OPERA: Optimal Routing Metric for Cognitive Radio Ad Hoc Networks

Marcello Caleffi, *Member, IEEE,* Ian F. Akyildiz, *Fellow, IEEE,* and Luigi Paura, *Member, IEEE*

***Abstract*—Two main issues affect the existing routing metrics for cognitive radio ad hoc networks: i) they are often based on heuristics, and thus they have not been proved to be optimal; ii) they do not account for the route diversity effects, and thus they are not able to measure the actual cost of a route. In this paper, an *optimal routing metric* for cognitive radio ad hoc networks, referred to as OPERA, is proposed. OPERA is designed to achieve two features: i) *Optimality*: OPERA is optimal when combined with both Dijkstra and Bellman-Ford based routing protocols; ii) *Accuracy*: OPERA exploits the route diversity provided by the intermediate nodes to measure the actual end-to-end delay, by taking explicitly into account the unique characteristics of cognitive radio networks. A closed-form expression of the proposed routing metric is analytically derived for both static and mobile networks, and its optimality is proved rigorously. Performance evaluation is conducted through simulations, and the results reveal the benefits of adopting the proposed routing metric for cognitive radio ad hoc networks.**

***Index Terms*—Cognitive radio, routing metric, optimality, route diversity, mobility.**

I. INTRODUCTION

COGNITIVE Radio (CR) paradigm has been proposed as a viable solution to counteract both spectrum inefficiency and spectrum scarcity problems. The CR paradigm exploits the concept of *spectrum hole*: a CR user is allowed to use a spectrum band licensed to a primary user (PU) when it is temporarily unused. The CR paradigm can be applied to ad hoc scenarios, and the resulting networks, referred to as CR Ad Hoc Networks (CRAHNs) [1], are composed by CR users that exploit the spectrum holes for establishing multi-hop communications in a peer-to-peer fashion. To fully unleash the potentials of such networks, new challenges must be addressed and solved at the network layer. In particular, effective routing metrics able to account for the distinguishable properties of the CR paradigm are needed [2].

The routing metrics for CRAHNs can be classified in two classes: a) the metrics originally proposed for multi-channel environments and then adapted to CR networks; b) the metrics specifically designed for CR networks. All the routing metrics in the first class, due to their origin, cannot fully account for the spectrum dynamics introduced by the PU activity. Indeed, in a CR network the routes of the CR users are highly affected by the spectrum dynamics, therefore these metrics fail in discovering the optimal route. The routing metrics in the second class, namely, the metrics explicitly designed for CR networks, either they are not optimal or they are not able to measure the actual path cost. We describe both the two drawbacks in the following.

A. Metric Optimality

A routing metric is a function that assigns a weight (i.e., a cost) to any given path [3]. A routing metric is defined *optimal* when *there exists an efficient shortest-path algorithm based on such a metric that always discovers the lowest-weight route between any pair of nodes in any connected network.*

We explain the concept of optimality with an example of a metric that is not optimal when combined with Belmann-Ford (or Dijkstra) shortest-path algorithm: the Weighted Cumulative Expected Transmission Time (WCETT), proposed in [4] to account for the multi-channel inter-flow interference. For a path p, WCETT is defined as:

$$WCETT(p) = (1-\beta) \sum_{\text{link } l \in p} \text{Delay}(l) + \beta \max_{\text{channel } j} X_j$$

where β is a tunable parameter, $\text{Delay}(l)$ denotes the transmission delay for link l, and X_j captures the inter-flow interference by counting the number of times that channel j is used along the path. Since WCETT metric is not optimal, there is no efficient shortest-path algorithm able to always discover the lowest-WCETT route.

Fig. 1 shows a simple network in which the Belmann-Ford algorithm based on WCETT fails in discovering the optimal route for $\beta > 4/9$. More in detail, if we assume $\beta = 2/3$, node u_1 announces the route u_1-u_2-u_d (with a cost $0.4/3 + 2/3$) as the optimal route to reach u_d, since it is preferred to the route u_1-u_d (with a cost $0.476/3 + 2/3$). Hence, node u_s does not have any chance to check the weight of the route u_1-u_d, and it incorrectly sets the sub-optimal route u_s-u_3-u_4-u_5-u_d (with a cost $1.7/3 + 2/3$) as the route to u_d, since it is preferred to the route u_s-u_1-u_2-u_d (with a cost $0.9/3 + 4/3$). However, the route u_s-u_1-u_d (with a cost $0.976/3 + 2/3$) is the actual

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M. Caleffi was with the Broadband Wireless Networking Laboratory, School of Electrical and Computer Engineering, Georgia Institute of Technology, USA. He is now with the Dept. of Biomedical, Electronics and Telecommunications Engineering, University of Naples Federico II, Italy (e-mail: marcello.caleffi@unina.it).

I. F. Akyildiz is with the Broadband Wireless Networking Laboratory, School of Electrical and Computer Engineering, Georgia Institute of Technology, USA (e-mail: ian.akyildiz@ee.gatech.edu).

L. Paura is with the Department of Biomedical, Electronics and Telecommunications Engineering, University of Naples Federico II, Italy (e-mail: paura@unina.it).

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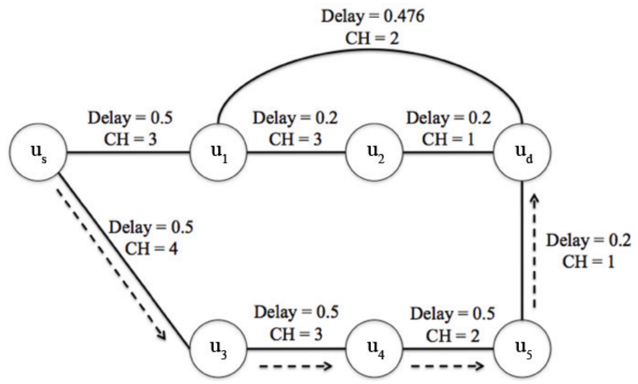

Fig. 1. Metric Optimality: WCETT metric fails in finding the shortest path between u_s and u_d whenever $\beta > 4/9$, since node u_s sets u_s-u_3-u_4-u_5-u_d as the optimal route, although u_s-u_1-u_d is the actual optimal one.

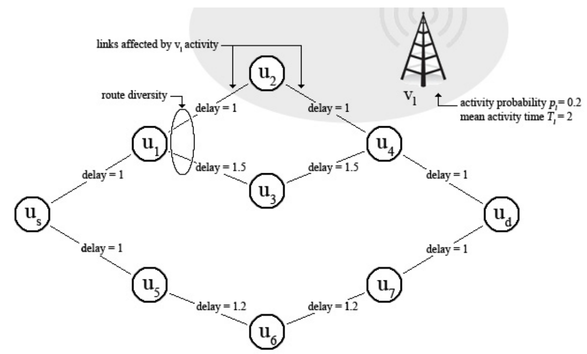

Fig. 2. Metric Accuracy: a metric that neglects the route diversity measures a delay equal to 4.5 for the route u_s-u_1-u_2-u_4-u_d, although the actual delay for the combined route u_s-u_1-(u_2 when v_l is not active, u_3 otherwise)-u_4-u_d is 4.2.

optimal route. A similar issue occurs when Dijkstra algorithm is applied.

The lack of the optimality property is not trivial: regardless of the adopted routing procedure, the packets can be routed either through sub-optimal routes, wasting the network resources, or even worse through route loops, causing unreachable destinations. Clearly, these issues become more severe in CR networks.

B. Metric Accuracy

A routing metric is defined *accurate* when *it always measures the actual route cost between every pair of nodes in any connected network*.

Most of the metrics explicitly designed for cognitive radio networks measure the cost of a route only with reference to the routing opportunities available when the route is not disconnected by the PU activity. Thus, they neglect the additional routing opportunities provided by the route diversity, failing so to measure the actual quality of a route.

We illustrate this issue with the example in Fig. 2, where three routes exist between the source u_s and the destination u_d: i) the 1st route is composed by the intermediate nodes u_1-u_2-u_4, and it is affected by the activity of PU v_l with activity probability $p_l = (1 - \bar{p}_l) = 0.2$ and with mean activity time $T_l = 2$; ii) the 2nd route is composed by the intermediate nodes u_1-u_3-u_4; iii) the 3rd route is composed by the intermediate nodes u_5-u_6-u_7.

If we neglect the route diversity, we have the following delays for each route: i) 1st route[1]: delay $= 4.5$; ii) 2nd route: delay $= 5$; iii) 3rd route: delay $= 4.4$. As a consequence, a metric that does not account for the route diversity would (incorrectly) announce the 3rd route as the lowest cost route, and u_s will forward the packets through the intermediate node u_5, regardless from the adopted routing procedure.

