On the Throughput of Wireless Underground Sensor Networks using Magneto-Inductive Waveguides

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***Abstract*—Wireless Underground Sensor Networks (WUSNs) present a variety of new research challenges. Recently, a magneto-inductive (MI) waveguide technique has been proposed to cope with the very harsh propagation conditions in WUSNs. This approach allows for an extension of the transmission range, which can be quite limited if relays are not deployed. In this paper, tree-based WUSNs are considered with sensors connected via MI-waveguides. The objective of our work is to determine the optimal system parameters and topology in order to avoid bottlenecks in the system and achieve optimal network throughput.**

I. Introduction

Wireless underground sensor networks (WUSNs) are an emerging and promising research area. For WUSNs, the goal is to establish an efficient wireless communication in the underground medium. Typical applications for such networks include soil condition monitoring, earthquake prediction, border patrol, etc. [1], [2]. Since the propagation medium consists of soil, rock, and sand, traditional wireless signal propagation techniques using electromagnetic (EM) waves can be only applied for very small transmission ranges due to the high pathloss and vulnerability to changes of soil properties, such as moisture [3].

Magnetic induction (MI)-based WUSNs were first introduced in [2] and make use of magnetic antennas implemented as coils, which are combined in waveguide structures with several passive relay devices between two transceiver nodes [4], [5], [6]. Similar to traditional wireless relaying concepts this approach is supposed to benefit from a lower pathloss. Hence, the transmission range can be greatly improved compared to the EM based approach.

The network throughput, also called network capacity, was intensively studied in the past. The most popular definition of the network capacity was originally given by [7]. This work was then extended to different types of wireless networks, such as cognitive radio networks [8], ad hoc networks with infrastructure [9], ad hoc networks with directional antennas [10], and magnetic induction based WUSNs [11]. In particular, [11] provides a scaling law for the MI based networks, adopting a channel model from [4] using several approximations, which enables a simple calculation of the network capacity based on the general equations for wireless networks. One of these approximations is based on the assumption of a weak coupling between the coils in an MI-waveguide, independent from the system parameters. However, as it was shown in [12], for an MI-waveguide with high relay density, the magnetic induction is very large and the system parameters can be adjusted to maximize the channel capacity for a given waveguide. Further differences to the traditional wireless networks are interference propagation and variations of the channel and noise characteristics depending on the topology of the network, which has been not taken into account in [11]. This leads to a significant difference in channel models and in network design. As it was discussed in [12], [13], and [14], the channel capacity of an MI based link depends on the choice of the system parameters, like size of the coils, carrier frequency f_0, and number of coil windings N. A practical sensor network may contain several links with different numbers of relays and therefore the optimal parameters may differ from link to link. In order to overcome the problems of individual manufacturing of each sensor node and implementing additional coils for multiple connected waveguides, one of our objectives in this work is the unification of these parameters, which results in an optimization problem as it will be formally discussed in Section III. In addition, a practical network differs from a single waveguide connection, which is optimized in [12], in its signal propagation characteristics and interference coming from other nodes transmitting simultaneously. These interfering signals cannot be avoided via frequency multiplexing, because all nodes transmit their data at the same frequency due to the parameter unification. Therefore, we assume a TDMA access scheme, such that high power interference signals can be avoided. This yields a routing optimization problem and extends the original problem towards the optimization of the network throughput. In this paper, we focus on tree-based networks with one sink, which collects the data from all nodes. The sink can be implemented as a node, which is connected wirelessly or via wireline with a mobile or removable aboveground device. This network structure is appropriate for most of the target applications with the primary goal of data collection. Each node transmits not only its own information, but also relays all received data from other nodes. We utilize the decode-and-forward relaying concept in this work. Also, we assume that no bit errors occur at the output of the decoder. According to [7] and [11], the traffic load of a link equals the throughput of an information stream multiplied by the number of streams (routes) to be served by the node. In

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sensor networks it is frequently assumed that the data rates of all streams are equal. In order to avoid a bottleneck and loss of data packets, the traffic load has to be less or equal to the available data rate at the node corresponding to the channel capacity. However, the transmission may be disturbed by interfering signals coming from the other nodes. Hence, a multinode scheduling needs to be established, thus reducing the data rate. The transceivers are operated in full-duplex mode. A simultaneous transmission and reception of signals is possible, if the known (transmitted) signal is subtracted from the sum of transmitted and received signals at the load impedance Z_L. In case of equality between the maximum available data rate and the traffic load, the throughput of a link is given by [11][1]:

$$\begin{aligned} T_i \cdot N_{\text{routes},i} &= \frac{C_{\text{ch},i}}{1+N_{\text{interferers},i}}, \\ T_i &= \frac{C_{\text{ch},i}}{N_{\text{routes},i}\cdot(1+N_{\text{interferers},i})}, \end{aligned} \quad (1)$$

where $C_{\text{ch},i}$ is the channel capacity of link i, $N_{\text{routes},i}$ is the number of data streams of link i, $N_{\text{interferers},i}$ is the number of interfering nodes, and T_i is the throughput of link i. The number of relevant interferer nodes for a particular link depends on the interference powers received from the different nodes, hence, on the system parameters and on the network topology, as discussed in Section II-B.

