fluid domain.

To counter this challenge, we adopt our developed asymptotic queueing analysis tools to prove the throughput optimality of LIFO-DMWS. In the following subsections, we first introduce the advantage of LIFO service discipline for HT traffic by showing the asymptotic queueing delay of LIFO queues under single-queue, single-server scenario in Section III-A. Then, we prove the throughput optimality in Section III-B by deriving the asymptotic queueing delay of each queue under the LIFO-DMWS policy.

A. Asymptotic Queueing Delay of LIFO Discipline

Informally speaking, LIFO discipline allows the waiting time or queueing delay to be "the same degree heavier" as the service time in the case of HT arrival. Specifically, as shown in [14], the tail distribution D of the queueing delay of a single queue with FIFO discipline follows $\Pr(D > x|\text{FIFO}) \sim \frac{\rho}{1-\rho}\Pr(B^r > x) = \frac{\rho}{1-\rho}\frac{1}{E[B]}\int_x^\infty \Pr(B > u)du$, where $f(x) \sim g(x)$ denotes $\lim_{x\to\infty} f(x)/g(x) = 1$, and B, B^r and ρ denotes the service time, the residual service time and the traffic load, respectively.

If traffic arrival $A(t)$ is HT with tail index $\kappa(A(t)$, i.e., $A(t) \in \mathcal{RV}(\kappa(A(t))$, the service time for $A(t)$ follows HT distribution which asymptotically behaves as $x^{-\kappa(A(t))}$, i.e.,

$$\lim_{t\to\infty}\frac{\log\Pr(B > t|\text{FIFO})}{\log t} = -\kappa(A(t)). \tag{5}$$

As a result, the queueing delay tail distribution behaves as $x^{1-\kappa(A(t))}$, i.e.,

$$\lim_{t\to\infty}\frac{\log\Pr(D > t|\text{FIFO})}{\log t} = -\kappa(A(t)) + 1, \tag{6}$$

which is one degree heavier than the service time tail distribution in (5). This implies by Definition 2 that if traffic arrival process has unbounded variance, i.e., $\kappa(A(t)) < 2$, then the queueing delay has unbounded mean. On the other hand, the queueing delay tail distribution with LIFO discipline becomes $\Pr(D > x|\text{LIFO}) \sim \frac{1}{1-\rho}\Pr(B > x(1-\rho))$. This means that if traffic arrival process is HT with tail index $\kappa(A(t))$, the service time for $A(t)$ follows HT distribution which asymptotically behaves as

$$\lim_{t\to\infty}\frac{\log\Pr(B > t|\text{LIFO})}{\log t} = -\kappa(A(t)). \tag{7}$$

As a result, the queueing delay tail distribution behaves as

$$\lim_{t\to\infty}\frac{\log\Pr(D > t|\text{LIFO})}{\log t} = -\kappa(A(t)). \tag{8}$$

It means that the queueing delay is as heavy as that of the service time which has the same tail index of arrival process. This implies by Definition 2 that the average queueing delay is bounded as long as service time is of bounded mean or has a tail index larger than one, i.e., $\kappa(A(t) > 1$.

B. Throughput Optimality Analysis of LIFO-DMWS

In this section, we first investigate the asymptotic queueing delay performance under general *work-conserving* scheduling policies in Lemma 3, which gives the upper bound of queueing delay tail index under LIFO-DMWS. In Lemma 2, we derive the asymptotic delay performance under LIFO-DMWS, which yields the lower bound of queueing delay tail index. Then, we prove the throughput optimality of LIFO-DMWS by showing that the upper and lower bounds coincide and are larger than one as long as the arrival traffic has tail index larger than one. This indicates that all queues are of bounded average queueing delay under LIFO-DMWS. Before going to the main theorems, we first introduce Lemma 1 that is used in the later proofs.