Differently, if we consider the effects of the route diversity in measuring the delay of the 1st route, we have also to consider the additional routing opportunities provided by the intermediate node u_3 when v_l is active. Thus, we have that the delay of the route u_1-(u_2 when v_l is not active, u_3 otherwise)-u_4, is[2] 4.2. As a consequence, a metric that accounts for the route diversity would (correctly) announce such a route as the lowest cost route, and u_s would forward the packets through the intermediate node u_1, i.e., through the combined route with the actual lowest cost.

The lack of the accuracy property is not trivial in CR networks: if the metric overestimates the route cost, the packets can be routed through sub-optimal routes, wasting the network resources.

In this paper, we propose a novel CR routing metric, called *Optimal Primary-aware routE quAlity* (OPERA), with the objective to overcome both the issues mentioned above: un-optimality and un-accuracy.

More specifically, OPERA is designed to achieve two distinguishable features: i) *Optimality*: OPERA is optimal when combined with both Dijkstra and Bellman-Ford based routing protocols; ii) *Accuracy*: OPERA exploits the route diversity provided by the intermediate nodes to measure the actual end-to-end delay of a route, by taking explicitly into account the unique characteristics of cognitive radio networks.

We analytically derive closed-form expressions of OPERA metric for static and mobile networks, and the optimality of OPERA is analytically proved by means of the routing algebra theory. Performance evaluation is conducted through simulations, and the results reveal the benefits of adopting the proposed routing metric for cognitive radio ad hoc networks.

The rest of the paper is organized as follows. Sec. II discusses related work. Sec. III describes the network model,

[1]The delay of the 1st route is the sum of two terms: i) the delay introduced by links u_s-u_1 and u_4-u_d, which is always equal to 2; ii) the delay introduced by links u_1-u_2 and u_2-u_4, which depends on v_l activity and grows linearly with T_l for each unsuccessful transmission. As a consequence, the delay of the 1st route is equal to:

$$\text{delay} = 2 + \bar{p}_l 2 + p_l \bar{p}_l (2 + T_l) + p_l^2 \bar{p}_l (2 + 2T_l) + \ldots =$$
$$= 2 + 2\,\bar{p}_l \sum_{n=0}^{\infty} p_l^n + T_l\,\bar{p}_l \sum_{n=0}^{\infty} p_l^n n = 2 + \left(2 + T_l \frac{p_l}{1 - p_l}\right)$$

[2]The delay of the combined route is the sum of three terms: i) the delay introduced by links u_s-u_1 and u_4-u_d, which is always equal to 2; ii) with probability $\bar{p}_l$, the delay introduced by links u_1-u_2 and u_2-u_3, which is equal to 2; iii) with probability p_l, the delay introduced by links u_1-u_3 and u_3-u_4, which is equal to 3. As a consequence, if we neglect the additional delay introduced by packet queueing at u_2, the delay of the route is equal to: delay $= 2 + \bar{p}_l 2 + p_l 3 = 4.2$.

whereas in Sec. IV the closed-form expression of OPERA is derived for both static and mobile networks. In Sec. V we prove the optimality of OPERA, and in Sec. VI we evaluate its performance. Finally, conclusions are drawn in Sec. VII and some proofs are gathered in the appendix.

II. Related Work

As mentioned in Sec. I, the routing metrics for CRAHNs can be classified as follows: i) the metrics originally proposed for multi-channel environments and then adapted to CR networks; ii) the metrics specifically designed for CR networks.

With reference to the metrics belonging to the first class, the works [5], [6], [7] propose to measure the quality of a route in terms of delay. The expressions of the metrics account for different delay components related to multi-channel environments, such as the channel switching delay or the link switching delay. However, none of them accounts for the spectrum dynamics, and thus they fail in describing the characteristics of a CR environment. Differently, OPERA is designed by explicitly accounting for the spectrum dynamics related to the PU activity and for the sensing process characteristics.

With reference to the metrics specifically designed for CR networks, the authors in [8] propose a metric that combines the link stability and the switching delays. However, it is possible to prove that the proposed metric is not optimal since it is not isotonic [9]. The paper [10] introduces a CR metric based on the route stability. The stability of a route is defined as a function of the route maintenance cost, which is measured in terms of channel switching and link switching delays. The paper [11] proposes a routing metric and a routing protocol to achieve high throughput efficiency, by allowing the CR users to opportunistically transmit according to the concept of spectrum utility. The authors consider several problems, such as spectrum and power allocation and distributed routing, and the proposed algorithm is proved to be computationally efficient and with bounded BER guarantees. In [12], the authors perform an asymptotic analysis of the capacity and delay for cognitive radio networks, by comparing shortest-path routing, multi-path routing and network coding techniques. However, they do not provide any heuristic or theory for measuring the quality of a route. In [13], the authors propose an opportunistic path metric that accounts for the route diversity effects. However, neither they provide a closed form expression or they prove the optimality of their metric. In [14] the authors propose a novel CR metric based on path stability and availability over time, while in [15], the authors propose a routing metric that aims to minimize the interference caused by the CR users to the PUs. Unlike OPERA, none of the cited works jointly addresses the optimality and accuracy issues.

III. Network Model

We model the cognitive network with a direct temporal graph:

$$G(t) = (U, E(t)) \tag{1}$$

in which the spectrum is organized in N distinct bands (channels), a vertex $u_i \in U$ denotes a CR user, and an edge $e_{ij}(t) \in E(t)$ denotes the presence of at least one communication link $e_{ij}^m(t)$ from CR user u_i to CR user u_j at time t through the spectrum band m:

$$e_{ij}(t) = 1 \Longleftrightarrow \exists\, m \in \{1, \ldots, N\} : e_{ij}^m(t) = 1 \tag{2}$$

Each link $e_{ij}^m(t)$ is characterized by the *link throughput* $\psi_{ij}^m(t)$, which measures the amount of information for unit time that u_i can successfully transmits to u_j through the m-th channel at time t. The link throughput jointly accounts for several factors, such as the channel bandwidth, the channel spectral efficiency, the wireless propagation conditions, the channel contention delays, the node queueing delays, etc.. The expected value of $\psi_{ij}^m(t)$ at time t is denoted with $\bar{\psi}_{ij}^m(t)$, and the CR users are able to estimate the expected values, i.e., through the past channel-throughput history.

The CR users, equipped with a single radio interface, are assumed heterogeneous, namely, they can have different transmission ranges. Also the PUs, denoted with $v_l \in \mathcal{V}$, are assumed heterogeneous, namely, they can have different transmission ranges, activity probabilities p_l, and mean activity times. The CR user time is organized into fixed-sized slots of duration T, as shown in Fig. 3[3]. Each time slot T is further organized in a *sensing period* T_s, which measures the portion of time slot assigned to spectrum sensing, and in a *transmission period* T_{tx}, which measures the portion of time slot devoted to unlicensed access to the licensed spectrum for packet transmission.

In the following we give some definitions adopted through the paper.

Definition 1. (Interfering Set) The *interfering set* $\mathcal{V}_i^m(t) \subseteq \mathcal{V}$ is the set of PUs that can prevent CR user u_i either to receive or transmit through the m-th channel at time t[4]:

$$\mathcal{V}_i^m(t) = \{v_l \in \mathcal{V} : ||u_i - v_l||(t) < \min\{R_{u_i}, R_{v_l}\}\} \tag{3}$$

where $||\cdot||(t)$ denotes the Euclidean distance at time t, and R_{u_i} and R_{v_l} denote the interference range of u_i and v_l, respectively. Similarly, we denote with $\mathcal{V}_{ij}^m(t) = \mathcal{V}_i^m(t) \cup \mathcal{V}_j^m(t)$ the set of PUs whose activities can interfere with the communications through link e_{ij}^m at time t.

Definition 2. (Throughput Sequence) The *throughput sequence* $(\tau_{ij}^1(t), \tau_{ij}^2(t), \ldots, \tau_{ij}^N(t))$, with $\tau_{ij}^m(t) \in \{1, \ldots, N\}$, is the sequence of channels ordered according to the decreasing expected link throughput $\bar{\psi}_{ij}^m(t)$ at time t:

$$\bar{\psi}_{ij}^{\tau_{ij}^m(t)}(t) \geq \bar{\psi}_{ij}^{\tau_{ij}^{m+1}(t)}(t) \tag{4}$$

Definition 3. (Link Availability Probability) The *link availability probability* $p_{ij}^m(t)$ is the probability of link e_{ij}^m being not affected by PU activity at time t[5], i.e., the probability that

[3] In the following, we assume for the sake of simplicity that the slot periods of the CR users are synchronized. Nevertheless, the results presented through the paper can be easily extended to the case of asynchronous slot periods.