This paper is organized as follows: In Section II the network infrastructure for WUSNs based on MI-waveguides is presented. In Section III the problem of optimizing the throughput is formulated and key optimization strategies are discussed. Section IV provides insight into the simulation results and Section V concludes the paper.

II. Framework

According to [12], MI-waveguides with low coil densities provide much lower channel capacity than the direct MI transmission scheme (no passive relays used). Hence, there are two strategies, which can be applied to the deployment of practical MI-based WUSNs: direct MI transmission and MI-waveguides with a coil density of at least $\frac{1}{3}\ \frac{\text{Relay}}{\text{m}}$. The advantage of the first strategy lies in the deployment effort, which is greatly reduced compared to the MI-waveguides. However, due to a good coupling between coils at low frequencies even for longer distances, the number of possible interferers is significantly higher than for using the second deployment strategy, yielding much lower throughput.

For MI-waveguides, we assume that all devices can be split in two groups: sensor nodes and relays, see Fig. 1. All devices of one group contain the same set of passive circuit elements, cf. [11], [12], see Fig. 2. Each circuit includes a magnetic antenna (which is assumed to be a multilayer air core coil), a capacitor C, a resistor R (which models the copper resistance of the coil) and a load resistor Z_L in the transceivers. These

[1]The interfering signals are separated by means of TDMA. Then, the own data is transmitted only in each $(1 + N_{\text{interferers},i})$ slot and the maximal available data rate decreases by this factor.

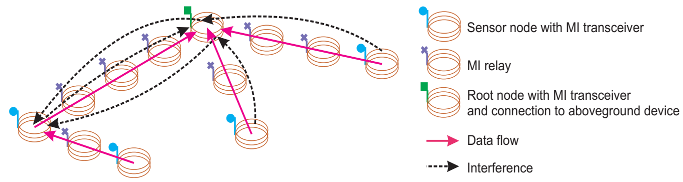

Fig. 1. Example of a tree-based network using magnetic induction.

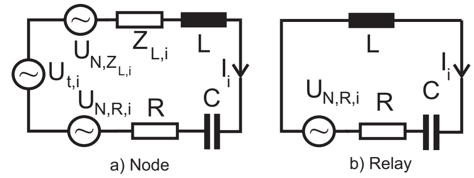

Fig. 2. Node and relay circuits.

passive elements are chosen according to [12]. We assume that all devices are deployed in a conductive environment (soil) with constant properties over space and time.

A. Advanced MI-Waveguides for WUSNs

The recently proposed single MI-waveguide system model [12] provides a good overview of the behavior of MI-based systems. However, the channel and noise models of the connected MI-waveguides in a network need to be investigated, because they may differ significantly from the channel and noise models of a single MI-waveguide. Due to many possible connections to every node, the pathloss of the transmission becomes too complicated for exact derivation. Therefore, we modify the existing channel and noise models of a single MI-waveguide by assuming that the receiver node is disconnected, such that no signal is reflected from the MI-waveguides connected to the receiver node. This approximation is meaningful, because the influence from such relays is very limited due to a high pathloss, especially after a node circuit with a matched impedance. However, the pathloss function in [12] needs to be changed accordingly, because due to the unification of the circuit elements the transmitter node circuit has additional load impedance, like the receiver node circuit. Starting with the voltage equation in the transmitter circuit and ignoring interwaveguide reflections, we obtain for link i

$$U_{t,i} = (Z + Z_L)\cdot I_{t,i} - j2\pi f M_i \cdot I_{1,i}\cdot N_{c,i}, \quad (2)$$

where $I_{t,i}$ is the current flowing in the transmitter, $I_{1,i}$ is the current in the first relay, $Z = R + j2\pi f L + \frac{1}{j2\pi f C}$, and $N_{c,i}$ is the number of MI-waveguides connected to the transmitter node. We exploit the fact that the induced current in the first relay coils is influenced by the magnetic field from the transmitter in a much stronger manner than by the relays close to the receiver due to the propagation distance. Therefore, the induced currents in the first relays are similar for all waveguides connected to the same transmitter, yielding equal induced voltage in the transmitter coil. With the results from [12], we obtain