Lemma 1: For any work-conserving scheduling with single-hop hybrid traffic, we have the following: Under FIFO,

$$\kappa(\sum_{f\in F} Q_f) = [\min_{f\in F}\kappa(A_f(t))] - 1; \tag{9}$$

under LIFO,

$$\kappa(\max_{f\in F} D_f) \geq \min_{f\in F}\kappa(A_f(t)). \tag{10}$$

Proof: The result for FIFO service discipline is obtained by the analysis of the fictitious queue. Specifically, consider a fictitious q_v, which has the arrival process $A_v(t) = \sum_{f\in F} A_f(t)$ under the original single-hop queueing system. As we adopt regularly varying distributions for HT flows, i.e., flow $f \in HT$ with $A_f(t) \in \mathcal{RV}(\beta_f)$ and $\kappa(A_f(t)) = \beta_f$, it implies by regular variation that the arrival $A_v(t) \in \mathcal{RV}(\min_{f\in F}\beta_f)$. Let D_v, Q_v, and $Q_v(t)$ denote the steady-state queueing delay, the steady-state queue backlog, and the queue backlog at time t of q_v, respectively. Regarding FIFO service discipline, it follows by Eq. (6) that $\kappa(D_v) = (\min_{f\in F}\beta_f) - 1$. Under any work-conserving scheduling policy in the original queueing system, we have $Q_v(t) = \sum_{f\in F} Q_f(t)$, which implies that $Q_v = \sum_{f\in F} Q_f$. By distributional Little's law, we have $\Pr(Q_v > t) \sim \Pr(E[A_v(t)]D_v > t)$, which indicates that $\kappa(\sum_{f\in F} Q_f) = (\min_{f\in F}\beta_f) - 1$, and prove the result for FIFO discipline.

The result for LIFO service discipline is obtained by union bound and regular variation. Specifically, we have that $\Pr(\max_{f\in F} D_f > t) = \Pr(D_1 > t \vee \cdots \vee D_F > t) \leq \Pr(D_1 > t) + \cdots + \Pr(D_F > t)$. This, combing with Eq. (2) and Definition 2, implies that $\kappa(\max_{f\in F} D_f) \geq \min_{f\in F}\kappa(A_f(t))$; thus, we complete the proof. ■

Lemma 2: Consider any work-conserving scheduling with arrival traffic satisfying $\min_{f\in F}\kappa(A_f(t)) > 1$. Under FIFO discipline, the steady-state queueing delay D_f of flow f with $\kappa(A_f(t))$ follows

$$\kappa(A_f(t)) - 1 \geq \kappa(D_f) \geq [\min_{f\in F}\kappa(A_f(t))] - 1; \tag{11}$$

under LIFO discipline, the steady-state delay D_f follows

$$\kappa(A_f(t)) \geq \kappa(D_f) \geq \min_{f\in F}\kappa(A_f(t)) \tag{12}$$

whenever incoming traffic rates are within the network capacity region.

Proof: By Definition 5 and the work in [1], the prerequisite that the incoming rates are within the convex hull of the set of all feasible schedules ensures the steady-state stability of queueing systems and thus the existence of steady-state queueing distributions. This implies that the steady-state queue backlog Q_f and delay D_f exist for all $f \in F$, and the asymptotic queueing delay analysis can be readily applied.

We prove the asymptotic results in Eq. (11) with queue q_f as follows. Specifically, it is evident that the queue backlog Q_f

is stochastically dominated by the composite queue backlog $\sum_{f\in F} Q_f$, which by Lemma 1 and distributional Little's law that $\Pr(D_f > t) \sim \Pr(Q_f/\lambda_f > t)$, proves the lower bound of Eq. (11), i.e., $\kappa(D_f) \geq [\min_{f\in F}\kappa(A_f(t))] - 1$. As to the upper bound, we consider the best scheduling policy for q_f, which allows q_f to receive the service whenever q_f is not empty. This scheduling policy yields the best asymptotic results for the queue q_f, since q_f does not have to compete with other queues for the service and thus behaves like a single flow queue with FIFO discipline. Invoking Eq. (6), the upper bound of Eq. (11) hods, i.e., $\kappa(D_f) \leq \kappa(A_f(t)) - 1$, and thus proves Eq. (11) for the FIFO discipline.

