On the Origins of Heavy-Tailed Delay in Dynamic Spectrum Access Networks

Pu Wang, *Student Member*, *IEEE*, and Ian F. Akyildiz, *Fellow*, *IEEE*

Abstract—This paper provides an asymptotic analysis of the transmission delay experienced by SUs for dynamic spectrum access (DSA) networks. It is shown that DSA induces only light-tailed delay if both the busy time of PU channels and the message size of SUs are light tailed. On the contrary, if either the busy time or the message size is heavy tailed, then the SUs' transmission delay is heavy tailed. For this latter case, it is proven that if one of either the busy time or the message size is light tailed and the other is regularly varying with index α, the transmission delay is regularly varying with the same index. As a consequence, the delay has an infinite mean provided $\alpha < 1$ and an infinite variance provided $\alpha < 2$. Furthermore, if both the busy time and the message size are regularly varying with different indices, then the delay tail distribution is as heavy as the one with the smaller index. Moreover, the impact of spectrum mobility and multiradio diversity on the delay performance of SUs is studied. It is shown that both spectrum mobility and multiradio diversity can greatly mitigate the heavy-tailed delay by increasing the orders of its finite moments.

Index Terms—Dynamic spectrum access, heavy-tailed delay, spectrum mobility, multiradio diversity.

1 Introduction

Dynamic spectrum access (DSA) enables the secondary users (SUs) to use or share the spectrum in an opportunistic manner [1]. Under such scheme, SUs access the spectrum during idle periods of the primary users (PUs), and cease transmissions when the PU channels become occupied by the PUs. Apparently, the dynamically changing PU activity has a nonnegligible impact on the QoS performance of SUs. This may become more evident when SUs demand for real-time services in order to support multimedia applications, such as voice over IP and online gaming.

The delay, as one of the key QoS metrics, has been widely studied for classical communication network paradigms the last several decades. So far, the delay with heavy-tailed distributions has drawn high attentions in the research community due to its significantly different behavior from that of the light-tailed (e.g., exponential) distribution [9]. More specifically, the heavy-tailed delay can have infinite moments of lower orders, e.g., mean and variance. In this case, the network can exhibit significant performance degradations including the considerably reduced network throughput, queue stability, and system scalability. Despite its importance, the tail behavior of the SUs' transmission delay is still an underexplored area, partially due to the dynamic and complex network environment. In this paper, we analyze the asymptotic tail distribution of the transmission delay experienced by SUs and discover the impact of DSA paradigm on the delay performance.

- *The authors are with the Broadband Wireless Networking Laboratory, School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, GA 30332. E-mail: {pwang40, ian}@ece.gatech.edu.*

Manuscript received 3 June 2011; revised 5 Aug. 2011; accepted 12 Aug. 2011; published online 18 Aug. 2011.
For information on obtaining reprints of this article, please send e-mail to: tmc@computer.org and reference IEEECS Log Number TMCSI-2011-06-0298.
Digital Object Identifier no. 10.1109/TMC.2011.187.

We consider a cognitive radio network in which an SU can exploit the spectrum holes of multiple stochastically independent channels. A PU channel is modeled by an alternating renewal process, which alternates between busy periods $\{B_i\}_{i\geq 1}$ and idle periods $\{I_i\}_{i\geq 1}$. An SU is only allowed to transmit during the idle periods, and avoid transmissions when the PU channels become busy. Upon the arrival of a message with size $L > 0$, the SU first splits it into multiple packets with constant size $L_p > 0$, which are then sent consecutively over PU channels. Accordingly, the total time an SU takes to complete the transmissions of a message is defined as the transmission delay. Apparently, under such generic settings, the transmission delay has a close relationship with the SU message size as well as the PU channel availability. The distributions of the message size and PU busy time can be either heavy tailed (HT) or light tailed (LT), depending on the underlying communication systems and the applications the SUs and PUs demand for. For example, in the earlier 2G voice-oriented cellular systems, empirical measurements show that the call holding times are light tailed, or more specifically, exponentially distributed [11]. On the contrary, heavy-tailed distributions have been widely observed in current data-oriented communication networks. For example, the file size on the Internet servers, the web access pattern, and the scene length distribution of variable bit rate (VBR) and MEPG video streams have shown HT statistical characteristics [9]. Moreover, recent empirical evidence shows that the call holding time or channel occupancy time in 3G cellular networks also exhibits the HT nature [14], [15].

In this paper, we first investigate the delay performance when only a single PU channel is utilized. Specifically, it is shown that the DSA induces only light-tailed delay as long as both the busy time of PU channels and the message size of SUs are light tailed. On the contrary, if either the busy time or the message size is heavy tailed, then the SUs' transmission delay is heavy tailed. For this case, we prove that if one of the busy time or the message size is light tailed and the other is regularly varying with index α, then the transmission delay is regularly varying with the same index α. As a consequence, the delay has an infinite variance provided $\alpha < 2$ and an infinite mean provided $\alpha < 1$. This implies that the SUs can experience extremely high delay variations and even stochastically zero throughout when transmitting messages with the finite mean size. Furthermore, if both the busy time and the message size are regularly varying with index α and β, respectively, then the tail distribution of the delay is as heavy as the one with the smaller index.

Moreover, we investigate the benefits of exploiting the transmission opportunities on multiple PU channels. More specifically, we consider two multiple channel access schemes, namely, spectrum mobility and multiradio diversity. Under spectrum mobility, if a PU appears in a channel currently used by an SU, the SU vacates the channel immediately and continues its transmission in another idle channel [1]. Under multiradio diversity, an SU is equipped with multiple radio interfaces so that it can simultaneously access multiple channels. We show that compared with the case in which only a single channel is used, both spectrum mobility and multiradio diversity can mitigate the degree of heavy-tailed delay by increasing the orders of its finite moments.

The rest of this paper is organized as follows. Section 2 summarizes the related work. Section 3 introduces system model and preliminaries. Section 4 presents the main results regarding the delay performance of SUs. The impact of spectrum mobility and multiradio diversity is studied in Section 5. The simulation results are presented in Section 6. Finally, Section 7 concludes this paper.

2 Related Work

Although the delay is an important QoS metric in wireless networks, the delay analysis for cognitive radio networks is still scarce to the best of our knowledge. In [6] and [13], the queuing delay of SUs in a multichannel cognitive network was investigated with different objectives. Specifically, using large deviation approximation, Laourine et al. [6] aimed to analyze the stationary queue distribution of SUs under the Markov chain-based PU traffic model. On the contrary, Wang et al. [13] studied the moments of the SUs' queue length under the PU traffic model as an alternating ON/OFF process, where the ON periods follow a general distribution and the OFF periods are exponentially distributed. Instead of studying the queuing delay as [6] and [13], we aim to investigate the transmission delay of SUs. To the best of our knowledge, little work on the analysis of such delay has been done for cognitive radio networks. Besides the above mentioned work, a different application that is related to our work is file fragmentation [8]. In this problem, files are partioned into fragments and transferred over wireless channels. The objective is to find the optimal fragmentation policies that minimize the mean transmission time. Different from the file fragmentation application, in which only one file fragment is sent each time the wireless channel is available, SUs will keep sending packets back-to-back as long as the PU channel is detected as idle. Moreover, in the file fragmentation problem, the channel busy time is assumed to be zero [8]. This assumption is not valid in cognitive radio networks due to the existence of PU activities. In particular, recent work, which is based on real-life measurement data, has identified the heavy-tailed behavior in the busy periods of PU channels [15]. This behavior was further shown to have a significant impact on the sensing performance of SUs. However, Wellens et al. [15] did not answer how this heavy-tailed behavior of PU channels affects the delay performance of SUs, which is one of the key research problems addressed in this paper.

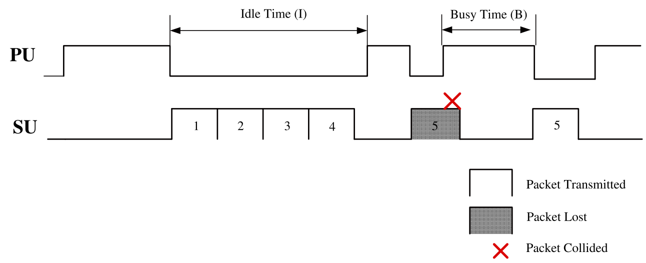

Fig. 1. System model.

3 System Model and Preliminaries

3.1 System Model

Consider a PU channel and an SU which transmits when the PU channel is idle. Without loss of generality, we assume that the PU channel is of unit capacity. This channel is modeled by an alternating renewal process, which alternates between busy periods $\{B_i\}_{i\geq 1}$ and idle periods $\{I_i\}_{i\geq 1}$. $\{B_i\}_{i\geq 1}$ and $\{I_i\}_{i\geq 1}$ are mutually independent random sequences of i.i.d. random variables with distributions F_B and F_I, respectively. Let $L > 0$ denote the size of the messages generated by the SU, and L is a random variable (r.v.) independent of $\{B_i\}_{i\geq 1}$ and $\{I_i\}_{i\geq 1}$. For each message, the SU divides it into packets with constant size $L_p > 0$, which are then sent over the PU channel. In each idle period I_i, the SU attempts to transmit, and if $I_i > L_p$, the SU sends packets consecutively until the remaining time of the idle period I_i is less than the packet size L_p. Otherwise, if $I_i < L_p$, the SU transmits unsuccessfully and waits for the next idle period for retransmission. An illustration of this model is given in Fig. 1.