$$\begin{aligned} I_{k_i} &= \frac{U_{t,i}}{j2\pi f M_i}\cdot\frac{1}{Q_i}, \quad (3) \\ Q_i &= (x_i + x_{L,i})\cdot S(x, x_L, k_i) - N_{c,i}\cdot S(x_i, x_{L,i}, k_i - 1), \end{aligned}$$

where $x_i = \frac{Z}{j2\pi f M_i}$, $x_{L,i} = \frac{Z_L}{j2\pi f M_i}$.
The functions $S(x_i, x_{L,i}, n) = F(x_i, n) + x_{L,i} \cdot F(x_i, n-1)$, $F(x_i, n) = x_i \cdot F(x_i, n-1) - F(x_i, n-2)$, $F(x_i, 1) = x_i$, $F(x_i, 0) = 1$ stem from [12]. I_{k_i} is the current of the k_ith coil (receiver coil of the waveguide consisting of $k_i + 1$ coils) after transmitter i. In addition to this direct signal propagation, there are signal reflections from the waveguides connected to the same transmitter (interwaveguide reflections). These reflected signals are more attenuated, due to a longer transmission route. If we assume that a signal is propagated through one of the connected waveguides c and then received by the coil n of the waveguide i the receiver is connected to, the current induced from this signal at the receiver is then approximately given by $I_{k_i,\text{refl.},n,c} = \frac{U_{t,i}}{j2\pi f M_i \cdot Q_i} \frac{1}{x'_{n,c}}$, cf. [12]. The current from all these reflected signals is

$$I_{k_i,\text{refl.}} = \frac{U_{t,i}}{j2\pi f M_i \cdot Q_i} \sum_{c=1}^{N_{c,i}-1} \sum_{n=1}^{\min(k_i,k_c)} \frac{1}{x'_{n,c}}, \quad (4)$$

where $x'_{n,c} = \frac{Z}{j2\pi f M'_n}$ with M'_n as mutual inductance between the two relays with the same place number n in the considered waveguide with the length k_i and a neighbor interfering waveguide c with the length k_c. The total number of such reflections is $\min(k_i, k_c)$ per waveguide pair and $N_{c,i} - 1$ interfering waveguides have to be accounted for per link. This results in $I_{k_i,\text{total}} = I_{k_i} + I_{k_i,\text{refl.}}$, and the pathloss can be calculated as (cf. derivations for a single waveguide in [12])

$$L_{p,i}(f) = \frac{|S(x_i, x_{L,i}, k_i) \cdot Q_i| \left| 1 + \sum_{c=1}^{N_{c,i}-1} \sum_{n=1}^{\min(k_i,k_c)} \frac{1}{x'_{n,c}} \right|}{|\text{Im}\{x_{L,i}\}|} \quad (5)$$

Because our objective is to use identical devices, the load impedances in all nodes' circuits are identical. These load impedances are therefore not exactly matched to the waveguides, which may vary in their length and other properties. The optimal system parameters for the network may differ from the optimal parameters for a particular waveguide, which leads to a decrease in magnetic induction, such that the approximation $F(x, k) \approx x^k$ holds and the matched impedance can be given by $Z_L = \text{Re}\{j2\pi f_0 M_i \cdot \frac{F(x_0,k+1)}{F(x_0,k)}\} \approx R$ with $x_0 = \frac{R}{j2\pi f_0 M_i}$ at the resonance frequency f_0.
Due to the increased number of coils in a network, the noise power at the receiver is significantly greater than for a single waveguide transmission. As discussed in [12], we assume the ambient EM waves based noise to be negligible compared to the thermal noise produced in the copper resistors of the coils. Due to a high pathloss we focus on the thermal noise produced in the waveguides, which are directly connected to the receiver of interest. We approximate the noise power of all MI-waveguides connected to one receiver node by the sum of the noise powers produced by these waveguides, when each of them is solely connected to the receiver. According to [12], every waveguide c_r of length $k_{c_r} + 1$ produces a noise power spectral density

$$\text{E}\{P_{N,c_r,R}(f)\} = \frac{1}{2} \frac{4KTRZ_L}{|j2\pi f M_{c_r}|^2} \sum_{n=0}^{k_{c_r}} |S(x_{c_r}, x_{L,c_r}, n)|^2 \times \left| \sum_{m=n}^{k_{c_r}} \frac{1}{S(x_{c_r}, x_{L,c_r}, m) S(x_{c_r}, x_{L,c_r}, m+1)} \right|^2 \quad (6)$$