We prove Eq. (12) for LIFO service discipline as follows. The lower bound is simply obtained from the result in Lemma 1. Specifically, it is obvious that the queueing delay D_f is stochastically dominated by the maximum delay $\max_{f\in F} D_f$, i.e., $\Pr(D_f > t) \leq \Pr(\max_{f\in F} D_f > t)$ for all $t \geq 0$. This, combing with Eq. (10), implies that $\kappa(D_f) \geq \kappa(\max_{f\in F} D_f) \geq \min_{f\in F}\kappa(A_f(t))$. The upper bound is obtained by using the best scheduling policy and Eq. (8). Specifically, consider the best scheduling for q_f, which implies that q_f has an exclusive access to the scheduler, like a single queue with LIFO discipline. Invoking Eq. (8), the upper bound of Eq. (12) hods, i.e., $\kappa(D_f) \leq \kappa(A_f(t))$; thus, we complete the entire theorem proof. ■

Now, we derive the upper bound of the tail index for queueing delay.

Lemma 3: Under LIFO-DMWS, the tail index $\kappa(D_f)$ of the steady-state queueing delay D_f is upper bounded by

$$\kappa(D_f) \leq \min_{f\in F}\kappa(A_f(t)). \tag{13}$$

Proof: We exploit our asymptotic queueing analysis tools [10] to study queueing delay for LIFO-DMWS. Specifically, we construct a fictitious queueing system with F flow queues $\{\bar{q}_f\}_{f\in F}$, where each queue $\bar{q}_f$ has the same input process as q_f. Consider a particular queue $\bar{q}_f$, and let all queues $\{\bar{q}_j\}_{j\neq f}$ except $\bar{q}_f$ have the exclusive access to their own scheduler without competing with each other (i.e., these queues operate as single flow queues with LIFO discipline). The queue $\bar{q}_f$ receives service if and only if $\bar{W}_f^{LIFO} = \max_{j\in F}\bar{W}_j^{LIFO}$. In such a system, it is easy to prove that the fictitious queue $\bar{q}_f$ has less queueing delay than the queue q_f in the original system, i.e., $D_f(t) \geq \bar{D}_f(t)$. We assume that the fictitious system is in the steady state. Let Φ_j denote the event where $\bar{q}_j$ is not empty and all the other queues excluding $\bar{q}_f$ are empty, i.e.,

$$\Phi_j := \{\bar{D}_j \neq 0 \wedge \bigcap_{k\neq f,j} \bar{D}_k = 0\} \tag{14}$$

and $\Pr(\Phi_j) := \Pr(\bar{D}_j > 0)\prod_{k\neq f,j}(1 - \Pr(\bar{D}_k > 0))$. Thus, we have the lower bound of the d^{th} moment of D_f as

$$E[D_f^d] \geq \sum_{j\neq f}\Pr(\Phi_j)E[\bar{D}_f^d|\Phi_j]. \tag{15}$$

In the rest of the proof, we will derive the lower bound of the conditional moments $E[\bar{D}_f^d|\Phi_j]$. Assume that the event Φ_j occurs at time t. Let $B_j^r(t)$ and $B_j^e(t)$ denote the residual and the expanded service time of the message currently in service for the queue $\bar{q}_j$. By renew theory and Eq. (8) under LIFO discipline, we have

$$\kappa(B_j^r(t)) = \kappa(B_j^e(t)) = \kappa(A_j(t)). \tag{16}$$

If the event Φ_j occurs, then three possible events, namely (i) Υ_1, (ii) Υ_2, and (iii) Υ_3, occur to $\bar{W}_f^{LIFO}(t)$. Specifically, we define $\Upsilon_1 := \{\bar{W}_f^{LIFO}(t) \geq \bar{W}_j^{LIFO}(t) + B_j^r(t)\}$, $\Upsilon_2 := \{\bar{W}_j^{LIFO}(t) + B_j^r(t) > \bar{W}_f^{LIFO}(t) \geq \bar{W}_j^{LIFO}(t)\}$, and $\Upsilon_3 := \{\bar{W}_j^{LIFO}(t) > \bar{W}_f^{LIFO}(t)\}$. If (i) Υ_1 occurs, we have

$$\bar{D}_f(t) \geq \bar{D}_j(t) + B_j^r(t). \tag{17}$$

If (ii) Υ_2 occurs, we have

$$\bar{D}_f(t) \geq \bar{D}_j(t). \tag{18}$$

In the case of (iii) Υ_3, let τ denote the last time before t that $\bar{q}_f$ receives service. This means that $\bar{W}_f^{LIFO}(\tau) > \bar{W}_j^{LIFO}(\tau)$. Combing with the fact that $\bar{W}_j^{LIFO}(t) > \bar{W}_f^{LIFO}(t) > \bar{W}_f^{LIFO}(\tau)$, it indicates that the burst being served at time t did not begin to receive service at time τ, i.e., $t - \tau > B_j^e(t)$. This implies that

$$\bar{D}_f(t) \equiv \bar{W}_f^{LIFO}(t) = \bar{W}_f^{LIFO}(\tau) + (t-\tau) > B_j^e(t) \tag{19}$$