Definition 1. *During an idle period I_i, the transmission time X_i of the SU is defined as*

$$X_i := \sup\{nL_p : nL_p \leq I_i\}, \tag{1}$$

the total number of idle periods the SU occupies for transmitting a message of size L is defined as

$$M := \inf\left\{m : \sum_{i=1}^{m} X_i \geq L\right\}, \tag{2}$$

and the total delay T of the SU transmitting a message of size L is defined as

$$T(L) := \sum_{i=1}^{M}\{I_i + B_i\}. \tag{3}$$

3.2 Preliminaries

In this paper, we use the following notations. For any two real functions $a(t)$ and $b(t)$, we let $a(t) \sim b(t)$ denote $\lim_{t\to\infty} a(t)/b(t) = 1$. We say that $a(t) \lesssim b(t)$ if $\limsup_{t\to\infty} a(t)/b(t) \le 1$, and $a(t) \gtrsim b(t)$ if $\liminf_{t\to\infty} a(t)/b(t) \ge 1$. Furthermore, we say that $a(t) = o(b(t))$ if $\lim_{t\to\infty} a(t)/\,b(t) = 0$. In addition, for any two nonnegative r.v.s X and Y, we say that $X \le_{a.s.} Y$ if $X \le Y$ almost surely, and $X \le_{s.t.} Y$ if X is stochastically dominated by Y, i.e., $P(X > t) \le P(Y > t)$ for all $t \ge 0$. We say $X \overset{d}{=} Y$ if X and Y are equal in distribution. Also, let $F(x) = P(X \le x)$ denote the cumulative distribution function (cdf) of a nonnegative r.v. X. Let $\overline{F}(x) = P(X > x)$ denote its tail distribution function.

Definition 2. *A r.v. X is heavy tailed if for all $\theta > 0$*

$$\lim_{x\to\infty} e^{\theta x}\overline{F}(x) = \infty, \tag{4}$$

or, equivalently, if for all $z > 0$

$$E[e^{zX}] = \infty. \tag{5}$$

Definition 3. *A r.v. X is light tailed if there exists $\theta > 0$ such that*

$$\lim_{x\to\infty} e^{\theta x}\overline{F}(x) = 0, \tag{6}$$

or, equivalently, if there exists $z > 0$ such that

$$E[e^{zX}] < \infty. \tag{7}$$

Remark 1. Generally speaking, a r.v. is HT if its tail distribution decreases slower than exponentially. Some typical HT distributions include Pareto, log-normal, Bur, and Weibull (with shape parameter less than 1) distributions. On the contrary, a r.v. is LT if its tail distribution decreases exponentially or faster. Some typical LT distributions cover exponential, Gamma, and Weibull (with shape parameter larger than 1) distributions. A key characteristic that distinguishes HT r.v.s from LT ones is that the moment generating function of any HT r.v. X is infinite, i.e., $E(e^{zX}) = \infty, \forall\, z > 0$.

An important subclass of HT distributions is the class of regularly varying distributions [2]. Its definition involves the slowly varying function which is defined as follows.

Definition 4. *A measurable positive function $\mathcal{L}(x)$ defined in some interval $[a, \infty)$ is called slowly varying if for all $y > 0$*

$$\lim_{x\to\infty} \frac{\mathcal{L}(yx)}{\mathcal{L}(x)} = 1. \tag{8}$$

For example, a constant and a logarithmic function are both slowly varying functions.

Lemma 1 (Properties of Slowly Varying Function [2]).

1. *If $\mathcal{L}(x)$ varies slowly, $\lim_{x\to 0} \log(\mathcal{L}(x))/\log x = 0$.*
2. *If $\mathcal{L}(x)$ varies slowly, so does $(\mathcal{L}(x))^a$ for every $a \in \mathbb{R}$.*
3. *If $\mathcal{L}_1(x)$ and $\mathcal{L}_2(x)$ vary slowly, so do $\mathcal{L}_1(x) + \mathcal{L}_2(x)$ and $\mathcal{L}_1(x)\mathcal{L}_2(x)$.*

Definition 5. *A r.v. X is called regularly varying with index $\alpha > 0$, denoted by $X \in \mathcal{RV}(\alpha)$, if*

$$\overline{F}(x) \sim x^{-\alpha}\mathcal{L}(x), \tag{9}$$

where $L(x)$ is a slowly varying function.

Remark 2. Regularly varying distributions are a generalization of power law distributions. The index α indicates how heavy the tail distribution is, where smaller values of α imply heavier tail. Moreover, for a r.v. $X \in \mathcal{RV}(\alpha)$, the exact values of α determine whether the moments of X are bounded or not. This is explained in the following lemma.

Lemma 2. *For any r.v. $X \in \mathcal{RV}(\alpha)$, the moments of order $m > \alpha$ is unbounded, i.e.,*

$$E[X^m] = \infty, \ \ \forall m > \alpha. \tag{10}$$

Remark 3. In particular, for any r.v. $X \in \mathcal{RV}(\alpha)$, if $\alpha < 1$, X has an infinite mean. If $1 < \alpha < 2$, X has a finite mean but an infinite variance.

The following preliminary Lemmas regarding regularly varying and light-tailed distributions are also useful in this paper. We first state the Lemmas, followed by their proofs in Section 3.3.

Lemma 3. *Let $X \in \mathcal{RV}(\alpha)$ and $Y \in \mathcal{RV}(\beta)$. If $\alpha > \beta$, then*

$$P(X > at) = o(P(Y > bt)),$$

with $a > 0$ and $b > 0$.

Lemma 4. *Let X and Y be nonnegative random variables. If $X \in \mathcal{RV}(\alpha)$ and $P(Y > t) = P(X > bt)$ with $b > 0$, then $Y \in \mathcal{RV}(\alpha)$.*

Lemma 5. *Let X be LT and $Y \in \mathcal{RV}(\alpha)$. Then,*

$$P(X > at) = o(P(Y > bt)),$$

with $a > 0$ and $b > 0$.

Lemma 6. *Let X and Y be nonnegative random variables. If $P(Y > t) = P(X > a(t + b))$ with $0 < a < \infty$ and $0 < b < \infty$, then Y is LT provided X is LT.*

Let $\{Y_i\}_{i\ge 1}$ be nonnegative i.i.d. random variables independent of the nonnegative random variable N. Define $S_N := \sum_{i=1}^{N} Y_i$. We have following Lemmas 7 [5] and 8.

Lemma 7.

1. *Assume $Y_1 \in \mathcal{RV}(\alpha)$, $E[N] < \infty$, and $P(N > t) = o(P(Y_1 > t))$. Then,*

$$P(S_N > t) \sim E[N]P(Y_1 > t).$$

2. *Assume $N \in \mathcal{RV}(\alpha)$, $E[Y_1] < \infty$, and $P(Y_1 > t) = o(P(N > t))$. Moreover, assume that $E[N] < \infty$ if $\alpha = 1$. Then,*

$$P(S_N > t) \sim P(N > (E[Y_1])^{-1}t).$$

Lemma 8. *Assume $N, Y_1 \in \mathcal{RV}(\alpha)$ with $E[N] < \infty$. Let $P(N > t) = t^{-\alpha}\mathcal{L}_1(t)$ and $P(Y_1 > t) = t^{-\alpha}\mathcal{L}_2(t)$. Then,*

$$P(S_N > t) \sim E[N]P(Y_1 > t) + (E[Y_1])^{\alpha}P(N > t). \tag{11}$$

Lemma 9 (Properties of LT Distributions [8]).