Additional noise results from the load impedance in transmitter circuits, corresponding to power spectral density

$$\text{E}\{P_{N,c_r,\text{Tx}}(f)\} = \frac{1}{2} \frac{4KTZ_L^2}{|j2\pi f M_{c_r}|^2} \frac{1}{|S(x_{c_r}, x_{L,c_r}, k_{c_r}+1)|^2}, \quad (7)$$

Assuming again that the induced current in the first relay connected to the receiver is influenced by the magnetic field from the receiver coil in a much stronger manner than by the relays far away from the receiver due to the propagation distance, we calculate the power spectral density from the load impedance in the receiver circuit using (2)

$$\text{E}\{P_{N,\text{Rx}}(f)\} \approx \frac{1}{2} \frac{4KTZ_L^2}{|j2\pi f M_i|^2} \times \frac{1}{\left| x_i + x_{L,i} - \frac{S(x_i, x_{L,i}, k_i - 1)}{S(x_i, x_{L,i}, k_i)} N_r \right|^2}, \quad (8)$$

where N_r is the number of the waveguide connections of the receiver node and index i indicates the considered waveguide. The resulting total noise power spectral density at the considered receiver is then given by

$$\text{E}\{P_N(f)\} = \text{E}\{P_{N,\text{Rx}}(f) + \sum_{c_r=1}^{N_r} (P_{N,c_r,R}(f) + P_{N,c_r,\text{Tx}}(f))\}. \quad (9)$$

The channel capacity of a link is then given according to [15]

$$C_{\text{ch},i} = \int_{-\infty}^{+\infty} \log_2 \left(1 + \frac{P_{t,i}(f)}{L_{p,i}(f) \cdot \text{E}\{P_{N,i}(f)\}} \right) \text{d}f, \quad (10)$$

where $P_{t,i}(f)$ stands for the transmit power spectral density for link i and can be found via water filling. $\text{E}\{P_{N,i}(f)\}$ denotes the total noise power spectral density at the receiver of link i.

B. Interference in MI-based WUSNs

An important issue in sensor networks is scheduling of the multinode transmissions, which is necessary due to the interference [7]. As it was studied in [12], direct MI transmission provides large channel capacity only at a very low carrier frequency and a high number of windings of the coil. It can be shown that for the optimal parameters of the MI-waveguide, the channel capacity of the direct MI transmission for intercoil distances above 20 m becomes very low, because the signal power of the direct link between the coils is equal or below the power of the thermal noise. Therefore, the MI-waveguides are highly directional and the number of interferers cannot be given by the nodes inside the coverage area like it is done for the EM-waves based sensor networks [7], [11]. Moreover, the interfering signals may be propagated through the whole network, see Fig. 1, and the number of interfering nodes depends on the topology, i.e., the connected waveguides. We characterize the interferers by the number of hops away from the receiver node. However, for a given set of system parameters, only a small part of all available interferer nodes needs to be taken into account, because the resulting pathloss for the interfering signals may be very high, especially for the interferers, which are a large number of hops away from the receiver.

In [12], it was shown that a high channel capacity can be achieved although the bandwidth of MI-waveguide based transmissions is quite limited. We deduce from this fact, that the signal-to-noise ratio for such links is high. Therefore, we assume that the interference power is much higher than the noise power at the receiver. To guarantee a certain signal-to-noise-plus-interference ratio (SINR), we assume the worst case, where the distance between each two hops of the interference route is the minimum internode distance of the whole network. This enables the approximation of the interference route by an MI-waveguide of the length $k_I + 1 = m \cdot k_{\min} + 1$ coils, where m is the number of hops (involved waveguides) and $k_{\min} - 1$ is the minimum number of relays (passive coils) in an MI-waveguide in the network. The interference power can be specified by

$$P_I = \int_{f_0 - 0.5B}^{f_0 + 0.5B} \frac{P_{t,I}(f)}{L_{p,I}(f,m)} \mathrm{d}f, \tag{11}$$

where $P_{t,I}(f)$ is the power spectral density of the interference source and $L_{p,I}(f,m)$ is the pathloss of the interfering signals, which is similar to (3) and (5) and can be given by

$$\begin{aligned} L_{p,I}(f,m) &\approx \frac{\left|S(\tilde{x}, \tilde{x}_L, m \cdot k_{\min}) \cdot \tilde{Q})\right|}{|\mathrm{Im}\{\tilde{x}_L\}|} \cdot \left|2^{m-1}\right|^2, \\ \tilde{Q} &= (\tilde{x} + \tilde{x}_L) \cdot S(\tilde{x}, \tilde{x}_L, m \cdot k_{\min}) \\ &- N_c \cdot S(\tilde{x}, \tilde{x}_L, m \cdot k_{\min} - 1), \end{aligned}$$