[4] The rationale for adopting a symmetric interference model is that, in a multi-hop environment, a CR user is expected to forward the traffic received by other CR users. Thus, both the transmission and the reception functionalities are required.

[5] We note that the time dependence of $p_{ij}^m(t)$ is due to the PU and/or CR user mobility, which can affect $\mathcal{V}_{ij}^m(t)$ in time.

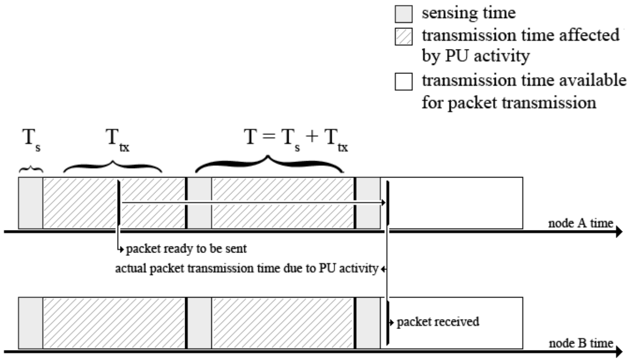

Fig. 3. Link Delay for packet transmission due to the presence of PU activity.

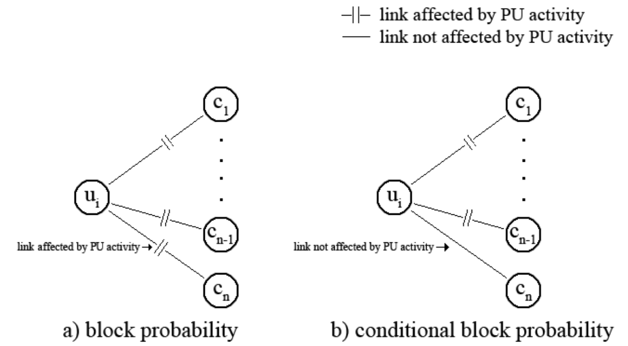

Fig. 4. Difference between Block Probability and Conditional Block Probability.

no PU belonging to the interfering set $\mathcal{V}_{ij}^m(t)$ is active at time t:

$$p_{ij}^m(t) \triangleq P(E_{ij}^m(t) = 1) = \prod_{v_l \in \mathcal{V}_{ij}^m(t)} (1 - p_l) \qquad (5)$$

where $E_{ij}^m(t)$ is the random process that models the PU activity on link e_{ij}^m, namely, $E_{ij}^m(t) = 1$ denotes the event 'link e_{ij}^m is not affected by PU activity at time t', and $\bar{p}_{ij}^m(t) \triangleq 1 - p_{ij}^m(t)$.

Definition 4. (Block Probability) Given a CR user u_i and an ordered set $C = \{c_1, \ldots, c_n\}$ of neighbors of u_i, the *block probability* $\bar{z}_{iC_n}^m(t)$ is the probability of the links $e_{ic_1}^m, \ldots, e_{ic_n}^m$ being affected by PU activity at time t:

$$\bar{z}_{iC_n}^m(t) \triangleq P\left(E_{ic_1}^m(t) = 0, \ldots, E_{ic_n}^m(t) = 0\right) \qquad (6)$$

Remark. (Independent PU Activity Condition) We observe that, if the random variables $\{E_{ic_j}^m(t)\}_{j=1}^n$ modeling the PU activity at time t are independent each other, we have that:

$$\bar{z}_{iC_n}^m(t) = \prod_{j=1}^{n} \bar{p}_{ic_j}^m(t) \qquad (7)$$

Definition 5. (Conditional Block Probability) Given a CR user u_i and an ordered set $C = \{c_1, \ldots, c_n\}$ of neighbors of u_i, the *conditional block probability* $\bar{z}_{i\setminus C_n}^m(t)$ is the probability of the links $e_{ic_1}^m, \ldots, e_{ic_{n-1}}^m$ being affected by PU activity at time t, given that the link $e_{ic_n}^m$ is not affected by PU activity at time t:

$$\bar{z}_{i\setminus C_n}^m(t) \triangleq P\left(E_{ic_1}^m(t) = 0, \ldots, E_{ic_{n-1}}^m(t) = 0 | E_{ic_n}^m(t) = 1\right) \qquad (8)$$

Remark. Fig. 4 highlights the difference between the *block probability* and the *conditional block probability*, which concerns link $e_{ic_n}^m$. In fact, the first probability measures the probability of all the n links ($e_{ic_n}^m$ included) being affected by PU activity. Differently, the conditional probability measures the probability of the first $n-1$ links being affected by PU activity, given that the n-th link $e_{ic_n}^m$ is not affected by PU activity.

In the following, we consider two scenarios: i) *static networks*; ii) *mobile networks*. In static networks, both the CR users and the PUs are static; as a consequence, the parameters e_{ij}^m, E_{ij}^m, $\bar{\psi}_{ij}^m$, p_{ij}^m, $\bar{z}_{iC_n}^m$, and $\bar{z}_{i\setminus C_n}^m$ do not depend on the time. In mobile networks, the CR users and/or the PUs are mobile.

IV. OPERA: Optimal Routing Metric

In this section we define OPERA and we analytically derive its expression, first with reference to static networks (Sec. IV-A), then with reference to mobile networks (Sec. IV-B).

A. OPERA for Static Networks

In this sub-section, we derive the expression of OPERA with reference to static networks (Theorem 1). To this aim, we first present Lemma 1.

Lemma 1. *The expected link delay l_{ij} experienced by a packet sent by u_i to the neighbor u_j is given by:*

$$l_{ij} = \frac{1}{1 - q_{ij}^N} \left(\sum_{m=1}^{N} q_{ij}^{m-1} p_{ij}^{\tau_{ij}^m} \frac{L}{\bar{\psi}_{ij}^{\tau_{ij}^m}} + q_{ij}^N T \right) \qquad (9)$$

where L is the packet length, τ_{ij}^m is defined in (4), $p_{ij}^{\tau_{ij}^m}$ *is defined in* (5), *and q_{ij}^m is given by:*

$$q_{ij}^m = \begin{cases} 1 & \text{if } m = 0 \\ \prod_{n=1}^{m} \bar{p}_{ij}^{\tau_{ij}^n} & \text{otherwise} \end{cases} \qquad (10)$$

Proof: See Appendix A. ■

Remark. The *expected link delay* (9) allows us to estimate the delay for a packet sent over a link by accounting for two main factors that affect the transmission of a packet over a CR link: i) the PU characteristics, via the probabilities $p_{ij}^{\tau_{ij}^m}$ and q_{ij}^m; ii) the sensing process characteristics, via the time parameter T. More in detail, the *expected link delay* is function of the weighted sum of two delay terms: i) the delay for packet transmission, which depends on the inverse of the channel capacity $\bar{\psi}_{ij}^{\tau_{ij}^m}$; ii) the delay introduced by PU activity, which depends on the time parameter T. The delay introduced by PU activity is weighted by the probability q_{ij}^N, which is the probability of at least one PU being active on each available channel.

Remark. By ordering the channels according to the decreasing expected link throughput $\bar{\psi}_{ij}^{m}$, the *expected link delay* expression (9) computes the actual minimum expected delay, as proved in [16].

By means of Lemma 1, we can now derive the expression of OPERA for static networks.

Theorem 1. ***(OPERA for static networks)*** *The expected end-to-end delay $\mathcal{D}_{ik}(\mathcal{C})$ for the source-destination (u_i, u_k) through the ordered set $\mathcal{C} = \{c_j\}_{j=1}^{n}$ of neighbors of u_i is given by:*

$$\mathcal{D}_{ik}(\mathcal{C}) = \frac{1}{1-\bar{q}_{ic_n}^{N}} \left(\sum_{j=1}^{n} \sum_{m=1}^{N} \bar{q}_{ic_j}^{m-1} p_{ic_j}^{\tau_{ic_j}^{m}} \left(\frac{L}{\bar{\psi}_{ic_j}^{\tau_{ic_j}^{m}}} + \mathcal{D}_{c_j k} \right) + + \bar{q}_{ic_n}^{N} T \right) \tag{11}$$

where:

$$\bar{q}_{ic_j}^{\tilde{m}} = \prod_{m=1}^{\tilde{m}} \bar{z}_{iC_j}^{\tau_{ic_j}^{m}} \bar{z}_{i\setminus C_j}^{\tau_{ic_j}^{\tilde{m}+1}} \prod_{m=\tilde{m}+2}^{N} \bar{z}_{iC_{j-1}}^{\tau_{ic_j}^{m}} \tag{12}$$

and where L is the packet length, $\tau_{ic_j}^{m}$ is defined in (4), $p_{ic_j}^{\tau_{ic_j}^{m}}$ is defined in (5), $\bar{z}_{iC_j}^{\tau_{ic_j}^{m}}$ is defined in (6), $\bar{z}_{i\setminus C_j}^{\tau_{ic_j}^{m}}$ is defined in (8), $\mathcal{D}_{c_j k}$ is the minimum expected end-to-end delay for the couple source destination (c_j, u_k).