1. *If X and Y are nonnegative LT random variables, then $X + Y$ is LT.*
2. *Let $\{X_i\}_{i\geq 1}$ be i.i.d. LT random variables, and N be integer LT random variable. Then, the random sum $\sum_{i=1}^{N} X_i$ is LT.*
3. *Let L be a nonnegative random variable and $\{X_i\}_{i\geq 1}$ be nonnegative i.i.d. random variables independent of L and satisfying $P(X_i > 0) > 0$. If L is LT, so is $\inf\{n : \sum_{i=1}^{n} X_i \geq L\}$.*

3.3 Proofs of the Preliminary Lemmas

Proof of Lemma 3 to 6. The proof follows easily from the definitions of LT and HT r.v.s. □

Proof of Lemma 8. We use techniques similar to those used in [5] to prove that the lower and upper bounds in (11) asymptotically coincide. For every fixed n_0, we obtain

$$P(S_N > t) = \sum_{n=1}^{n_0} P(N = n)P(S_n > t) + \sum_{n=n_0}^{\infty} P(N = n)P(S_n > t).$$

Since $Y_1 \in \mathcal{RV}(\alpha)$, Y_1 is subexponentially distributed. By the subexponentiality of Y_1 and the independence of N and Y_1, we obtain

$$\sum_{n=1}^{n_0} P(N = n)P(S_n > t) \sim \sum_{n=1}^{n_0} P(N = n)nP(Y_1 > t) \sim E[N]P(Y_1 > t), n_0 \to \infty.$$

For any $1 > \delta > 0$, we obtain for large enough t

$$\sum_{n=n_0+1}^{\infty} P(N = n)P(S_n > t) = \left(\sum_{n=n_0+1}^{t(1-\delta)/E[Y_1]} + \sum_{n=t(1-\delta)/E[Y_1]}^{\infty}\right) P(N = n)P(S_n > t) := I + II.$$

For Term II, we obtain

$$II = \left(\sum_{n=t(1-\delta)/E[Y_1]}^{t(1+\delta)/E[Y_1]} + \sum_{n=t(1+\delta)/E[Y_1]}^{\infty}\right) P(N = n)P(S_n > t) := J_1 + J_2. \tag{12}$$

By the law of large numbers and letting $\delta \downarrow 0$, we obtain

$$J_1 \leq \sum_{n=t(1-\delta)/E[Y_1]}^{t(1+\delta)/E[Y_1]} \left(P(N = n)P\left(\sum_{i=1}^{t(1+\delta)/E[Y_1]} Y_i > t\right)\right) \sim P\left(N > \frac{t(1-\delta)}{E[Y_1]}\right) - P\left(N > \frac{t(1+\delta)}{E[Y_1]}\right) = o(1).$$

For J_2, we have

$$J_2 \leq \sum_{n=t(1+\delta)/E[Y_1]}^{\infty} P(N = n) \sim P\left(N > \frac{1+\delta}{E[Y_1]}t\right), \tag{13}$$

and by the law of large numbers

$$J_2 \geq \sum_{n=t(1+\delta)/E[Y_1]}^{\infty} P(N = n)P\left(\sum_{i=1}^{t(1+\delta)/E[Y_1]} Y_i > t\right) \sim P\left(N > \frac{1+\delta}{E[Y_1]}t\right). \tag{14}$$

Combining (13) and (14) and letting $\delta \downarrow 0$, we have

$$J_2 \sim P\left(N > \frac{t}{E[Y_1]}\right) \sim (E[Y_1])^{\alpha}P(N > t). \tag{15}$$

For Term I, we have

$$I = \sum_{n=n_0+1}^{t(1-\delta)/E[Y_1]} P(N = n)P(S_n - nE[Y_1] > t - nE[Y_1]).$$

Since $n < t(1-\delta)/E[Y_1]$, we obtain that $y := t - nE[Y_1] > nE[Y_1]((1-\delta)^{-1} - 1)$. By large deviations theory [3], [7], it follows that for any $\varepsilon > 0$

$$\lim_{n\to\infty} \sup_{y>\varepsilon n}\left|\frac{P(S_n - nE[Y_1] > y)}{nP(Y_1 > y)} - 1\right| = 0,$$

which implies that there exists some positive constant C such that

$$\lim_{n_0\to\infty} \limsup_{t\to\infty} I \leq \lim_{n_0\to\infty} C \sum_{n=n_0+1}^{\infty} P(N = n)nP(Y_1 > y) = 0.$$

This, in conjunction with (12), (13), and (15), completes the proof. □

4 Asymptotic Analysis of the Transmission Delay

In this section, we study the tail asymptotics for the transmission delay experienced by SUs with PU idle times $\{I_i\}_{i\geq 1}$ following LT distribution.

Theorem 1. *If the message size L is heavy tailed, then the number M of idle periods for sending such file is heavy tailed.*

Theorem 2. *If either the busy period B_i or the message size L is heavy tailed, then the transmission delay $T(L)$ is heavy tailed.*

Theorem 3. *If both the busy period B_i and the message size L are light tailed, then the transmission delay $T(L)$ is light tailed.*

Remark 4. From these results, we see that under the DSA paradigm, SUs can experience light-tailed transmission delay if and only if both the message size of SUs and the busy time of PUs are light tailed. In other words, the heavy-tailed delay originates not only from the heavy-tailed file size, but also from the heavy-tailed busy time. In this case, the SUs' transmission delay probably has infinite moments of certain orders, e.g., mean and variance, and definitely has an infinite moment generating function, i.e., infinite exponential moments of all orders.

Proof of Theorem 1. From (2), we have

$$P(M > t) = P\left(L > \sum_{i=1}^{t} X_i\right). \tag{16}$$

Let $\mu := E[X_1]$. For $\varepsilon \in (0, \mu)$, by the law of large numbers, we obtain

$$\begin{aligned}
&P(M > t)\\
&= P\left(L > \sum_{i=1}^{t} X_i\right)\\
&\geq P\left(L > \sum_{i=1}^{t} X_i \wedge t(\mu - \varepsilon) < \sum_{i=1}^{t} X_i < t(\mu + \varepsilon)\right)\\
&\geq P(L > t(\mu + \varepsilon))P\left(t(\mu - \varepsilon) < \sum_{i=1}^{t} X_i < t(\mu + \varepsilon)\right)\\
&\sim P(L > t(\mu + \varepsilon)).
\end{aligned}$$

Letting $\varepsilon \downarrow 0$ yields $P(M > t) \gtrsim P(L > \mu t)$. Let $t' = \mu t$. For any $\theta > 0$,

$$\begin{aligned}
\lim_{t\to\infty} e^{\theta t} P(M > t) &\geq \lim_{t\to\infty} e^{\theta t} P(L > \mu t)\\
&= \lim_{t'\to\infty} e^{\frac{\theta}{\mu} t'} P(L > t')\\
&= \infty.
\end{aligned}$$

The last equation holds since L is HT. Thus, M is HT by the Definition 2. □

Proof of Theorem 2. We first consider the case where L is HT. For any $\delta > 0$, we have

$$\begin{aligned}
P(T(L) > t) &= P\left(\sum_{i=1}^{M} I_i + B_i > t\right)\\
&\geq P\left(M \geq \frac{t(1+\delta)}{E[X_1]}\right)\\
&\quad - P\left(\sum_{i=1}^{M} I_i < t \wedge M \geq \frac{t(1+\delta)}{E[X_1]}\right)\\
&\geq P\left(M \geq \frac{t(1+\delta)}{E[X_1]}\right) - P\left(\sum_{i=1}^{t(1+\delta)/E[X_1]} I_i < t\right).
\end{aligned}$$

Let $\widetilde{I}_i := E[I_1] - I_i$. Since I_i is LT, then $\widetilde{I}_i$ is LT. Thus, by applying Chernoff bound, we can argue that there exists a positive constant λ such that for large enough t

$$P\left(\sum_{i=1}^{t(1+\delta)/E[X_1]} I_i < t\right) = P\left(\sum_{i=1}^{t(1+\delta)/E[X_1]} \widetilde{I}_i > \delta t\right) < e^{-\lambda t}.$$

Since L is HT, then M is HT by Theorem 2. Thus, for any $0 < \theta < \lambda$

$$\lim_{t\to\infty} e^{\theta t} P(T(t) > t) = \infty. \tag{17}$$

For any $\theta > \lambda$, there always exists a constant $0 < \widetilde{\theta} < \lambda$ such that

$$\lim_{t\to\infty} e^{\theta t} P(T(t) > t) > \lim_{t\to\infty} e^{\widetilde{\theta t}} P(T(t) > t) = \infty. \tag{18}$$

Combining (17) and (18), we have for any $\theta > 0$,

$$\lim_{t\to\infty} e^{\theta t} P(T(t) > t) = \infty. \tag{19}$$

This implies that $T(L)$ is HT by Definition 2.

We will next consider the case where B_i is HT. Since we prove in the previous case that if L is HT, then $T(L)$ is HT, we assume that L is LT. It is easy to see

$$P(T(L) > t) = P\left(\sum_{i=1}^{M} I_i + B_i > t\right) \geq P\left(\sum_{i=1}^{M} B_i > t\right). \tag{20}$$

Which implies $T(L)$ is HT provided one can prove $Z := \sum_{i=1}^{M} B_i$ is HT. Toward this, by the independence between M and B_i, we obtain the moment generating function $M_Z(x)$ of Z, i.e.,

$$M_Z(x) = E\left[e^{x \sum_{i=1}^{M} B_i}\right] = E\left[(E[e^{B_1}])^{xM}\right].$$

Since function $f(y) = a^y$ is convex and B_1 is HT, by Jensen's inequality [10], for all $x > 0$

$$M_Z(x) = E\left[\left(E\left[e^{B_1}\right]\right)^{xM}\right] \geq \left(E\left[e^{B_1}\right]\right)^{xE[M]} = \infty.$$

Thus, it follows that $T(L)$ is HT by the Definition. □

Proof of Theorem 3. By Definition 1, we have $X_i = N_i L_p$ with N_i as a positive integer random variable, where

$$N_i = \sup\left\{n : \sum_{i=1}^{n} n L_p \leq I_i\right\}. \tag{21}$$

It is easy to see

$$P(N_i > n) = P(I_i \geq (n+1)L_p). \tag{22}$$

This implies that N_i is LT by Lemma 6. Accordingly, it follows easily from Definition 3 that $X_i = N_i L_p$ is LT. Therefore, invoking Lemma 9(3), we obtain that M is LT. Since $I_i + B_i$ is LT by Lemma 9(1), we finally obtain that $T(L)$ is LT using Lemma 9(2). □

The above theorems state the conditions under which the SUs' transmission delay exhibits heavy-tailed behavior. The following theorems present the exact asymptotic results for this delay under the regularly varying busy time of PUs and message size of SUs.