Combining Eq. (15) and Eqs. (17)-(19), we obtain

$$\begin{aligned} E[D_f^d] \geq & \sum_{j\neq f}\Pr(\Phi_j)\Big\{\Pr(\Upsilon_1)E[\big(\bar{D}_j(t) + B_j^r(t)\big)^d] \\ & + \Pr(\Upsilon_2)E[\bar{D}_j(t)^d] + \Pr(\Upsilon_3)E[B_j^e(t)^d]\Big\} \\ \geq & \sum_{j\neq f}\Pr(\Phi_j)\big\{\Pr(\Upsilon_1)(E[\bar{D}_j(t)^d] + E[B_j^r(t)^d]) \\ & + \Pr(\Upsilon_2)E[\bar{D}_j(t)^d] + \Pr(\Upsilon_3)E[B_j^e(t)^d]\big\}. \end{aligned} \tag{20}$$

This, combing with Eq. (8) and Eq. (16), implies that if the order of the moments $d \geq \min_{j\neq f}\kappa(A_j(t))$, then at least one of the terms on the right-hand side of Eq. (20) is infinite, which implies

$$\kappa(D_f) \leq \min_{j\neq f}\kappa(A_j(t)). \tag{21}$$

Moreover, since under any work-conserving scheduling policy, D_f is lowered bounded by that of a single exclusive queue with LIFO discipline. This implies that

$$\kappa(D_f) \leq \kappa(A_f(t)), \tag{22}$$

which, combing with Eq. (21) completes the proof. ■

Upon this stage, the throughput optimality of LIFO-DMWS is presented in the following Theorem 2.

Proof of Theorem 2: Given that incoming traffic rates are within the network capacity region, this implies that the network is steady-state stable. Hence, we are ready to apply the asymptotic queueing analysis for LIFO-DMWS as follows. First, the upper bound of $\kappa(D_f)$ under LIFO-DMWS is given by Lemma 3. Since LIFO discipline is work-conserving, by Theorem 2, the lower bound of $\kappa(D_f)$ under LIFO-DMWS is obtained, i.e., $\kappa(D_f) \geq \min_{f\in F} \kappa(A_f(t))$. Therefore, it follows that the upper and lower bounds of $\kappa(D_f)$ coincide under LIFO-DMWS, i.e., $\kappa(D_f) = \min_{f\in F} \kappa(A_f(t))$. This, combing with the given condition $\min_{f\in F} \kappa(A_f(t)) > 1$ and Eq. (1), completes the proof. ■