Proof: See Appendix B. ■

Remark. Similarly to the *expected link delay* (9), the expression (11) of OPERA explicitly accounts for two main factors that affect the transmission of a packet over a CR link: i) the PU characteristics, via the probabilities $p_{ic_j}^{\tau_{ic_j}^{m}}$, $\bar{q}_{ic_j}^{\tilde{m}}$, $\bar{z}_{iC_j}^{\tau_{ic_j}^{m}}$, and $\bar{z}_{i\setminus C_j}^{\tau_{ic_j}^{m}}$; ii) the sensing process characteristics, via the time parameter T. Unlike (9), the delay for packet transmission in OPERA is defined as the inverse of the channel capacities $\bar{\psi}_{ic_j}^{\tau_{ic_j}^{m}}$ toward forwarder c_j plus the *expected end-to-end delay* $\mathcal{D}_{c_j k}$ from c_j to the final destination u_k. The delay introduced by PU activity (T) is weighted by the probability $\bar{q}_{ic_n}^{N}$ that represents the probability that all the available channels towards all the forwarders $c_1, \ldots, c_n$ are affected by PU activity. Clearly, the *expected end-to-end delay* depends on the set of forwarders $\mathcal{C}$; with Theorem 3 we give the optimal choice for the set of forwarders $\mathcal{C}$.

B. OPERA for Mobile Networks

In this sub-section, we derive (Theorem 2) the expression of OPERA with reference to mobile networks. Since the proof of Theorem 2 requires three definitions, Definition 6-8, and one lemma, Lemma 2, we first present these intermediate results.

In this scenario, the expected end-to-end delay can change dramatically due to the effect of the PU and/or CR user mobility. Therefore, OPERA measures the expected end-to-end delay by explicitly accounting for the effect of the relative movement between two CR users (13) and between a CR user and a PU (14). Moreover, OPERA measures the delay with reference to the time interval δ, which represents the route update period. In such a way, OPERA can be efficiently adopted by any routing protocol based on periodic route updates.

Definition 6. (Expected Link Utilization) The *expected link utilization* factor $\rho_{ij}(t,n)$ measures the fraction of the time interval $[t+nT, t+(n+1)T)$ during which u_i and u_j are able to communicate, measured at time t:

$$\rho_{ij}(t,n) = \begin{cases} 1 & \text{if } t_{ij}(t,n) = T \\ t_{ij}(t,n)/T & \text{if } 0 \leq t_{ij}(t,n) < T \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

where $t_{ij}(t,n)$ is the expected cumulative contact time period between u_i and u_j in time interval $[t+nT, t+(n+1)T)$, measured at time t.

Definition 7. (Expected Link Interference) The *expected link interference* factor $\sigma_{ij}^{l}(t,n)$ measures the fraction of the time interval $[t+nT, t+(n+1)T)$ during which the activity of PU v_l can interfere with the communications between u_i and u_j, measured at time t:

$$\sigma_{ij}^{l}(t,n) = \begin{cases} 1 & \text{if } t_{ij}^{l}(t,n) = T \\ t_{ij}^{l}(t,n)/T & \text{if } 0 \leq t_{ij}^{l}(t,n) < T \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

where $t_{ij}^{l}(t,n)$ is the expected cumulative contact time period between v_l and either u_i or u_j in time interval $[t+nT, t+(n+1)T)$, measured at time t.

Definition 8. (Link Availability Probability) The *link availability probability* $p_{ij}^{m}(t,n)$ between u_i and u_j is the probability of link e_{ij}^{m} being: i) connected, ii) not affected by PU activity, in the time interval $[t+nT, t+(n+1)T)$, measured at time t:

$$p_{ij}^{m}(t,n) \triangleq \rho_{ij}(t,n) \prod_{v_l \in \mathcal{V}_{ij}^{m}(t,n)} \left(1 - \sigma_{ij}^{l}(t,n)\bar{p}^{l}\right) \tag{15}$$

where $\mathcal{V}_{ij}^{m}(t,n)$ is the interfering set for link $e_{i,j}^{m}$ in the time interval $[t+nT, t+(n+1)T)$, measured at time t.

Remark. The expression (15) of the *link availability probability* allows us to account for the dynamics introduced in the link connectivity by: i) the relative movement between two CR users, through the *expected link utilization* factor $\rho_{ij}(t,n)$ (13); ii) the relative movement between a CR user and a PU, through the *expected link interference* factor $\sigma_{ij}^{l}(t,n)$ (14).

By means of Definition 8, we derive now the expression of the expected link delay for mobile networks in the time interval δ.

Lemma 2. *The expected link delay $l_{ij}(t,\delta)$ experienced by a packet sent by u_i to the neighbor u_j in the time interval $[t, t+\delta)$ measured at time t is shown in* (16)*, where L is the packet length, $\tau_{ij}^{m}(t)$ is defined in* (4)*, $p_{ij}^{\tau_{ij}^{m}(t)}(t,e)$ is defined in* (15)*, e denotes the number of transmission attempts failed due to the presence of PU activity on each available channel, and $q_{ij}^{m}(t,e)$ is given by:*

$$q_{ij}^{m}(t,e) = \begin{cases} 1 & \textit{if } m = 0 \\ \prod_{n=1}^{m} \bar{p}_{ij}^{\tau_{ij}^{m}(t)}(t,e) & \textit{otherwise} \end{cases} \tag{17}$$

$$l_{ij}(t,\delta) = \sum_{e=0}^{\lfloor \frac{\delta}{T} \rfloor} \left(\prod_{r=1}^{e} q_{ij}^{N}(t,r) \sum_{m=1}^{N} q_{ij}^{m-1}(t,e) p_{ij}^{\tau_{ij}^{m}(t)}(t,e) \frac{L}{\bar{\psi}_{ij}^{\tau_{ij}^{m}(t)}(t)} \right) + \sum_{e=1}^{\lfloor \frac{\delta}{T} \rfloor} \left(\prod_{r=1}^{e} q_{ij}^{N}(t,r)\, e\, T \left(1 - q_{ij}^{N}(t,e)\right) \right) \tag{16}$$

Proof: It is straightforward to prove the theorem by following the reasoning adopted to prove Lemma 1. ∎

Remark. Similarly to static networks (see (9)), the expression of the *expected link delay* for mobile networks (16) accounts for both the PU characteristics and the sensing process characteristics. Unlike (9), expression (16) accounts also for the dynamics introduced in the link delay by the movement of the CR users and/or the PUs. Moreover, expression (16) depends on the time interval t and on the time horizon δ, which can be interpreted as the route update period. The shorter is δ, the more accurate is the estimation of the expected link delay provided by (16), but the higher is the routing overhead generated by the adopted routing protocol.

By means of Lemma 2, we can now derive the expression of OPERA for mobile networks.

Theorem 2. ***(OPERA for mobile networks)*** *The expected end-to-end delay $\mathcal{D}_{ik}(t,\delta,\mathcal{C})$ for the source-destination (u_i,u_k) through the ordered set $\mathcal{C}=\{c_j\}_{j=1}^{n}$ of neighbors of u_i in the time interval $[t,t+\delta)$ measured at time t is shown in* (18) *reported at the top of the next page, where*

$$\bar{q}_{ic_j}^{\tilde{m}}(t,e) = \prod_{m=1}^{\tilde{m}} \bar{z}_{iC_j}^{\tau_{ic_j}^{m}(t)}(t,e)\, \bar{z}_{i\setminus C_j}^{\tau_{ic_j}^{\tilde{m}+1}(t)}(t,e) \cdot \\ \cdot \prod_{m=\tilde{m}+2}^{N} \bar{z}_{iC_{j-1}}^{\tau_{ic_j}^{m}(t)}(t,e) \tag{19}$$

$$\bar{z}_{iC_n}^{m}(t,e) \triangleq P\left(E_{ic_1}^{m}(t,e)=0,\ldots,E_{ic_n}^{m}(t,e)=0\right) \tag{20}$$

$$\bar{z}_{i\setminus C_n}^{m}(t,e) \triangleq \\ P\left(E_{ic_1}^{m}(t,e)=0,\ldots,E_{ic_{n-1}}^{m}(t,e)=0 | E_{ic_n}^{m}(t,e)=1\right) \tag{21}$$

and where L is the packet length, $\tau_{ic_j}^{m}(t)$ is defined in (4), *$p_{ic_j}^{\tau_{ic_j}^{m}(t)}(t,e)$ is defined in* (15), *$E_{ic_j}^{m}(t,e)$ denotes the random process that models the PU activity on link e_{ij}^{m} in the time interval $[t+eT,t+(e+1)T)$, and e denotes the number of transmission attempts failed due to the presence of PU activity on each available channel towards each forwarder c_j.*

Proof: It is straightforward to prove the theorem by following the same reasoning adopted to prove Theorem 1. ∎

Remark. Differently from the expression of OPERA derived for static networks (11), the expression (18) depends on the time interval t and on the time horizon δ, which can be interpreted as the route update period. The shorter is δ, the more accurate is the estimation of the expected end-to-end delay provided by (18), but the higher is routing overhead generated by the adopted routing protocol.