Theorem 4. *If $L \in \mathcal{RV}(\alpha)$, then $M \in \mathcal{RV}(\alpha)$ and*

$$P(M > t) \sim P(L > E[X_1]t). \tag{23}$$

Remark 5. Comparing with Theorem 1, Theorem 4 provides more refined results regarding the total number of idle periods an SU occupies to transmit a message. Specifically, if the message size is regularly varying, then the number of idle periods for transmitting such message is also regularly varying with the same index. This implies that if the message size has infinite mean and variance, so does the number of idle periods occupied by SUs.

Theorem 5. *If $L \in \mathcal{RV}(\alpha)$ and B_i is LT, then $T(L) \in \mathcal{RV}(\alpha)$ and*

$$P(T(L) > t) \sim P\left(L > \frac{E[X_1]}{E[I_1] + E[B_1]} t\right). \tag{24}$$

Theorem 6. *If $B_i \in \mathcal{RV}(\alpha)$ and L is LT, then $T(L) \in \mathcal{RV}(\alpha)$ and*

$$P(T(L) > t) \sim E[M]P(B_1 > t). \tag{25}$$

Remark 6. The preceding results establish the relationship between the tail asymptotics of L, B_i, and $T(L)$. Specifically, if one of either the busy time or message size is light tailed and the other is regularly varying, then the tail distribution of the transmission delay is asymptotically proportional to the one with regularly varying distribution. This result implies that the SUs can experience extremely high delay variance and stochastically zero throughput even when the transmitting messages are of finite mean size. For example, if the message size is LT, then its mean is finite. In this case, by Theorem 6, when $2 > \alpha > 1$, the transmission delay does not have finite variance, and when $1 > \alpha > 0$, it does not have finite mean, which implies approximately zero throughput on the average.

Theorem 7. *Assume that $B \in \mathcal{RV}(\alpha_b)$, $L \in \mathcal{RV}(\alpha_l)$, and $E[L] < \infty$. Then, we have*

1. *If $\alpha_b < \alpha_l$, then $T(L) \in \mathcal{RV}(\alpha_b)$ and*

$$P(T(L) > t) \sim E[M]P(B_1 > t). \tag{26}$$

2. *If $\alpha_b \geq \alpha_l$,*

$$\lim_{t\to\infty} \frac{\log[P(T(L) > t)]}{\log t} = -\alpha_l. \tag{27}$$

Corollary 1. *If $B \in \mathcal{RV}(\alpha_b)$, $L \in \mathcal{RV}(\alpha_l)$, and $E[L] < \infty$, then*

$$\lim_{t\to\infty} \frac{\log[P(T(L) > t)]}{\log t} = -\min(\alpha_b, \alpha_l),$$

and accordingly, the moments of orders $m > \min(\alpha_b, \alpha_l)$ is unbounded, i.e.,

$$E[T(L)^m] = \infty.$$

Remark 7. Comparing the Theorem 7 and Theorems 4 to 6, we observe that the exact asymptotic tail for the transmission delay is not available in the case of $\alpha_b \geq \alpha_l$. Instead, Corollary 1 states that if both the busy time and the message size are regularly varying, then the tail heaviness of the transmission delay is asymptotically equivalent to the one with smaller index. In this case, it follows directly from in [4, Theorem 2] that the transmission delay still has infinite moments of orders larger than the index $\min(\alpha_b, \alpha_l)$, even through this delay does not strictly follow regular varying distributions.

The proof of Theorem 4 relies on Lemma 10, which we state and prove first.

Lemma 10. *Let $\widetilde{T}(L) = \sum_{i=1}^{M} I_i$. If $L \in \mathcal{RV}(\alpha)$, then*

$$P(\widetilde{T}(L) > t) \sim P(L > \delta t), \tag{28}$$

where $\delta = E[X_1]/E[I_1]$.

The proof of Lemma 10 relies on Theorem 8 [5]. This technique is similar to the one used in the proof for optimal file fragmentation [8].

Theorem 8 ([5]). *Let $L \in \mathcal{RV}(\alpha)$. Let $R(t)$ be a nonnegative, almost surely nondecreasing random process independent of L. If $R(t)$ satisfies following conditions:*

1. *$R(t)/t \to \gamma$ almost surely as t goes to infinity, with $0 < \gamma < 1$.*
2. *There exists a positive and finite constant K such that $P(R(t)/t < K) = o(P(L > t))$.*

Then, $P(L > R(t)) \sim P(L > \gamma t)$.

Proof of Lemma 10. We define $N_t := \sup\{n : \sum_{i=1}^{n} I_i < t\}$ and $R(t) := \sum_{i=1}^{N_t} X_i$. It is easy to see that $P(\widetilde{T}(L) > t) = P(L > R(t))$. Thus, to prove Lemma 10, it is sufficient to prove Conditions 1 and 2 of Theorem 8 are satisfied. By renewal theory, we have

$$\lim_{t\to\infty} \frac{R(t)}{t} = \frac{E[X_1]}{E[I_1]} = \gamma, \tag{29}$$

almost surely. Since $X_1 \leq_{a.s.} I_1$, we conclude $E[X_1] < E[I_1]$ and $0 < \gamma < 1$, implying that Condition 1 of Theorem 8 is satisfied. Next, we will prove that Condition 2 of Theorem 8 is also satisfied. Let $K = (1-\delta)E[X_1]/((1+\delta)E[I_1])$. Then, for any $1 > \delta > 0$, we have

$$\begin{aligned} P(R(t) < Kt) &\leq P\left(N_t < \frac{t(1-\delta)}{E[I_1]}\right) \\ &\quad + P\left(\sum_{i=1}^{N_t} X_i < Kt \wedge N_t > \frac{t(1-\delta)}{E[I_1]}\right) \\ &:= J_1 + J_2. \end{aligned}$$

For J_1, since I_i is LT, by Chernoff bound, there exists $\lambda_1 > 0$ such that

$$J_1 \leq P\left(\sum_{i=1}^{t(1-\delta)/E[I_1]} I_i > t\right) \leq e^{-\lambda_1 t}. \tag{30}$$

For J_2, let $\widetilde{X}_i = E(X_i) - X_i$, we obtain

$$\begin{aligned} J_2 &< P\left(\sum_{i=1}^{t(1-\delta)/E[I_1]} X_i < Kt\right) \\ &= P\left(\sum_{i=1}^{t(1-\delta)/E[I_1]} \widetilde{X}_i > \frac{\delta(1-\delta)E[X_1]}{(1+\delta)E(I_1)} t\right). \end{aligned}$$

Since X_i is LT, by Chernoff bound, we can always find $\lambda_2 > 0$ such that

$$J_2 \leq e^{-\lambda_2 t}. \tag{31}$$

By (30) and (31), we conclude

$$P(R(t) < Kt) \leq e^{-\lambda_1 t} + e^{-\lambda_2 t}. \tag{32}$$

Since $L \in \mathcal{RV}(\alpha)$, we have

$$\limsup_{t\to\infty} \frac{P(R(t) < Kt)}{P(L > t)} \leq \limsup_{t\to\infty} \frac{e^{-\lambda_1 t} + e^{-\lambda_2 t}}{t^{-\alpha}\mathcal{L}(t)} = 0. \tag{33}$$

The last equality holds since regularly varying distributions are a subclass of HT distributions. Accordingly, for any $\lambda > 0$, $\lim_{t\to\infty}(e^{\lambda t}t^{-\alpha}\mathcal{L}(t)) = \infty$. By (33), we conclude $P(R(t) < Kt) = o(P(L > t))$. Therefore, both Conditions 1 and 2 of Theorem 8 are satisfied. This completes the proof. $\square$

Proof of Theorem 4. Let $\widetilde{T}(L) = \sum_{i=1}^{M} I_i$. By Lemma 10, (23) follows provided one can show that

$$P(\widetilde{T}(L) > t) \sim P\left(M > \frac{t}{E[I_1]}\right). \tag{34}$$

For all $1 > \delta > 0$, we obtain

$$\begin{aligned} P(\widetilde{T}(L) > t) &\le P\left(M \ge \frac{t(1-\delta)}{E[I_1]}\right) \\ &\quad + P\left(\sum_{i=1}^{M} I_i > t \wedge M \le \frac{t(1-\delta)}{E[I_1]}\right) \\ &\le P\left(M \ge \frac{t(1-\delta)}{E[I_1]}\right) + P\left(\sum_{i=1}^{t(1-\delta)/E[X_1]} I_i > t\right) \\ &\sim P\left(M \ge \frac{t(1-\delta)}{E[I_1]}\right). \end{aligned} \tag{35}$$