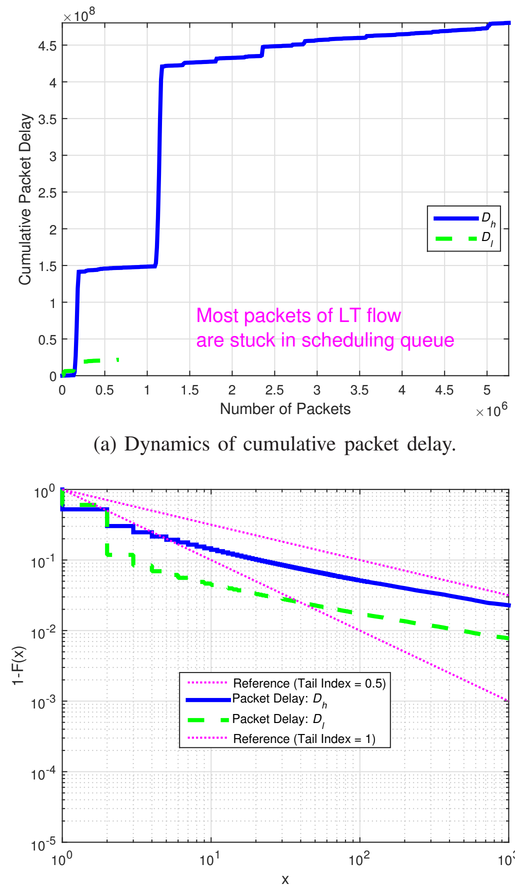
IV. Performance Evaluation

In this section, we validate our asymptotic queueing analysis. We choose Pareto and Poisson distributions to depict HT and LT distributions, respectively. Specifically, a r.v. $X \in \mathcal{PAR}(\beta, x_m)$, if it follows Pareto distribution with a shape parameter $\beta > 0$ and a scale parameter $x_m > 0$, i.e., $P(X > x) = (x_m/x)^\beta$. A r.v. $X \in Poiss(\lambda)$, if it follows Poisson distribution with mean $\lambda > 0$, i.e., $P(X > x) = 1 - e^{-\lambda} \sum_{i=0}^{\lfloor x \rfloor} \lambda^i/i!$, where $\lfloor x \rfloor$ is the floor function. In the following, we validate the throughput optimality of LIFO-DMWS with single-hop hybrid traffic. All simulation results are evaluated over 10^5 time slots.

A. Throughput-optimal LIFO-DMWS

We consider a scenario where a HT flow and a LT flow sharing a single channel, i.e., $A_h(t) \in PAR(1.5, 1)$ and $A_l(t) \in Poiss(3)$. All the following tail distribution results are plotted on log-log coordinates, by which a HT distribution with tail index κ manifests itself as a straight line with the slope equal to $-\kappa$. We first examine the performance of hybrid traffic under the classic MWS policy. To enable a fair comparison, we also adopt delay as the weight metric for MWS instead of queue length and denote it by (FIFO-)DMWS. Fig. 1a shows that during 10^5 time slots, only around 10% of packets (as compared to HT traffic) from LT traffic leave the scheduler and contribute to the cumulative packet delay, given the same packer arrival rate for both LT and HT flows. The reason is that most of packets from LT traffic are stuck in the queue due to the competitions with the HT flows. Moreover, Fig. 1b shows that under the DMWS policy, the queueing delay of LT flow follows heavy tailed distribution with a tail index smaller than one, as its tail distribution decays slower than the reference Pareto distribution with tail index one. This means that the LT traffic also has unbounded average delay as that of HT traffic. Hence, under the DMWS policy with hybrid traffic, the LT flow necessarily has infinite average queueing delay, and DMWS is not throughput optimal.

We next show that the bounded delay and thus throughput optimality can be achieved for hybrid traffic by applying the LIFO-DMWS policy. Fig. 2a shows that LT traffic can receive sufficient service opportunities to sent a comparable number of packets as HT traffic flows. Moreover, as shown in Fig. 2b that under the LIFO-DMWS policy, the queueing delay of LT flow has a tail index greater than one, as their tail distributions decay faster than the reference Pareto distribution with tail index one. This means the LT traffic has bounded queueing delay. Furthermore, while the queueing delay of HT flows are also of bounded mean. Finally, a more complicated scenario under LIFO-DMWS is studied with three hybrid flows in Fig. 3, i.e., $A_h(t) \in PAR(1.5, 1)$, $A_{l1}(t) \in Poiss(3)$, and $A_{l2}(t) \in Poiss(2)$, and the same conclusion is reached accordingly. In particular, Fig. 3a shows that most of packets from LT flows can exit the system. Fig. 3b further indicates that for the tail distributions of queueing delay of all LT and HT flows have a slope or decaying rate greater than one, indicating the average bounded delay for all traffic flows. Above results are consistent with Theorem 2, which implies that under hybrid HT and LT traffic, LIFO-DMWS is throughput-optimal by achieving strong stability.