V. OPERA Optimality

In this section, we first define the conditions for route metric optimality with a preliminary lemma in Sec. V-A. Then, in Sec. V-B we prove (Theorem 3) that OPERA metric is optimal when combined with both Dijkstra and Bellman-Ford based routing protocols.

A. Preliminaries

For the source-destination (u_i,u_k), each forwarder can route the packets through a set of different candidate routes. Such a set can be regarded as a rooted tree, called CR-tree, where all the leaves coincide with the destination u_k. Accordingly, we use the following definition and lemma based on theorems (1) and (4) in [3] to prove the optimality of OPERA.

Definition 9. (**Optimality**) A routing protocol is optimal if it always routes packets along the path with the minimum route cost between every pair of nodes in any connected network.

Lemma 3. ***(Conditions for Optimality)*** *Both a Dijkstra-based and a Bellman-Ford-based CR routing protocols are optimal when adopted by a CR algebra with the following properties: relay beneficial condition, strictly preference preservation, and relay order optimality.*

In the following, we first define the concept of routing algebra, and then we define the three properties listed by Lemma 3.

Definition 10. (**CR algebra**) A *CR algebra* is a 4-tuple:

$$\mathcal{CR} = (\mathcal{P},\oplus,\mathcal{L},\leq) \tag{22}$$

where $\mathcal{P}$ is the set of CR-trees, $\oplus$ is the operator that concatenates two CR-trees in a single CR-tree, $\mathcal{L}:\mathcal{P}\rightarrow\mathbb{R}$ is a function that assigns a cost to a CR tree, and $\leq$ is the ordering relation over the CR-trees in terms of costs.

We use the following notations. We denote with $\mathcal{P}_{ik}(\mathcal{C})\in\mathcal{P}$ the CR-tree for the source-destination pair (u_i,u_k) through the forwarder set $\mathcal{C}$. The cost of the CR-tree is denoted with $\mathcal{L}(\mathcal{P}_{ik}(\mathcal{C}))$. We say that $\mathcal{P}_{ik}(\mathcal{C})$ is strictly preferred to $\mathcal{P}_{ik}(\tilde{\mathcal{C}})$ if $\mathcal{L}(\mathcal{P}_{ik}(\mathcal{C}))<\mathcal{L}(\mathcal{P}_{ik}(\tilde{\mathcal{C}}))$.

Definition 11. (**Relay Beneficial Condition**) The algebra $\mathcal{CR}$ is relay-conditionally-beneficial if, $\forall\, u_i,u_k\in U$ and $\forall\, c_{\tilde{n}}\in\mathcal{C}$, it results:

$$\mathcal{L}(\mathcal{P}_{ik}(c_{\tilde{n}}))<\mathcal{L}(\mathcal{P}_{ik}(\tilde{\mathcal{C}})) \Longleftrightarrow \mathcal{L}(\mathcal{P}_{ik}(\tilde{\mathcal{C}}\oplus c_{\tilde{n}}))<\mathcal{L}(\mathcal{P}_{ik}(\tilde{\mathcal{C}})) \tag{23}$$

where $\mathcal{C}$ denotes the set of forwarders and $\tilde{\mathcal{C}}\subset\mathcal{C}$. Intuitively, the relay beneficial conditional property requires that a neighbor c_j is selected as forwarder if and only if it can decrease the delay from u_i to u_k.

$$\mathcal{D}_{ik}(t,\delta,\mathcal{C}) = \sum_{e=0}^{\lfloor \frac{\delta}{T} \rfloor} \left(\prod_{r=1}^{e} \bar{q}_{ic_n}^{N}(t,r) \sum_{j=1}^{n} \sum_{m=1}^{N} \left(\bar{q}_{ic_j}^{m-1}(t,e) p_{ic_j}^{\tau_{ic_j}^{m}(t)}(t,e) \left(\frac{L}{\bar{\psi}_{ic_j}^{\tau_{ic_j}^{m}(t)}(t)} + \mathcal{D}_{c_j k}(t,\delta) \right) \right) \right) + \sum_{e=1}^{\lfloor \frac{\delta}{T} \rfloor} \left(\prod_{r=1}^{e} \bar{q}_{ic_n}^{N}(t,r)\, e\, T \left(1 - \bar{q}_{ic_n}^{N}(t,e)\right) \right) \quad (18)$$

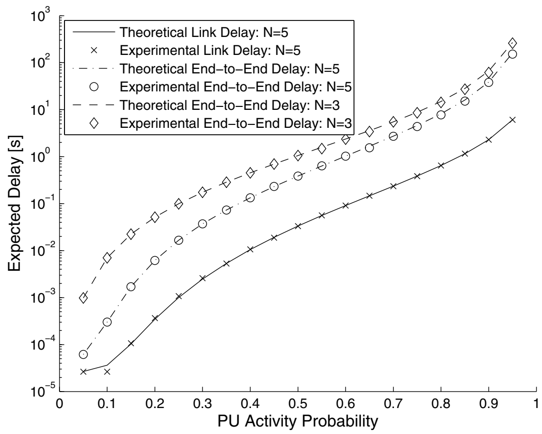

Fig. 5. OPERA validation: theoretical delays vs experimental delays for expressions (9) and (11). Logarithmic scale for axis y.

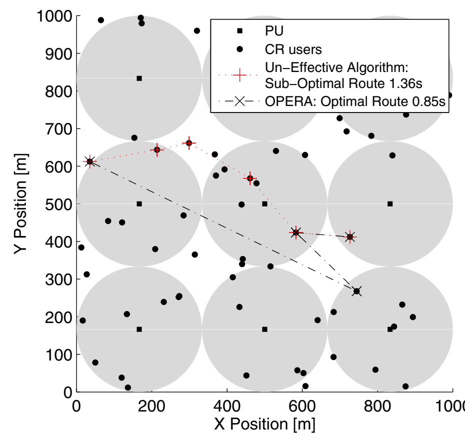

Fig. 6. Considered topology: the figure shows two different routes and the respective delays between the same couple source-destintion, the route singled out by OPERA (black line) and the route singled out by the un-effective algorithm.

Definition 12. (**Strictly preference preservation**) The algebra $\mathcal{CR}$ is strictly preference-preservable if, $\forall\, u_i, u_k \in U$ and $\forall\, c_{\tilde{n}} \in \mathcal{C}$, it results:

$$\mathcal{L}(\mathcal{P}_{ik}(c_{\tilde{n}})) < \mathcal{L}(\mathcal{P}_{ik}(\tilde{\mathcal{C}})) \iff \mathcal{L}(\mathcal{P}_{ik}(c_{\tilde{n}})) < \mathcal{L}(\mathcal{P}_{ik}(\tilde{\mathcal{C}} \oplus c_{\tilde{n}})) \quad (24)$$

where $\mathcal{C}$ denotes the set of forwarders and $\tilde{\mathcal{C}} \subset \mathcal{C}$. Intuitively, a CR algebra is strictly-preference preservable if the concatenation operation preserves the preference.

Definition 13. (**Relay order optimality**) The algebra $\mathcal{CR}$ is relay-order-optimal if, $\forall\, u_i, u_k \in U$, it results:

$$\mathcal{L}(\mathcal{P}_{ik}(\mathcal{C})) \leq \mathcal{L}(\mathcal{P}_{ik}(\tilde{\mathcal{C}})) \quad (25)$$

where $\mathcal{C}$ is the set of forwarders ordered according to the preferences associated with the corresponding CR-trees and $\tilde{\mathcal{C}}$ is any permutation of the set $\mathcal{C}$. Intuitively, a CR algebra is relay-order-optimal if it exists a unique criterion to order the set of forwarders.