The last step follows from Chernoff bounds. Letting $\delta \downarrow 0$, this proves the upper bound in (34). As to the lower bound, for all $\delta > 0$, letting $\widetilde{I}_i := E[I_1] - I_i$ yeilds

$$\begin{aligned} P(\widetilde{T}(L) > t) &\ge P\left(M \ge \frac{t(1+\delta)}{E[I_1]}\right) \\ &\quad - P\left(\sum_{i=1}^{M} I_i < t \wedge M \ge \frac{t(1+\delta)}{E[I_1]}\right) \\ &\ge P\left(M \ge \frac{t(1+\delta)}{E[I_1]}\right) - P\left(\sum_{i=1}^{t(1+\delta)/E[X_1]} \widetilde{I}_i > \delta t\right) \\ &\sim P\left(M \ge \frac{t(1+\delta)}{E[I_1]}\right). \end{aligned} \tag{36}$$

Letting $\delta \downarrow 0$, this proves the lower bound in (34). By (35) and (36), we obtain

$$P\left(M > \frac{t}{E[I_1]}\right) \sim P(\widetilde{T}(L) > t) \sim P\left(L > \frac{E[X_1]}{E[I_1]}t\right),$$

which implies $P(M > t) \sim P(L > E[X_1]t)$. This completes the proof of (23) and implies $M \in \mathcal{RV}(\alpha)$ by Lemma 4. $\square$

Proof of Theorem 5. The proof follows easily by the similar arguments used in proving Lemma 10. $\square$

To facilitate the proofs of Theorems 6 and 7, we define $T_I := \sum_{i=1}^{M} I_i$ and $T_B := \sum_{i=1}^{M} B_i$. This implies $T(L) = T_I + T_B$.

Proof of Theorem 6. To prove Theorem 6, we first show that T_I is LT and $P(T_B > t) \sim E[M]P(B_1 > t)$. First, we argue that T_I is LT. Since L is LT, from Lemma 9(3), we conclude that M is LT. This implies that $T_I := \sum_{i=1}^{M} I_i$ is LT using Lemma 9(2).

We now show that $P(T_B > t) \sim E[M]P(B_1 > t)$. Since M is independent of B_i and $B_i \in \mathcal{RV}(\alpha)$, it follows that $P(M > t) = o(P(B_i > t))$ invoking Lemma 5. From Lemma 7(1), we see that

$$P(T_B > t) \sim E[M]P(B_1 > t) \tag{37}$$

which, in turn, implies $T_B \in \mathcal{RV}(\alpha_l)$ by invoking Lemma 1(3).

We are now ready to prove the upper bound in (25). For any $0 < \delta < 1$

$$\begin{aligned} P(T(L) > t) &= P(T_I + T_B > t) \\ &\le P(T_B > (1-\delta)t) + P(T_I > \delta t) \\ &\sim P(T_B > (1-\delta)t). \end{aligned}$$

The last step follows since $P(T_I > \delta t) = o(P(T_B > (1-\delta)t))$ using Lemma 5. Letting $\delta \downarrow 0$, this proves the upper bound in (25). As to the lower bound, it is easy to see

$$P(T(L) > t) = P(T_I + T_B > t) \ge P(T_B > t),$$

which, combining with the upper bound, completes the proof of (25). Moreover, (25) implies $T(L) \in \mathcal{RV}(\alpha)$ using Lemma 1(3). This completes the proof. $\square$

Proof of Theorem 7. We first consider the case where $\alpha_b < \alpha_l$. Since $L \in \mathcal{RV}(\alpha_l)$ and $E[M] < \infty$, using Theorem 4, we obtain that $M \in \mathcal{RV}(\alpha_l)$ and $E[M] < \infty$. This, combining with $\alpha_b < \alpha_l$, implies that $P(M > t) = o(P(B_1 > t))$ using Lemma 3. Invoking Lemma 7(1), we conclude that

$$P(T_B = \sum_{i=1}^{M} B_i > t) \sim E[M]P(B_1 > t),$$

which in turn implies that $T_B \in \mathcal{RV}(\alpha_b)$ by Lemma 4. By Lemma 10, we can see that $T_I \in \mathcal{RV}(\alpha_l)$ since $L \in \mathcal{RV}(\alpha_l)$.

We are now ready to prove the upper bound in (26). For any $1 > \delta > 0$, we obtain that

$$P(T(L) > t) \le P(T_B > (1-\delta)t) + P(T_I > \delta t). \tag{38}$$

Since $T_I \in \mathcal{RV}(\alpha_l)$, $T_B \in \mathcal{RV}(\alpha_b)$, and $\alpha_b < \alpha_l$, using Lemma 3, we obtain that $P(T_I > \delta t) = o(P(T_B > (1-\delta)t))$. This implies that $P(T(L) > t) \lesssim P(T_B > (1-\delta)t)$ from (38). Letting $\delta \downarrow 0$, we verify the upper bound in (26). As to the lower bound, it is easy to see that $P(T(L) > t) \ge P(T_B > t)$. Since the lower and upper bounds coincide, this completes the proof of (26).

We will next consider the case where $\alpha_b \ge \alpha_l$. Since $L \in \mathcal{RV}(\alpha_l)$, from Lemma 10 and regular variations, we obtain that

$$P(T_I > t) \sim \left(\frac{E[I_1]}{E[X_1]}\right)^{\alpha_l} P(L > t). \tag{39}$$

From Theorem 4, we conclude that $M \in \mathcal{RV}(\alpha_l)$ and

$$P(M > t) = P(L > E[X_1]t).$$

If $\alpha_b > \alpha_l$, it follows that $P(B_i > t) = o(P(M > t))$. This implies, using Lemma 7(2), that

$$P(T_B > t) \sim \left(\frac{E[B_1]}{E[X_1]}\right)^{\alpha_l} P(L > t). \quad (40)$$

If $\alpha_b = \alpha_l$, using Lemma 8, we obtain that

$$P(T_B > t) \sim E[M]P(B_1 > t) + (E[B_1])^{\alpha_l} P(L > t). \quad (41)$$

Combining (38), (39), (40), and (41), we obtain that

$$\limsup_{t\to\infty} \frac{\log[P(T(L) > t)]}{\log t} \le -\alpha_l,$$

which, in conjunction with

$$\liminf_{t\to\infty} \frac{\log[P(T(L) > t)]}{\log t} \ge \liminf_{t\to\infty} \frac{\log[P(T_I > t)]}{\log t} \ge -\alpha_l,$$

completes the proof. □

All the above theorems consider the case where the idle periods $\{I_i\}_{i\ge1}$ are LT r.v.s. The following theorem computes the logarithmic asymptotics for the delay under regularly varying idle periods, i.e., $\{I_i\}_{i\ge1} \in \mathcal{RV}(\alpha_I)$.

Theorem 9. *Assume that $B_i \in \mathcal{RV}(\alpha_b)$. If $L \in \mathrm{LT}$ or $L \in \mathcal{RV}(\alpha_l)$ with $\alpha_b < \alpha_l$, we have*

$$\lim_{t\to\infty} \frac{\log[P(T(L) > t)]}{\log t} = -\alpha_b. \quad (42)$$

Assume that $L \in \mathcal{RV}(\alpha_l)$ and $E[L] < \infty$. If $B_i \in \mathrm{LT}$ or $B_i \in \mathcal{RV}(\alpha_l)$ with $\alpha_l \le \alpha_b$, we have

$$\lim_{t\to\infty} \frac{\log[P(T(L) > t)]}{\log t} = -\alpha_l. \quad (43)$$

Remark 8. From the above results, we can see that the tail heaviness (i.e., logarithmic decaying rate) of the delay distribution only depends on either the message size or the busy period, whichever has the heavier tail distribution or the smaller decaying rate, i.e., $\min(\alpha_l, \alpha_b)$. This is consistent with the conclusions made in the case where idle periods are LT r.v.s. This implies that the tail behavior of the idle period distribution has no impact on the tail heaviness of the delay distribution.

Proof of Theorem 9. The proof follows the similar arguments used in proving the asymptotic results under the case where idle periods are LT r.v.s. □

5 Impact of Spectrum Mobility and Multiradio Diversity

In this section, we study the impact of spectrum mobility and multiradio diversity on the delay performance of SUs. By spectrum mobility, we mean that if a PU appears in a channel currently used by an SU, the SU should vacate the channel immediately and continue its transmission in another idle channel. By multiradio diversity, we mean that an SU is equipped with multiple radio interfaces so that it can simultaneously access multiple channels.

5.1 System Model

Assume that there exist $K \ge 1$ PU channels, which are modeled by K independent alternating renewal processes as defined in Section 2. Each channel $K \ge j \ge 1$ is denoted by $CH^j = \{(B_i^{(j)}, I_i^{(j)})\}\}_{i\ge1}$ and channels $\{CH^j\}_{K\ge j\ge1}$ are heterogenous, i.e., $\{B_1^{(j)}\}_{K\ge j\ge1}$ (or/and $\{I_1^{(j)}\}_{K\ge j\ge1}$) are not identically distributed. To simplify the analysis, we assume that the idle periods are light tailed.