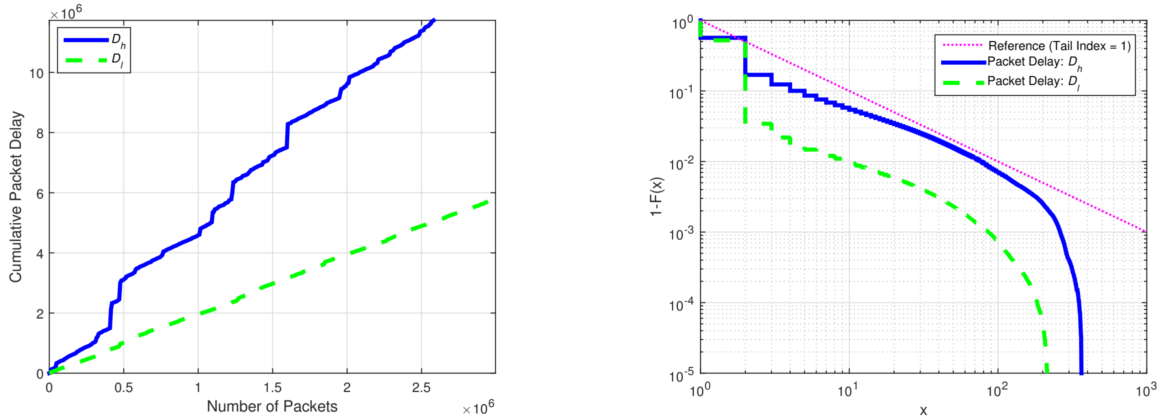
(a) Dynamics of cumulative packet delay.

(b) Tail distribution of packet delay.

Fig. 1: Queueing delay of a HT flow, i.e., $A_h(t) \in \mathcal{PAR}(1.5, 1)$, and a LT flow, i.e., $A_l(t) \in Poiss(3)$, under (FIFO-)DMWS [12].

V. Conclusion

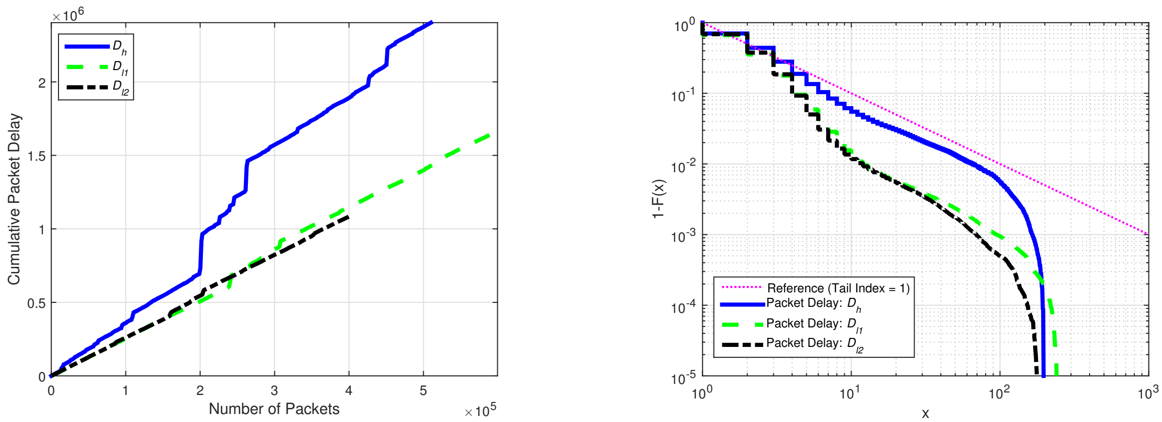
LIFO-DMWS is proposed to achieve throughput-optimality with heavy-tailed traffic. In particular, LIFO-DMWS guarantees bounded average delay for hybrid HT and LT flows under any admissible traffic arrivals without any knowledge of statistical information of the arrivals. Performance evaluations validate our theoretical findings. The future research will be the extension of LIFO-DMWS to multi-hop traffic flows with possibly feedback loops.



(a) Dynamics of cumulative packet delay.

(b) Tail distribution of packet delay.

Fig. 2: Queueing delay of a HT flow, i.e., $A_h(t) \in \mathcal{PAR}(1.5, 1)$, and a LT flow, i.e., $A_l(t) \in Poiss(3)$, under LIFO-DMWS.



(a) Dynamics of cumulative packet delay.

(b) Tail distribution of packet delay.

Fig. 3: Queueing delay of a HT flow, i.e., $A_h(t) \in \mathcal{PAR}(1.5, 1)$, and two LT flows, i.e., $A_{l1}(t) \in Poiss(3)$ and $A_{l2}(t) \in Poiss(2)$, under LIFO-DMWS.

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