B. Optimality

Theorem 3. *Any Dijkstra-based or Bellman-Ford-based routing protocol combined with OPERA is optimal when the independent PU activity condition* (7) *holds, and when the set of forwarders $\mathcal{C}$ for an arbitrary couple source-destination (u_i, u_k) is given by:*

$$\mathcal{C}_{ik} = \{c_j \in \mathcal{N}_i \;: \mathcal{W}_{ik}(c_j) < \mathcal{W}_{ik}(c_{j+1}) \text{ and } \mathcal{D}_{ik}(c_j) < \mathcal{D}_{ik}(\{c_1,\ldots,c_{j-1},c_j\})\} \quad (26)$$

where

$$\mathcal{W}_{ik}(u_{c_j}) = \frac{1}{1-q_{ic_j}^{N}} \sum_{m=1}^{N} q_{ic_j}^{m-1} p_{ic_j}^{\tau_{ic_j}^{m}} \left(\frac{L}{\bar{\psi}_{ic_j}^{\tau_{ic_j}^{m}}} + \mathcal{D}_{c_j k} \right) \quad (27)$$

and where L is the packet length, τ_{ij}^{m} is defined in (4)*, $p_{ij}^{\tau_{ij}^{m}}$ is defined in* (5)*, q_{ij}^{m} is defined in* (10)*, and $\mathcal{D}_{ik}$ is defined in* (11).

Proof: The proof follows from Lemmas 4-6. ■

Lemma 4. *OPERA is relay-conditionally-beneficial.*

Proof: See Appendix C. ■

Lemma 5. *OPERA is strictly preference-preservable.*

Proof: See Appendix D. ■

Lemma 6. *OPERA is relay order optimal.*

Proof: See Appendix E. ■

VI. Performance Evaluation

In this section, we evaluate the performance of OPERA in terms of *optimality* and *accuracy*.

More in detail, first we validate the theoretical results derived in Sec. IV, i.e., the expressions of the *expected link delay* (9) and *expected end-to-end delay* (11). Then, we validate the optimality property of OPERA derived in Sec. V (Theorem 3), by comparing the performance of OPERA with that provided by the exhaustive search of the minimum end-to-end delay. Finally, we evaluate the benefits of the accuracy property by

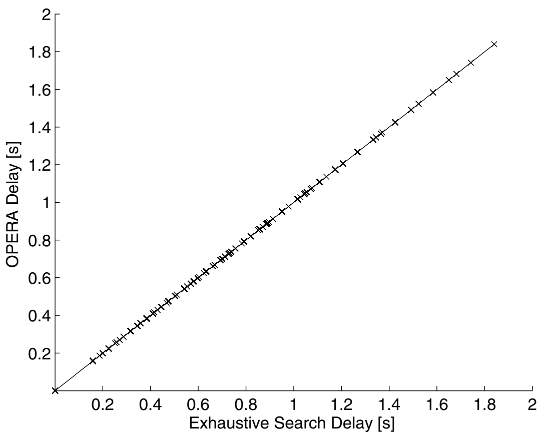

Fig. 7. OPERA optimality: OPERA delay vs Exhaustive Search delay for the topology shown in Fig. 6. For each dot, coordinate (x) represents the delay computed through exhaustive search, whereas coordinate (y) represents the delay computed through OPERA expression.

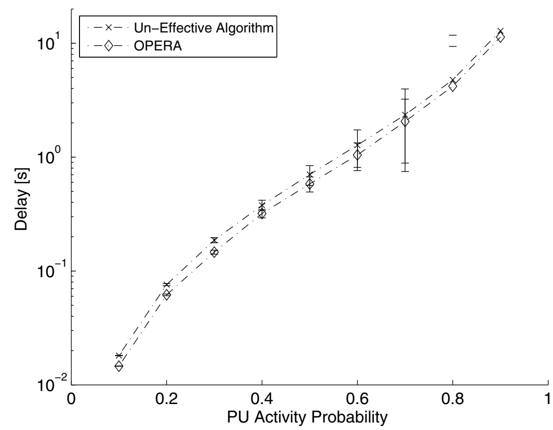

Fig. 8. OPERA accuracy: average delay vs PU activity probability p_l. Logarithmic scale for axis y.

comparing the performance of OPERA with that provided by the algorithm that neglects the route diversity effects.

OPERA Validation

In Fig. 5, we validate the theoretical expressions of the *expected link delay* (9) and *expected end-to-end delay* (11). More in detail, we compare the theoretical delays with those obtained through Montecarlo simulations as the PU activity probability increases.

The adopted simulation set is as follows: the packet length is $L = 1500$B, the expected link throughput is $\bar{\psi} = 54$Mbps, the number of channels N is reported in the legend, each available channel is affected by a PU with an activity provability $p_l \in [0.05, 0.95]$, and the sensing process is characterized by the times $T = 2$s and $T_s = 0.001$s. For the expected link delay, we consider two neighbors, while for the expected end-to-end delay, we consider a set $\mathcal{C}$ composed by two forwarders, each of them one hop away from the destination.

First, we note that there is a very good agreement between the theoretical and the experimental results for both the expressions. Moreover, we observe that, for the lowest values of probability p_l, the delays are dominated by the packet transmission delay $\frac{L}{\bar{\psi}}$. Differently, as probability p_l increases, the delays are dominated by the time period T. This result agrees with the intuition: when the PU activity is negligible, the path delay depends on the throughputs of the links belonging to the path, while when the PU activity is dominant, the delay depends on T. Finally, we observe that the end-to-end delay decreases as the number of channels increases. This result is reasonable, since, as the number of channels increases, the probability $\bar{q}^N_{ic_n}$ (11) of unsuccessful transmission due to the PU activity decreases. Thus, the delay $\frac{\bar{q}^N_{ic_n}}{1-\bar{q}^N_{ic_n}}T$ introduced by the PU activity decreases as well.

OPERA Optimality

In this subsection, we validate the optimal property of OPERA stated by Theorem 3. More in detail, for every couple source-destination, we compare the delays computed with expression (11) with those obtained by exhaustive search of the minimum end-to-end-delay.

The adopted simulation set is as follows: the network topology is shown in Fig. 6 and it is similar to the one used in [15], with 64 CR users spread in a square region of side 1000m. The CR user transmission standard is IEEE 802.11g, the packet length is $L = 1500$B, the expected link throughput is $\bar{\psi}^m = 54$Mbps, the transmission range is equal to 50m, the number of channels is $N = 2$, and the sensing process is characterized by the times $T = 0.7$s and $T_s = 0.1$s as in [15]. The PU interference range R_{v_l} is shown in Fig. 6, and $p_l = 0.5$.

Fig. 7 presents the difference between the OPERA delays and the exhaustive search delays by showing a dot for each couple of CR users. The (x) coordinate of the dot represents the delay computed through exhaustive search, whereas the (y) coordinate of the dot represents the delay computed through expression (11). Clearly, if $y = x$, then the two delays are exactly the same, meaning that OPERA actually finds the minimum end-to-end delay. Since Fig. 7 clearly shows that for each couple of CR users we have $y = x$, then OPERA is optimal according to Definition 9. We further observe that the delays shown in Fig. 7 have the same order of magnitude of those shown in Fig. 5 for the same value of p_l. The reason is that, since $p_l = 0.5$ and since the topology is relatively small, the delay $\frac{\bar{q}^N_{ic_n}}{1-\bar{q}^N_{ic_n}}T$ introduced by the PU activity dominates the end-to-end delay.

OPERA Accuracy

In this subsection, we analyze the benefits provided by the route diversity in terms of actual delay of a route. More in detail, we compare the delays computed with expression (11) with those obtained by the algorithm that neglects the route diversity effects by recursively computing the path delay through (9), called Un-Effective Algorithm. We consider three scenarios: i) static networks, as probability p_l increases; ii) static networks, as time period T increases; iii) mobile

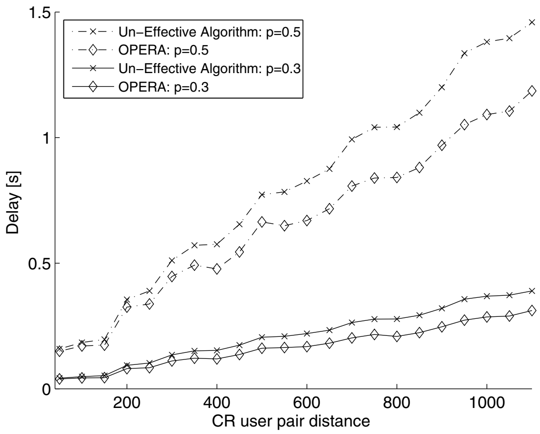

Fig. 9. OPERA accuracy: delay vs CR user pair distance for different values of PU activity probability p_l.