where $\tilde{x} = \frac{Z}{j2\pi f \tilde{M}}$ and $\tilde{x}_L = \frac{Z_L}{j2\pi f \tilde{M}}$ with $\tilde{M}$ standing for the mutual inductance between coils in 3 m distance (worst case assumption). An additional weight of 2^{m-1} is due to the load impedance $Z_L = R$ in every node of the interference route, yielding $\frac{Z+Z_L}{j2\pi f M} \approx \frac{2 \cdot R}{j2\pi f M} = 2 \cdot \tilde{x}$ if $|Z - Z_L| \approx 0$. The interference power is maximal, if

$$P_{t,I}(f) = \frac{1}{L_{p,I}(f,m)} \cdot \frac{P}{\int_{f_0-0.5B}^{f_0+0.5B} \frac{1}{L_{p,I}(f,m)} \mathrm{d}f}, \tag{12}$$

with the total transmission power P per node (worst case assumption). The power of the useful signal is given by:

$$P_{S,i} = \int_{f_0 - 0.5B}^{f_0 + 0.5B} \frac{P_{t,i}(f)}{L_{p,i}(f)} \mathrm{d}f, \tag{13}$$

where $P_{t,i}(f)$ is given by the water filling algorithm used for maximizing the channel capacity in (10) and $L_{p,i}(f)$ stems from (5). The interferer nodes, which are m hops away from the receiver may not be taken into account for scheduling, if $\frac{P_{S,i}}{P_I \cdot N_i(m)} \geq \gamma$, where $N_i(m)$ is the number of nodes, which are m hops away from the receiver i, and γ is a chosen threshold for the SINR, which is 10 dB in this work.

In addition to the directional signal propagation, each node receives signals from all coils of the network due to a quasi-omnidirectional magnetic field propagation. However, due to the above arguments, those signals can be neglected in the consideration of the SINR.

III. Throughput Optimization

If every link operates on a different frequency, an individual design of all waveguide circuits (nodes and relays) for this link is needed. Each node needs then as many circuits and therefore coils, as waveguides are connected to it. Such a system becomes impractical with increasing number of nodes. Therefore, we propose to choose a set of system parameters, which are identical for all used circuits. The optimal solution maximizing the throughput of the network may depend on the topology of the network, which is discussed in the following as well. In this work we focus on spanning trees as a special case of the network topology [16]. A fully connected spanning tree is a graph, which connects multiple nodes such that one and only one route between any two nodes exists. This approach is beneficial compared to circular connected trees (with possibly more than one route between any two nodes). The circularity of the network needs to be avoided, because the old data may disturb the transmission of the new data in an unpredictable way.

A. Problem Formulation

The optimization problem can be formulated as follows:

$$\begin{aligned} &\underset{\forall f_0, N, M_{\text{links}}}{\operatorname{argmax}} \ \min_i T_i, \qquad (14) \\ \text{s.t.:} \quad &(1)\ P_i = P\ \forall i,\ (2)\ \frac{1}{(2\pi f_0)^2 L} \geq C_0, \end{aligned}$$

where f_0 is the carrier frequency (identical for all links) and N is the number of windings (equal for all used coils). M_{links} corresponds to a set of links, which form a fully connected spanning tree and i stands for a particular link of this tree. T_i is the throughput according to (1). In addition, we assume equal transmit power in all nodes (constraint 1, P_i: transmit power of ith node) and that the smallest used capacitance is bounded by C_0, cf. [12] (constraint 2).

As it is shown in [12], finding the optimal system parameters for maximizing the channel capacity of an MI-link is a non-convex problem, which cannot be solved using convex optimization tools from [17]. Because the problem in [12] is obviously a subproblem of (14), (14) is also non-convex.

B. Minimum Spanning Tree

The minimum spanning tree (MST) results from a fully connected weighted graph with the minimum sum of weights. It can be found using the iterative method of Prim [18]. Unfortunately, we cannot use (1) as weight for the graph of nodes to be connected, because this preassumes the knowledge about the network topology for the calculation of the numbers of routes and interferers. Hence, it is not possible to optimize the throughput directly. We propose to use the number of relays in MI-waveguides as weights, which minimizes the total number of used relay devices, cf. [11]. However, this approach may not maximize the throughput. It has been observed in [12] that due to a suboptimal frequency of MI-waveguides and the aforementioned capacitor constraint, even an MI-waveguide with a high relay density behaves like a weakly coupled waveguide with a small bandwidth and its pathloss function can be approximated by $|x|^{2k}$. Further approximation exploits the narrowband transmission and a relatively low level of the thermal noise, resulting in

$$C_{\mathrm{ch},i} \approx B \cdot \log_2\left(\frac{U_i}{|x_i|^{2k_i}}\right), \quad (15)$$
$$U_i = \frac{1}{B}\int_{f_0-0.5B}^{f_0+0.5B} \frac{P_{t,i}(f)}{\mathrm{E}\{P_{N,i}(f)\}}\mathrm{d}f,$$