5.2 Spectrum Mobility

By spectrum mobility, an SU can switch to the idle channels when its current operating channel is occupied by a PU. As a consequence, the SU sees K channels as a single virtual channel, which stays idle if one of K channels is idle and stays busy if all K channels are busy. This virtual channel can be modeled by a random process that alternates between busy $\{B_i^s\}_{i\ge1}$ and idle $\{I_i^s\}_{i\ge1}$ periods. (Note that neither $\{B_i^s\}_{i\ge1}$ nor idle $\{I_i^s\}_{i\ge1}$ are necessarily i.i.d. random sequences.) The idle period I_i^s of the virtual channel is formed through a sequence of idle periods $\{I_{n_1}^{(c_1)}, I_{n_2}^{(c_2)}, \ldots, I_{n_l}^{(c_k)}\}$ from multiple channels $\{c_1, c_2, \ldots, c_k\}$. The actual idle time $A_i^{(j)}$ an SU can utilize from a particular idle period $I_i^{(j)}$ of channel j depends on channel switching policies, which specify whether and when the SU should switch to channel j if the current channel becomes busy. Obviously, we have $0 \le A_i^{(j)} \le I_i^{(j)}$ and $\{A_i^{(j)}\}_{i\ge1}$ are independent, but not necessarily equally distributed. The delay under spectrum mobility is defined as follows.

Definition 6 (Spectrum Mobility). *Consider a channel $1 \le j \le K$ with busy periods $\{B_i^{(j)}\}_{i\ge1}$, idle periods $\{I_i^{(j)}\}_{i\ge1}$, and the corresponding actual idle times $\{A_i^{(j)}\}_{i\ge1}$. During each $A_i^{(j)}$, we define the transmission time $Y_i^{(j)}$ as*

$$Y_i^{(j)} := \sup\{nL_p : nL_p \le A_i^{(j)}\}. \quad (44)$$

Furthermore, we define

$$N_s^{(j)}(t) := \sup\left\{n_j : \sum_{i=1}^{n_j} \left(I_i^{(j)} + B_i^{(j)}\right) < t\right\}, \quad (45)$$

and the total delay $T_s(L)$ under the spectrum mobility is defined as

$$T_s(L) := \inf\left\{t : \sum_{j=1}^{K} \sum_{i=1}^{N_s^{(j)}(t)} Y_i^{(j)} > L\right\}. \quad (46)$$

5.3 Multiradio Diversity

By multiradio diversity, an SU is equipped with K radio interfaces with each one operating on a different channel. With this feature, there exist two transmission policies: *static multiradio diversity* and *dynamic multiradio diversity*. Under the *static* policy, before transmitting a message, the SU divides it into K fragments with each fragment segmented into packets and sent over a preassigned interface. The total transmission delay is the time for the SU to finish sending all fragments. On the contrary, under the *dynamic* policy, without fragmenting the message before transmission, the SU directly divides the message into packets and dynamically assigns each packet to an interface whenever the channel associated with this interface is idle. The total transmission delay is the time for the SU to finish sending all the packets over multiple interfaces. The transmission delay under the two policies is defined, respectively, as follows.

Definition 7 (Static Multiradio Diversity). *Consider a message of size L, which is divided into fragments of sizes $\{r_i L\}_{K \geq i \geq 1}$ such that $0 \leq r_i \leq 1$ and $\sum_{i=1}^{K} r_i = 1$. Let $T_i(r_i L)$ be the delay of sending a fragment of size $r_i L$ over interface i. Then, the total delay $T_m^s(L)$ under static multiradio diversity is defined as*

$$T_m^s(L) := \max_{K \geq i \geq 1} T_i(r_i L). \tag{47}$$

Definition 8 (Dynamic Multiradio Diversity). *Given a channel $1 \leq j \leq K$ with busy periods $\{B_i^{(j)}\}_{i \geq 1}$ and idle periods $\{I_i^{(j)}\}_{i \geq 1}$. During an idle period $I_i^{(j)}$ of the channel j, we define the transmission time $X_i^{(j)}$ as*

$$X_i^{(j)} := \sup\{nL_p : nL_p \leq I_i^{(j)}\}. \tag{48}$$

Furthermore, we define

$$N_m^{(j)}(t) := \sup\left\{ n_j : \sum_{i=1}^{n_j} \left(I_i^{(j)} + B_i^{(j)}\right) < t \right\}, \tag{49}$$

and the total delay $T_m^d(L)$ under dynamic multiradio diversity is defined as

$$T_m^d(L) := \inf\left\{ t : \sum_{j=1}^{K} \sum_{i=1}^{N_m^{(j)}(t)} X_i^{(j)} > L \right\}. \tag{50}$$

5.4 Asymptotic Delay Analysis

Theorem 10. *Given K channels, where $\{B_1^{(j)}\}_{K \geq j \geq 1}$ are regularly varying random variables with indices $\alpha_1, \alpha_2, \ldots, \alpha_K$, respectively. Define $\alpha^{\Sigma} := \sum_{j \leq K} \alpha_j$, $\alpha^- := \min_{K \geq j \geq 1} \alpha_j$, and $\alpha^+ := \max_{K \geq j \geq 1} \alpha_j$.*

1. *Under spectrum mobility, there exists a channel switching policy such that if $L \in LT$, then*

$$\lim_{t \to \infty} \frac{\log[P(T_s(L) > t)]}{\log t} \leq -\alpha^+. \tag{51}$$

If $L \in \mathcal{RV}(\alpha_l)$ and $E[L] < \infty$, then

$$\lim_{t \to \infty} \frac{\log[P(T_s(L) > t)]}{\log t} \leq -\min(\alpha^+, \alpha_l). \tag{52}$$

2. *Under static multiradio diversity, if $L \in LT$, then*

$$\lim_{t \to \infty} \frac{\log[P\left(T_m^s(L) > t\right)]}{\log t} = -\alpha^-. \tag{53}$$

If $L \in \mathcal{RV}(\alpha_l)$ and $E[L] < \infty$, then

$$\lim_{t \to \infty} \frac{\log[P\left(T_m^s(L) > t\right)]}{\log t} = -\min(\alpha^-, \alpha_l). \tag{54}$$

3. *Under dynamic multiradio diversity, if $L \in LT$,*

$$\lim_{t \to \infty} \frac{\log[P\left(T_m^d(L) > t\right)]}{\log t} = -\alpha^{\Sigma}. \tag{55}$$

If $L \in \mathcal{RV}(\alpha_l)$ and $E[L] < \infty$, then

$$\lim_{t \to \infty} \frac{\log[P\left(T_m^d(L) > t\right)]}{\log t} = -\min(\alpha^{\Sigma}, \alpha_l). \tag{56}$$

Remark 9. From the above results, we see that both the spectrum mobility and the dynamic multiradio diversity can greatly improve the delay performance of SUs, while static multiradio diversity can deteriorate it. Particularly, Theorem 10(3) implies that under dynamic multiradio diversity, the delay distribution decays at a rate equal to the sum of the indices of all channels, i.e., $\alpha^{\Sigma} := \sum_{i \leq K} \alpha_i$. This rate is much higher than the one under the single channel case, which, as implied by Theorem 6 and Corollary 1, is equal to the index α_i of a particular channel i. On the other hand, Theorem 10(1) implies that the decaying rate of the delay distribution under spectrum mobility is lower bounded by that of the best channels, which have the largest index α_j among all channels. On the contrary, Theorem 10(2) indicates that the delay distribution under static multiradio diversity decays as faster as that of the worst channels, which have the smallest index α_j among all channels. As a consequence, compared with the single channel case, spectrum mobility and dynamic multiradio diversity can mitigate the heavy-tailed delay by increasing the orders of its finite moments at least to $\max_{K \geq j \geq 1} \alpha_j$ and exactly to $\alpha^{\Sigma} := \sum_{i \leq K} \alpha_i$, respectively, while static multiradio diversity can aggravate it by decreasing the orders of its finite moments to $\min_{K \geq j \geq 1} \alpha_j$.

Corollary 2. *If $L \in \mathcal{RV}(\alpha_l)$ and $\alpha^{\Sigma} \geq \alpha_l$, then we have*

$$\lim_{t \to \infty} \frac{\log[P\left(T_m^d(L) > t\right)]}{\log t} = -\alpha_l.$$

Remark 10. This corollary directly follows from Theorem 10(3) and implies that as the number of channels increases, dynamic multiradio diversity can achieve the optimum delay performance by maximizing the orders of finite moments. In other words, dynamic multiradio diversity can guarantee the delay with finite moments up to order α_l, which is the highest order that we can expect when transmitting heavy-tailed messages of index α_l by using any multiple channel schemes.

Corollary 2 characterizes the logarithmic asymptotics of the delay distribution for dynamic multiradio diversity. The following Theorem 11 computes the exact asymptotic results under some confined conditions.