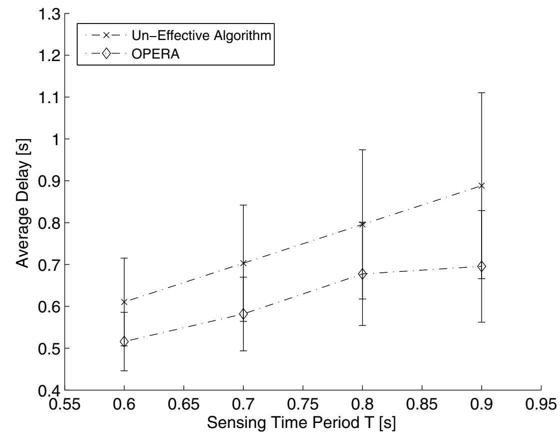

Fig. 10. OPERA accuracy: average delay vs time period T.

networks, as p_l increases. For the static scenarios, the simulation set is the same adopted in previous subsection, with $p_l \in [0.1, 0.9]$ and with $T \in [0.6, 0.9]$. For the mobile scenario, we adopt the simulation set used to validate OPERA.

Fig. 8 presents both the mean and the variance values of the delays for $T = 0.6$, as probability p_l increases. Each point represents the average of the delays of all the couples source-destination for a certain value of p_l. First, we observe that the higher is p_l, the lower is the confidence of the mean values. This result is reasonable, since the PU activity probability is a measure of the "noise" introduced in the delay estimation. Moreover, we observe that for each value of p_l, the difference between the average delays computed by OPERA and those computed by Un-Effective Algorithm is significant (axis y is log-scale). This result highlights that, also for low number of channels ($N = 2$), it exists a significant route diversity that must be exploited to effectively measure the route delays.

To better understand the effects of the route diversity on the end-to-end delay, in Fig. 9 we report the delays for two different values of p_l as the distance between the source and destination increases. First, we observe that the difference between the delays computed by OPERA and those computed by Un-Effective Algorithm increases with the distance. This result is reasonable, since the longer is the path, the more numerous are the PUs affecting it, and the more significant are the effects of the route diversity. Moreover, we observe that the Un-Effective Algorithm measures delays that are in average the 30% longer than those measured by OPERA. As a consequence, a routing protocol based on Un-Effective Algorithm forwards the packets on sub-optimal routes, as shown in Fig. 6.

Fig. 10 presents both the mean and the variance values of the delays for $p_l = 0.5$, as the time period T increases. First, we observe that, unlike the results of Fig. 8, T has no impact on the confidence of the mean values. This result confirms the consideration made for Fig. 8, since in this scenario the PU activity probability is fixed. Moreover, we observe that the delays grow roughly linearly with T. This result confirms the consideration made in the previous subsection: for $p_l = 0.5$ the delay $\frac{\bar{q}_{icn}^N}{1-\bar{q}_{icn}^N}T$, introduced by the PU activity, dominates the end-to-end delay. Finally, we observe that, as T grows, the difference between the delays computed by OPERA and those computed by Un-Effective Algorithm increases as well. This result highlights the importance of the route diversity effects in CRAHNs.

In Fig. 11, we report the delays for two different values of the time period T as the distance between the source and the destination increases. We observe that the difference between the delays computed by OPERA and those computed by Un-Effective Algorithm increases as the distance between the nodes increases. This result confirms the consideration made for Fig. 9: the longer is the path, the more significant are the effects of the route diversity.

Finally, in Fig. 12 we report the delays obtained by (18) as p_l increases for three different values of the route update period δ normalized to the sensing process parameter T. In this experiment, we adopt the simulation set used to validate OPERA, since it allows us to better analyze the impact of the route update period on the delay estimation. First, we observe that, surprisingly, the delays for mobile networks are significantly lower than those obtained for static networks (Fig. 5). The reason is that relation (18) accounts only for the delays of the packets that can be delivered in the time interval $[t, t + \delta)$, since packets with longer delays do not contribute to the delay estimation. Moreover, we observe that a similar effect is observable by comparing the delays for different values of δ: the longer is the route update period, the larger are the delays. Finally, we observe that for the lowest values of p_l, the delays are dominated by the packet transmission delay $\frac{L}{\psi}$, while, as p_l increases, the delays are dominated by the route update period δ. This result confirms the considerations made previously.

VII. Conclusions

In this paper, an optimal routing metric for cognitive radio ad hoc networks, OPERA, is proposed. OPERA has been designed to achieve two features: i) *Optimality*: OPERA is optimal when combined with both Dijkstra and Bellman-Ford based routing protocols; ii) *Accuracy*: OPERA exploits the route diversity provided by the intermediate nodes to

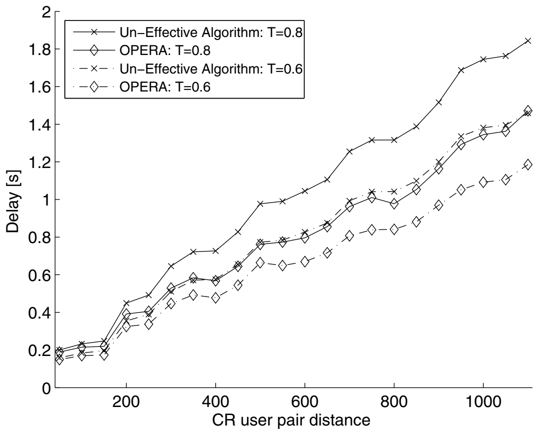

Fig. 11. OPERA accuracy: delay vs CR user pair distance for different values of time period T.

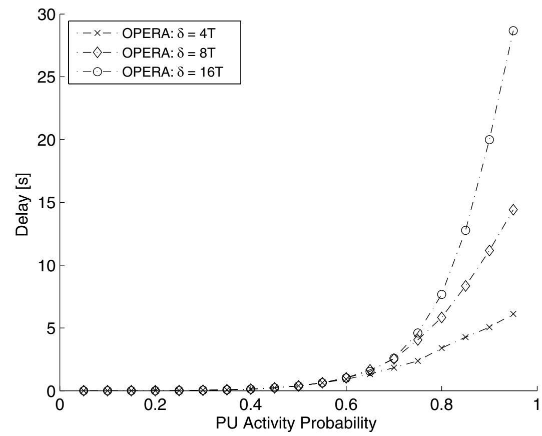

Fig. 12. Mobile networks: OPERA delay vs PU activity probability p_l for different values of the routing update period δ normalized to the time period T.

measure the actual end-to-end delay, by taking explicitly into account the unique characteristics of cognitive radio networks. A closed-form expression of OPERA has been analytically derived for both static and mobile networks, and its optimality has been proved rigorously. The performance evaluation confirmed the benefits of adopting the proposed routing metric for cognitive radio ad hoc networks.

Acknowledgement

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Appendix

A. Lemma 1

Proof: First, we observe that the additional delay for each unsuccessful transmission is[6] T, as shown in Fig. 3. Therefore, it is possible to express the *expected link delay* l_{ij} as:

$$l_{ij} = \sum_{e=0}^{+\infty} \left(q_{ij}^N\right)^e \sum_{m=1}^{N} \left(q_{ij}^{m-1} p_{ij}^{\tau_{ij}^m} \frac{L}{\bar{\psi}_{ij}^{\tau_{ij}^m}} \right) + \\ + \sum_{e=1}^{+\infty} \left(q_{ij}^N\right)^e e\,T \sum_{m=1}^{N} \left(q_{ij}^{m-1} p_{ij}^{\tau_{ij}^m} \right) \quad (28)$$

where e denotes the number of transmission attempts failed due to the presence of PU activity on each available channel. By using the notable relations $\sum_{n=0}^{\infty} nx^n = x/(1-x)^2$ and $\sum_{n=0}^{\infty} x^n = 1/(1-x)$ if $|x| < 1$, and by exploiting the relation $\sum_{m=1}^{N} \left(q_{ij}^{m-1} p_{ij}^{\tau_{ij}^m} \right) = 1 - q_{ij}^N$, we obtain (9). ■

[6]For the sake of simplicity, we have assumed that the arrival times of the packets are synchronized with the slot period T. Nevertheless, it is easy to derive (28) for the more general case of arrival times uniformly distributed in the transmission period T_{tx}, by setting the delay introduced by the first unsuccessful transmission equal to $\frac{T_{tx}}{2} + T_s$.