B is the bandwidth and $x_i \approx x\ \forall i$ yielding $U_i \approx U\ \forall i$. This equation can be transformed into

$$C_{\mathrm{ch},i} \approx B\log_2(U) - 2B \cdot \log_2(|x|) \cdot k_i. \quad (16)$$

Hence, the sum of channel capacities can be expressed as

$$\sum_i C_{\mathrm{ch},i} \approx (N_{\mathrm{nodes}} - 1)B\log_2(U) - 2B \cdot \log_2(|x|) \cdot \sum_i k_i. \quad (17)$$

Therefore, by minimizing the total number of relays, which is given by $\sum_i k_i$, the sum of channel capacities over all links is maximized. It has been shown in [19], that the MST minimizes not only the sum of the weights, but also the maximum weight occuring in the tree. Correspondingly, the minimum link capacity is maximized with the same approach. In this work we assume a uniform distribution of the nodes, which yields a uniform distribution of the routes and interferers, yielding similar values among all links for N_{routes} and $N_{\mathrm{interferers}}$. The problem (14) can be then split into two subproblems:

1) $\underset{f_0,N}{\operatorname{argmax}}\, Y(f_0, N)$, (18)
2) $Y(f_0, N) = V(f_0, N) - W(f_0, N) \cdot \min_{M_{\mathrm{links}}} \max_i k_i$, (19)

where $V(f_0, N) = \frac{B\cdot\log_2(U)}{N_{\mathrm{routes}}\cdot(1+N_{\mathrm{interferers}})}$ and $W(f_0, N) = \frac{2B\cdot\log_2(|x|)}{N_{\mathrm{routes}}\cdot(1+N_{\mathrm{interferers}})}$. If the network tree is a weighted graph and its weights are set to k_i, then the solution for (19) is given by the MST. Hence, this approach is an optimal solution for the given assumptions. The subproblem (18) is solved using a full search in the $\{f_0, N\}$-space, see Section III-D.

C. Advanced Spanning Tree

The previous discussion on the MST is based on the approximations of the channel capacity and that of the number of interferers and routes in a network. However, due to a random distribution of nodes and further optimization of the system parameters, there might be cases, when the numbers of interferers and routes are not equal for all nodes and also cases, when the system is operated at frequencies, at which the above approximations are only partially valid. Then, the MST becomes a suboptimal approach, which performs, however, still very close to the optimum. The optimal solution for the topology can be given by performing a full search over all possible fully connected spanning trees for the given node positions. However, the corresponding effort becomes very high with increasing numbers of nodes. Therefore, we propose an iterative algorithm, which finds the optimal solution with significantly reduced complexity.

The starting point for the algorithm is an MST. For the initial calculation of the MST, every node is allowed to be connected to every other node. We calculate the throughput for all links of the system, list them in increasing order, and store their indices in lists L_c and $L_{c,\min}$. In addition, an empty list $L_{\mathrm{forbidden}}$ is created in order to save the outliers. For the following steps, $L_{c,\min}$ remains unchanged, because it is used as a reference for the extended search, see below.

In each iteration, the first link from L_c is taken. In the given constellation this link is not only disturbed by a high amount of interference and loaded by a high number of information streams, due to the symmetry, it also provides interference to a high number of nodes and loads its direct receiver node with a high number of streams to be served. Therefore, this link is said to be the most disturbing one. Hence, we exclude it from the MST finding procedure by setting the number of relays between the corresponding nodes to infinity and saving the link's index in $L_{\mathrm{forbidden}}$. This link is then avoided by the Prim's algorithm. Then, we calculate a new MST and L_c. If the minimum throughput of the new tree is higher than the highest minimum throughput so far, the new tree is stored as a candidate for the optimal solution.

This strategy usually leads to an increase of the network throughput. However, the resulting tree depends on the choice of the first forbidden link, which determines all further steps of the algorithm. Since it may happen that the worst link by means of the network throughput is not the most disturbing one (due to e.g. low channel capacity and low number of the relevant interferers and routes), we also need to investigate the cases, where the algorithm starts with any other link as a forbidden connection. Hence, if the number of forbidden connections, which belong to the same node and are stored in $L_{\mathrm{forbidden}}$, is higher than X_f (e.g. $X_f = 5$ in this work), $L_{\mathrm{forbidden}}$ is cleared and the second element from $L_{c,\min}$ is taken as the first forbidden link. The optimization terminates after X_I iterations or if all elements from $L_{c,\min}$ have been used once as the first forbidden link. The stored tree with the highest network capacity is returned. We call this method an advanced spanning tree (AST) approach.