Theorem 11. *Given K channels, where $\{B_1^{(j)}\}_{K \geq j \geq 1}$ are regularly varying r.v.s with indices $\alpha_1, \alpha_2, \ldots, \alpha_K$, respectively. Define $\rho = \sum_{j=1}^{K} E[I_1^{(j)}]/(E[I_1^{(j)} + B_1^{(j)}])$ and $\alpha^* := \sum_{j \leq K : \alpha_j > 1} (\alpha_j - 1)$. Assume that $L \in \mathcal{RV}(\alpha_l)$. If $\rho < 1$ and $\alpha^* > \alpha_l$, then*

$$P(T_m^d(L) > t) \sim P\left(L > \sum_{j=1}^{K} \frac{E\left[X_1^{(j)}\right]}{E\left[I_1^{(j)} + B_1^{(j)}\right]} t \right).$$

Remark 11. The preceding result indicates that as more channels are employed, the tail distribution of the delay under dynamic multiradio diversity is asymptotically equivalent to that of the message size L scaled by a constant, i.e., $(\sum_{j=1}^{K} E[X_1^{(j)}]/E[I_1^{(j)}+B_1^{(j)}])^{-\alpha_l}$.

Proof of Theorem 10. Define $T_j(L)$ as the total delay of sending a message of size L over a particular channel $K \geq j \geq 1$. By Definition 1, we have $T_j(L) := \sum_{i=1}^{M_j}\{I_i^{(j)} + B_i^{(j)}\}$, where $M_j := \inf\{m : \sum_{i=1}^{m} X_i^{(j)} \geq L\}$ and $X_i^{(j)} := \sup\{nL_p : nL_p \leq I_i^{(j)}\}$. To prove (51) and (52), we consider a priority-based channel switching policy, where if the currently used channel becomes busy, an SU always switches to the channel j^+ with the maximum index $\alpha^+ = \max_{K\geq j\geq 1}\alpha_j$ provided that this channel is idle. This implies from (44) that $A_i^{(j)} = I_i^{(j)}$. Since the SU cannot perform channel switching in the middle of a packet being transmitted, it follows from (44) that $Z_i^{(j)} \leq Y_i^{(j)} \leq X_i^{(j)}$, where $Z_i^{(j)} := (X_i^{(j)} - L_p)1(I_i^{(j)} > L_p)$, from which it follows that $T_s(L) \leq_{a.s.} T_{j^+}(L)$, where $T_{j^+}(L) := \sum_{i=1}^{M'_j}\{I_i^{(j)} + B_i^{(j)}\}$ and $M'_j := \inf\{m : \sum_{i=1}^{m} Z_i^{(j)} \geq L\}$. This implies that

$$P(T_s(L) > t) \leq P\big(T_{j^+}(L) > t\big). \tag{57}$$

If $L \in \mathrm{LT}$, using Theorem 6, we obtain $T_{j^+}(L) \in \mathcal{RV}(\alpha^+)$. This implies from (57)

$$\lim_{t\to\infty}\frac{\log[P(T_s(L) > t)]}{\log t} \leq -\alpha^+.$$

which completes the proof of (51).

If $L \in \mathcal{RV}(\alpha_l)$, from Corollary 1, we conclude

$$\lim_{t\to\infty}\frac{\log[P(T_{j^+}(L) > t)]}{\log t} = -\min(\alpha_l, \alpha^+),$$

which implies from (57) that

$$\lim_{t\to\infty}\frac{\log[P(T_s(L) > t)]}{\log t} \leq -\min(\alpha_l, \alpha^+).$$

This completes the proof of (52).

We will next prove Theorem 10(2). By the definition of $T_m^s(L)$, we obtain

$$P(T_m^s(L) > t) = P\left(\bigcup_{i=1}^{K} T_i(r_iL) > t\right).$$

This, using the union bound, implies

$$P(T_j(r_jL) > t) \leq P(T_m^s(L) > t) \leq \sum_{j=1}^{K} P(T_j(r_jL) > t). \tag{58}$$

If L is LT, by Theorem 6, we have $T_j(r_jL) \in \mathcal{RV}(\alpha_j)$. If $\alpha_j > \alpha_i$, using Lemma 3, we obtain $P(T_j(r_jL) > t) = o(P(T_i(r_iL) > t)$. This implies from (58) that

$$-\min_{K\geq j\geq 1}\alpha_j \leq \lim_{t\to\infty}\frac{\log[P\big(T_m^s(L) > t\big)]}{\log t} \leq -\min_{K\geq j\geq 1}\alpha_j,$$

which completes the proof of (53).

If $L \in \mathcal{RV}(\alpha_l)$, from Corollary 1, we have

$$\lim_{t\to\infty}\frac{\log[P(T_j(r_jL) > t)]}{\log t} = -\min(\alpha_l, \alpha_i),$$

which implies from (58) that

$$\lim_{t\to\infty}\frac{\log[P(T_m^s(L) > t)]}{\log t} = -\min_{K\geq i\geq 1}\min(\alpha_l, \alpha_i).$$

This completes the proof of (54).

We will now prove Theorem 10(3). Let

$$T_m^{(j)}(L) := \inf\left\{t : \sum_{i=1}^{N_m^{(j)}(t)} X_i^{(j)} > L\right\}, \tag{59}$$

which, combining (48) and (49), defines the total delay of sending a message of size L over a single channel $K \geq j \geq 1$. This implies that $T_m^{(j)}(L) \stackrel{d}{=} T_j(L)$. Since $T_m^d(L) \leq_{a.s.} T_m^{(j)}(L)\ \forall 1 \leq j \leq K$, letting $S_n^{(j)} := \sum_{i=1}^{n} I_i^{(j)} + B_i^{(j)}$ and $M^+ := \max_{1\leq j\leq K} M_j$, we have

$$\begin{aligned}
P(T_m^d(L) > t) &\leq P\left(\min_{1\leq j\leq K} T_m^{(j)}(L) > t\right)\\
&= P\left(\bigcap_{j=1}^{K}\left\{\sum_{i=1}^{M_j}(I_i^{(j)} + B_i^{(j)}) > t\right\}\right)\\
&\leq \sum_{n=1}^{n_0} P(M^+ = n) P\left(\bigcap_{j=1}^{K} S_n^{(j)} > t\right)\\
&\quad + \sum_{n=n_0+1}^{\infty} P(M^+ = n) P\left(\bigcap_{j=1}^{K} S_n^{(j)} > t\right)\\
&:= I + II.
\end{aligned}$$

For Term I, by Lemma 7 (1), we have

$$\begin{aligned}
I &= \sum_{n=1}^{n_0} P(M^+ = n)\prod_{j=1}^{K} P\big(S_n^{(j)} > t\big)\\
&\sim E[(M^+)^K]\prod_{j=1}^{K} P\big(B_1^{(j)} > t\big),\ n_0 \to \infty.
\end{aligned} \tag{60}$$

For Term II, for any $0 < \delta < 1$, we obtain

$$\begin{aligned}
II &\leq \sum_{n=n_0+1}^{\infty} P(M^+ = n) P\big(S_n^{(j)} > t\big)\\
&= \left(\sum_{n=n_0+1}^{\delta t} + \sum_{n=\delta t}^{\infty}\right) P(M^+ = n) P\big(S_n^{(j)} > t\big)\\
&:= J_1 + J_2.
\end{aligned} \tag{61}$$

If $\alpha_j \leq 1$, let $\mu := 0$. Otherwise, let $\mu := E[I_1^{(j)}] + E[B_1^{(j)}]$. For $n < \delta t$, we have $y := t - n\mu > n(\delta^{-1} - \mu)$. Letting $Y_1^{(j)} := I_1^{(j)} + B_1^{(j)}$, it follows from large deviations [3], [7] theory that for any $\varepsilon > 0$

$$\lim_{n\to\infty}\sup_{y>\varepsilon n}\left|\frac{P(S_n^{(j)} - n\mu > y)}{nP(Y_1^{(j)} > y)} - 1\right| = 0.$$

This implies that there exists $C > 0$ such that as $n_0 \to \infty$

$$\limsup_{t\to\infty} J_1 \le \lim_{n_0\to\infty} C \sum_{n=n_0+1}^{\infty} P(M^+ = n) n P\big(Y_1^{(j)} > y\big) = 0. \quad (62)$$

For term J_2, by the union bound, we have

$$J_2 \le P(M^+ > \delta t) \le \sum_{i=1}^{K} P(M_j > \delta t).$$

This, combining with (60), (61), and (62), proves the upper bound of $T_m^d(L)$, i.e.,

$$P\big(T_m^d(L) > t\big) \lesssim c_1 \prod_{j=1}^{K} P\big(B_1^{(j)} > t\big) + \sum_{i=1}^{K} P(M_j > \delta t),$$

where $c_1 := E[(M^+)^K]$. If $L \in LT$, by Lemma 9(3), it follows that $M_j \in LT$, which implies

$$\lim_{t\to\infty} \frac{\log[P\big(T_m^d(L) > t\big)]}{\log t} \le -\sum_{j=1}^{K} \alpha_j. \quad (63)$$

If $L \in \mathcal{RV}(\alpha_l)$, it follows from Theorem 4 that $M_j \in \mathcal{RV}(\alpha_l)$, which implies that

$$\lim_{t\to\infty} \frac{\log[P\big(T_m^d(L) > t\big)]}{\log t} \le -\min\left(\alpha_l, \sum_{j=1}^{K} \alpha_j\right). \quad (64)$$

As to the lower bound, by the similar arguments as the proof of upper bound, we have

$$\begin{aligned} P(T_m^d(L) > t) &\ge P\left(\bigcap_{j=1}^{K} \left\{ T_j\left(\frac{L}{K}\right) > t \right\}\right) \\ &\gtrsim c_2 \prod_{j=1}^{K} P\big(B_1^{(j)} > t\big), \end{aligned} \quad (65)$$

where c_2 is a constant. Given K channels, we have $T_m^d(L) > L/K$ surely, which implies that $P(T_m^d(L) > t) > P(L/K > t)$. This, combining with (65), proves the lower bound of (55) and (56). This, in conjunction with the upper bound (63) and (64), completes the proof. □

The proof of Theorem 11 relies on Lemma 11, which corresponds to the of [7, Corollaries 1.6 and 1.8].