B. Theorem 1

Proof: Similarly to the proof of Lemma 1, it is possible to express $\mathcal{D}_{ik}(\mathcal{C})$ as in :

$$\mathcal{D}_{ik}(\mathcal{C}) = \sum_{e=0}^{+\infty} \left(\bar{q}_{ic_n}^N\right)^e \sum_{j=1}^{n} \sum_{m=1}^{N} \bar{q}_{ic_j}^{m-1} p_{ic_j}^{\tau_{ic_j}^m} \left(\frac{L}{\bar{\psi}_{ic_j}^{\tau_{ic_j}^m}} + \mathcal{D}_{c_j k} \right) + \\ \sum_{e=1}^{+\infty} \left(\bar{q}_{ic_n}^N\right)^e e\,T \sum_{j=1}^{n} \sum_{m=1}^{N} \bar{q}_{ic_j}^{m-1} p_{ic_j}^{\tau_{ic_j}^m} \quad (29)$$

where e denotes the number of transmission attempts failed due to the presence of PU activity on each available channel towards each available neighbor c_j. By using the notable relations $\sum_{n=0}^{\infty} nx^n = x/(1-x)^2$ and $\sum_{n=0}^{\infty} x^n = 1/(1-x)$ if $|x| < 1$, and by exploiting the relation $\sum_{j=1}^{n} \sum_{m=1}^{N} \left(\bar{q}_{ic_j}^{m-1} p_{ic_j}^{\tau_{ic_j}^m} \right) = 1 - q_{ic_n}^N$, we obtain (11). ■

C. Lemma 4

Proof: We consider the CR algebra $\mathcal{CR}$ (22), where the path cost $\mathcal{L}$ is defined according to OPERA metric (11), since the proof for mobile networks follows accordingly. Since the independent PU activity condition (7) holds, we have:

$$\mathcal{D}_{ik}(\tilde{\mathcal{C}}) = \frac{1}{1 - q_{i\tilde{\mathcal{C}}}^N} \left(\tilde{\mathcal{D}}_{\mathcal{C}} + q_{i\tilde{\mathcal{C}}}^N T \right) \quad (30)$$

$$\mathcal{D}_{ik}(c_{\tilde{n}}) = \frac{1}{1 - q_{ic_{\tilde{n}}}^N} \left(\tilde{\mathcal{D}}_{c_{\tilde{n}}} + q_{ic_{\tilde{n}}}^N T \right) \quad (31)$$

$$\mathcal{D}_{ik}(\tilde{\mathcal{C}} \oplus c_{\tilde{n}}) = \frac{1}{1 - q_{i\tilde{\mathcal{C}}}^N q_{ic_{\tilde{n}}}^N} \left(\tilde{\mathcal{D}}_{\mathcal{C}} + q_{i\tilde{\mathcal{C}}}^N \tilde{\mathcal{D}}_{c_{\tilde{n}}} + q_{i\tilde{\mathcal{C}}}^N q_{ic_{\tilde{n}}}^N T \right) \quad (32)$$

where $q_{ic_j}^N$ is defined in (10) and $q_{i\tilde{\mathcal{C}}}^N = \prod_{u_{c_j} \in \tilde{\mathcal{C}}} q_{ic_j}^N$.

Case 1 (⇒): We prove the case with a *reductio ad absurdum* by supposing that:

$$\exists\, c_{\tilde{n}} \in \mathcal{C}_{ik} \;:\; \mathcal{D}_{ik}(c_{\tilde{n}}) < \mathcal{D}_{ik}(\tilde{\mathcal{C}}) \Longrightarrow \mathcal{D}_{ik}(\tilde{\mathcal{C}} \oplus c_{\tilde{n}}) > \mathcal{D}_{ik}(\tilde{\mathcal{C}}) \quad (33)$$

By substituting (30) and (32) in the right side of (33), we obtain:

$$\mathcal{D}_{ik}(c_{\tilde{n}}) > \frac{1}{1-q^N_{i\tilde{\mathcal{C}}}} \left(\tilde{\mathcal{D}_{\mathcal{C}}} + \frac{1-q^N_{i\tilde{\mathcal{C}}} q^N_{ic_{\tilde{n}}}}{1-q^N_{ic_{\tilde{n}}}} T \right) \quad (34)$$

and, since $q^N_{i\tilde{\mathcal{C}}} \leq 1$ and $\mathcal{D}_{ik}(\tilde{\mathcal{C}}) > \mathcal{D}_{ik}(c_{\tilde{n}})$, (34) constitutes a *reductio ad absurdum*.

Case 2 ($\Leftarrow$): Since, from (26), we have that $\forall\, c_{\tilde{n}} \in \mathcal{C}_{ik} \;:\; \mathcal{D}_{ik}(c_{\tilde{n}}) < \mathcal{D}_{ik}(\tilde{\mathcal{C}} \oplus c_{\tilde{n}})$, and since, for hypothesis $\mathcal{D}_{ik}(\tilde{\mathcal{C}} \oplus c_{\tilde{n}}) < \mathcal{D}_{ik}(\tilde{\mathcal{C}})$, it results that $\forall\, c_{\tilde{n}} \in \mathcal{C}_{ik} \;:\; \mathcal{D}_{ik}(c_{\tilde{n}}) < \mathcal{D}_{ik}(\tilde{\mathcal{C}})$. ■

D. Lemma 5

Proof: From (26), we have that:

$$\forall\, c_{\tilde{n}} \in \mathcal{C}_{ik} : \mathcal{D}_{ik}(c_{\tilde{n}}) < \mathcal{D}_{ik}(\tilde{\mathcal{C}} \oplus c_{\tilde{n}}) \quad (35)$$

■

E. Lemma 6

Proof: We consider the CR algebra $\mathcal{CR}$ (22), where the cost $\mathcal{L}$ is defined according to OPERA metric (11), since the proof for mobile networks follows accordingly. We prove the lemma with a *reductio ad absurdum* by supposing that:

$$\mathcal{W}_{ik}(c_j) < \mathcal{W}_{ik}(c_{j+1}) \Longrightarrow \mathcal{D}_{ik}(\mathcal{C}) > \mathcal{D}_{ik}(\tilde{\mathcal{C}}) \quad (36)$$

where $\mathcal{W}_{ik}(c_j)$ is defined in (26), and $\mathcal{C} = \{c_1, \ldots, c_j, c_{j+1}, \ldots, c_n\}$ and $\tilde{\mathcal{C}} = \{c_1, \ldots, c_{j+1}, c_j, \ldots, c_n\}$ are ordered sets of neighbors. Since we can re-write the left side of (36) as $\mathcal{W}_{ik}(c_j) > \mathcal{W}_{ik}(c_{j+1})$, we have a *reductio ad absurdum*. ■

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Marcello Caleffi received the Dr. Eng. degree summa cum laude in Computer Science Engineering from University of Lecce in 2005, and the Ph.D. degree in Electronic and Telecommunications Engineering from University of Naples Federico II in 2009. Since 2008, he is with the Dept. of Biomedical, Electronic and Telecommunications Engineering, University of Naples Federico II, as postdoctoral research fellow. From 2010 to 2011, he was with Broadband Wireless Networking Laboratory, Georgia Institute of Technology, Atlanta, as well as with the NaNoNetworking Center in Catalunya (N3Cat), Universitat Politcnica de Catalunya (UPC), Barcelona, as a visiting researcher. His research interests are in cognitive radio networks and human-enabled wireless networks.

Ian F. Akyildiz received the B.S., M.S., and Ph.D. degrees in Computer Engineering from the University of Erlangen-Nürnberg, Germany, in 1978, 1981 and 1984, respectively. Currently, he is the Ken Byers Chair Professor with the School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, the Director of Broadband Wireless Networking Laboratory and Chair of the Telecommunication Group at Georgia Tech. In June 2008, Dr. Akyildiz became an honorary professor with the School of Electrical Engineering at Universitat Politècnica de Catalunya (UPC) in Barcelona, Spain. He is also the founding Director of the N3Cat (NaNoNetworking Center in Catalunya) at UPC in Barcelona. He is the Editor-in-Chief of *Computer Networks* (Elsevier) Journal, and the founding Editor-in-Chief of the *Ad Hoc Networks* (Elsevier) Journal, the *Physical Communication* (Elsevier) Journal, and the *Nano Communication Networks* (Elsevier) Journal. Dr. Akyildiz serves on the advisory boards of several research centers, academic departments, high tech companies, journals and conferences. He is an IEEE Fellow (1996) and an ACM Fellow (1997). He received numerous awards from IEEE and ACM. His research interests are in nano-networks, cognitive radio networks and wireless sensor networks.

Luigi Paura received the Dr. Eng. degree summa cum laude in Electronic Engineering in 1974 from University of Napoli Federico II. From 1979 to 1984, he was with the Dept. of Biomedical, Electronic and Telecom. Engineering, University of Naples Federico II, first as an Assistant Professor and then as an Associate Professor. Since 1994, he has been a Full Professor of Telecom.: first, with the Dept. of Mathematics, University of Lecce, Italy; then, with the Dept. of Information Engineering, Second University of Naples; and, finally, from 1998 he has been with the Dept. of Biomedical, Electronic and Telecom. Engineering, University of Naples Federico II. He also held teaching positions at University of Salerno, at University of Sannio, and at University Parthenope of Naples. In 1985-86 and 1991 he was a visiting researcher at Signal and Image Processing Lab, University of California, Davis. His research interests are mainly in digital communication systems and cognitive radio networks.