D. Optimal System Parameters

As it was shown in [12], system parameters like f_0 and N need to be optimized to achieve the maximum channel capacity. In [12] this optimization is performed using a multiscale search in the two-dimensional parameter space. However, this algorithm is time consuming and inaccurate, because f_0 is a continuous variable.

Since this work is focused on establishing a communication between the nodes using MI-waveguides, we exploit the property of the waveguides, that the optimization under the capacitor constraint according to [12] leads to a significant degradation of the channel capacity. Moreover, it was shown

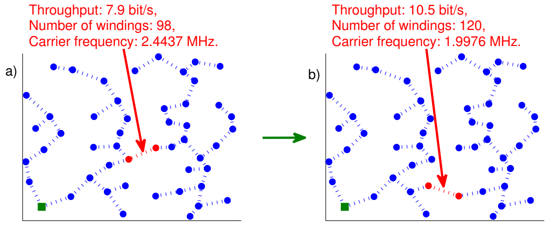

Fig. 3. Examples of WUSNs using MST and AST approaches.

that with increasing carrier frequency and/or increasing number of windings the channel capacity increases monotonically. Therefore, the optimal solution meets the capacitor constraint with equality. Hence, we can express the optimum f_0 as a function of N:

$$C_0 = \frac{1}{(2\pi f_0)^2 L(N)} \Rightarrow f_0 = \frac{1}{2\pi\sqrt{L(N)C_0}}, \qquad (20)$$

where $L(N)$ indicates that the inductivity L depends on N. With this information, the optimization problem for MI-waveguides in [12] and (18) can be solved using a full search in one integer variable N.

IV. Simulation Results

In this section, we discuss numerical results for the network throughput. In our simulations, we assume a total transmit power of P = 10 mW per node. Furthermore, we utilize coils with wire radius 0.5 mm and coil radius 0.15 m. The conductivity and permittivity of dry soil are, respectively, σ = 0.01 S/m and $\epsilon = 7\epsilon_0$, where $\epsilon_0 \approx 8.854 \cdot 10^{-12}$ F/m. Since the permeability of soil is close to that of air, we use $\mu = \mu_0$ with the magnetic constant $\mu_0 = 4\pi \cdot 10^{-7}$ H/m. As proposed in [11], we assume that the coil axes are turned to the ground surface. This enables an omnidirectional communication range of the node coils [11], which is needed to connect two or more MI-waveguides with different waveguide axes directions. In most applications of WUSNs, the density of sensor nodes needs to be high to ensure that a meaningful data collection can be made. However, as it was shown in [3], [12], with a small distance between two nodes, MI based transmission is outperformed by EM waves based transmission. Therefore, we restrict the minimum distance between each two nodes to be at least 21 m, which corresponds to $k_i \geq 7$ with a relay density of k_i/d = 1/(3 m). We assume a square field of the size $F_x \times F_x$. Within this field, a random uniformly distributed set of N_{nodes} sensor nodes is acquired for each network optimization. In this set, a root node is selected, which is the closest node to the lower left field corner. Fig. 3a) shows an example of a tree based sensor network with 50 nodes and $F_x = 300$ m, which is established using MST. The bottleneck of this network is marked with an arrow. Due to a high number of relevant interferers and information streams, the resulting throughput is very low even with the optimized system parameters. In the further optimization steps from Section III-C, this connection is avoided and an other connection is found, which is more beneficial, see Fig. 3b). Also, the system parameters change according to this new bottleneck. A throughput gain results.

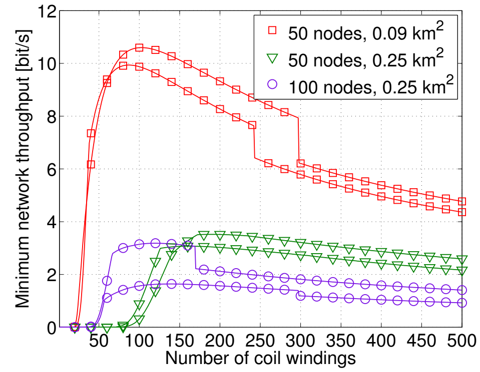

Fig. 4. Examples of the network throughput in system parameter space.

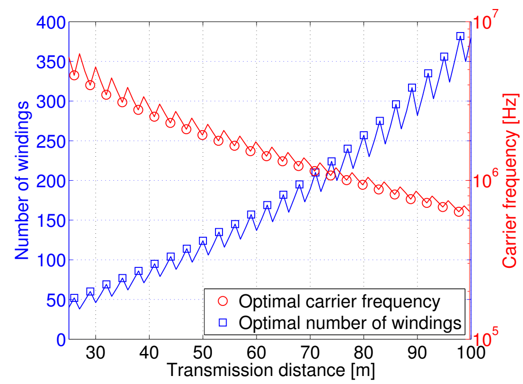

Fig. 5. Optimal system parameters for MI-waveguides with C_0 = 1 pF.

In order to examine the properties of the optimum solution we show examples of the minimum network throughput for different network constellations over the integer variable N, which corresponds to a carrier frequency, as discussed in Section III-D, see Fig. 4. This solution is obtained using the MST. Obviously, with increasing number of nodes or increasing size of the deployment field, the minimum network throughput decreases, which is expected according to the theoretical investigations in [7] and the scaling law from [11]. In all curves we observe points of discontinuity, which are due to the enhanced interference power yielding additional relevant interfering signals, which violate the SINR $\geq \gamma$ constraint. The optimal solution for networks with 50 nodes and F_x = 300 m results for lower values of N than for networks with 50 nodes and F_x = 500 m. This can be explained by estimating the average length of the worst link, which increases with the size of the deployment field. Fig. 5 shows the optimal parameters for the waveguides with different lengths according to [12]. Here, with increasing length of the waveguides, the optimal value of N increases as well.

Our investigations have shown that for ≈50% of the considered random sensor networks, no throughput gain for AST compared to MST can be achieved, because the MST approach is already optimal. For the remaining 50%, the expectation

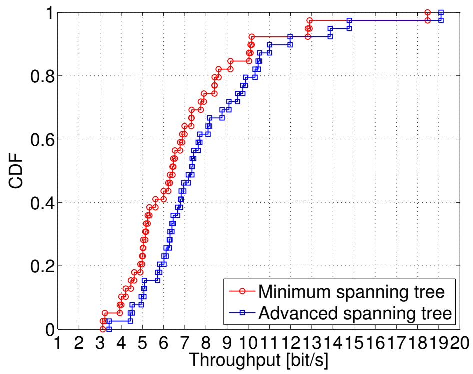

Fig. 6. CDF of the minimum throughput of random networks.

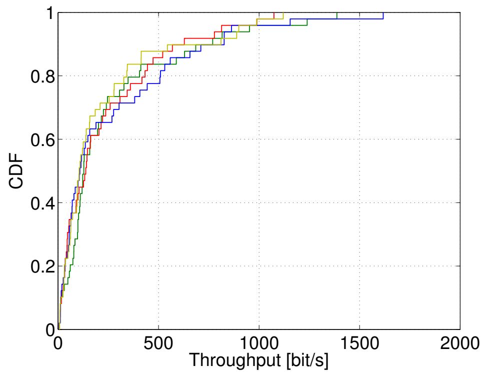

Fig. 7. Examples on CDF for network throughput of all links.

value for the throughput gain is ≈17.5% with 57% peak gain. Fig. 6 shows the cumulative distribution function of the throughput for such cases for $F_x = 300$ m. We observe a very low throughput for the MI-waveguides based WUSNs, which is due to a high number of interferers as discussed before, high numbers of noise sources, and a relatively low channel capacity due to a capacitor constraint and a conductivity-based loss in the medium [12].

Although the objective of this work is to maximize the minimum throughput, it is also interesting to consider the potential of the MI-waveguides based WUSNs by investigating a cumulative distribution function for the throughput of all network links, see Fig. 7. We observe that a big portion of the links achieves a throughput above 200 bit/s. This lets us conclude that this system can be still improved by means of e.g. energy harvesting, which could level out the difference between the minimum and the maximum network throughput. Then, the worst link receives more energy from the scheduled interfering signals and can adjust its transmit power accordingly, thus improving the channel capacity. The increased power at the receiver would also improve the SINR and reduce the number of the relevant interferers. However, this approach is out of the scope of this work and remains to be investigated. Also, the efficiency of a large number of passive relays in MI-waveguides for WUSNs still has to be analyzed.

V. Conclusion

In this paper we derived channel, noise, and interference models for WUSNs based on MI-waveguides. These models differ from the single MI-waveguide characteristics according to [12] due to additional couplings between relay coils of the MI-waveguides, which are connected to the same nodes. Taking into account the different lengths of the used MI-waveguides, an optimized set of system parameters is found, which maximizes the throughput of the network. In addition, a routing problem is considered, which is solved using an iterative algorithm based on a minimum spanning tree approach. Furthermore, the properties of the optimization process are discussed and potential improvements of the network throughput by using e.g. energy harvesting have been mentioned.

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