Lemma 11. *Let $X_1, X_2, \ldots, X_n$ be independent random variables with $E[X_i] = 0$, for $i = 1, 2, \ldots, n$ and define $A_t^+ := \sum_{i=1}^n \int_{u\ge 0} u^t dP(X_i < u)$.*

1. *If $1 \ge t \ge 2$ and $A_t^+ < \infty$, then for $y^t \ge 4A_t^+$ and $x > y$*

$$P\left(\sum_{i=1}^{n} X_i \ge x\right) \le \sum_{i=1}^{n} P(X_i > y) + \left(\frac{e^2 A_t^+}{x y^{t-1}}\right)^{x/2y}.$$

2. *If $t \ge 2$ and $A_t^+ < \infty$, then*

$$P\left(\sum_{i=1}^{n} X_i \ge x\right) \le c_t^{(1)} A_t^+ x^{-t} + exp\left\{\frac{-c_t^{(2)} x^2}{B_n^2}\right\},$$

where $c_t^{(1)} = (1 + 2/t)^t$, $c_t^{(2)} = 2(t+2)^{-2} e^{-t}$, and $B_n^2 = \sum_{i=1}^n E[(X_i)^2]$.

Proof of Theorem 11. By the definition of $T_m^d(L)$ in (50), we have

$$P\big(T_m^d(L) > t\big) = P\left(\sum_{j=1}^{K} \sum_{i=1}^{M_j(t)} X_i^{(j)} \le L\right).$$

Define $R(t) := \sum_{j=1}^K \sum_{i=1}^{M_j(t)} X_i^{(j)}$. Using renewal theory, we obtain $\lim_{t\to\infty} R(t)/t = \gamma$, where $\gamma = \sum_{j=1}^K E[X_1^{(j)}]/(E[I_1^j + B_1^{(j)}])$. To prove Theorem 11, it is sufficient to show that the conditions of Theorem 8 are satisfied. Since $X_1^{(j)} <_{a.s.} I_1^{(j)}$, this implies that $E[X_1^j] < E[I_1^j]$. It follows from the assumption $\sum_{j=1}^K E[I_1^{(j)}/(E[I_1^j + B_1^{(j)}])] < 1$ that $\gamma < 1$, which verifies the first condition of Theorem 8. To verify the second condition, we define

$$\begin{cases} j^+ = \arg\max_{1\le j\le K} \big(E\big[I_1^{(j)} + B_1^{(j)}\big]\big), \\ j^- = \arg\max_{1\le j\le K} E\big[X_1^{(j)}\big]. \end{cases} \quad (66)$$

Let $\varepsilon := (1-\delta)E[X_1^{(j^-)}]/((1+\delta)(E[I_1^{(j^+)} + B_1^{(j^+)}]))$ and $\sigma := (1-\delta)/E[I_1^{(j^+)} + B_1^{(j^+)}]$. Then, for any $0 < \delta < 1$, we obtain

$$\begin{aligned} P(R(t) < \varepsilon t) &\le P\left(\max_{1\le j\le K} \{N_m^{(j)}(t)\} \le \sigma t\right) \\ &\quad + P\left(\sum_{j=1}^{K} \sum_{i=1}^{N_m^{(j)}(t)} X_i^{(j)} \le \varepsilon t \wedge \max_{1\le j\le N} \{N_m^{(j)}(t)\} > \sigma t\right) \\ &:= J_1 + J_2. \end{aligned} \quad (67)$$

For term J_1, it follows from the independence of $\{N_m^{(j)}(t)\}_{j=1}^N$ that

$$J_1 = P\left(\bigcap_{j=1}^{K} N_m^{(j)}(t) \le \sigma t\right) = \prod_{j=1}^{K} P\big(N_m^{(j)}(t) \le \sigma t\big). \quad (68)$$

For any $B_i^{(j)}$ with $\alpha_j > 1$, let $Z_i^{(j)} := I_i^{(j)} + B_i^{(j)} - E[I_i^{(j)} + B_i^{(j)}]$. By (49), we obtain the following upper bound under the condition $\alpha_j > 1$, i.e.,

$$\begin{aligned} P(N_m^{(j)}(t) \le \sigma t) &= P\left(\sum_{i=1}^{\sigma t} \big(I_i^{(j)} + B_i^{(j)}\big) > t\right) \\ &= P\left(\sum_{i=1}^{\sigma t} Z_i^{(j)} > t - \frac{(1-\delta)\big(E\big[I_i^{(j)} + B_i^{(j)}\big]\big)}{E\big[I_1^{(j^+)} + B_1^{(j^+)}\big]} t\right) \\ &\le P\left(\sum_{i=1}^{\sigma t} Z_i^{(j)} > \delta t\right). \end{aligned}$$

Since $I_1^{(j)} \in$ LT and $B_1^{(j)} \in \mathcal{RV}(\alpha_j)$, an argument similar to the proof of Theorem 6 yields $P(Z_i^{(j)} > t) \sim P(B_i^{(j)} > t)$. This implies $Z_i^{(j)} \in \mathcal{RV}(\alpha_j)$. Let $\alpha_j^\Delta := \alpha_j - \Delta$. For an arbitrary small $\Delta > 0$, we have $E[(Z_i^{(j)})^{\alpha_j^\Delta}] < \infty$. If $1 \le \alpha_j \le 2$, letting $y = \delta t/2$, an application of Lemma 11(1) and Markov inequality yields

$$\begin{aligned} P\left(\sum_{i=1}^{\sigma t} Z_i^{(j)} \ge \delta t\right) &\le \frac{2^{\alpha_j^\Delta} \sigma E\big[\big(Z_i^{(j)}\big)^{\alpha_j^\Delta}\big]}{\delta^{\alpha_j^\Delta} t^{\alpha_j^\Delta - 1}} + \frac{\sigma e^2 E\big[\big(Z_i^{(j)}\big)^{\alpha_j^\Delta}\big]}{\delta^{\alpha_j^\Delta} 2^{1-\alpha_j^\Delta} t^{\alpha_j^\Delta - 1}} \\ &\le C_j^{(1)} t^{-(\alpha_j^\Delta - 1)}, \end{aligned} \quad (69)$$

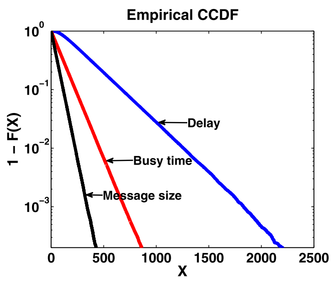

Fig. 2. Delay under LT message size and LT busy time.

where $C_j^{(1)}$ is a constant. If $\alpha_j > 2$, by Lemma 11 (2), we obtain

$$P\left(\sum_{i=1}^{\sigma t} Z_i^{(j)} > \delta t\right) \le \frac{\sigma c_t^{(1)} E\left[\left(Z_i^{(j)}\right)^{\alpha_j^{\Delta}}\right]}{\delta^{\alpha_j^{\Delta}} t^{\alpha_j^{\Delta}-1}} + exp\left\{\frac{-c_t^{(2)}(\delta t)^2}{\sigma t E\left[\left(Z_i^{(j)}\right)^2\right]}\right\} \le C_j^{(2)} t^{-(\alpha_j^{\Delta}-1)}, \tag{70}$$

where $C_j^{(2)}$ is a constant, $c_t^{(1)} = (1 + 2/\alpha_j^{\Delta})^{\alpha_j^{\Delta}}$, and $c_t^{(2)} = 2(\alpha_j^{\Delta} + 2)^{-2} e^{-\alpha_j^{\Delta}}$. Combining (68), (69), and (70) yields

$$J_1 = O\left(t^{-\sum_{i \le K: \alpha_i > 1}(\alpha_i - 1)}\right) = O(t^{-\alpha^*}). \tag{71}$$

For J_2, using the union bound, we obtain

$$\begin{aligned} J_2 &\le P\left(\sum_{j=1}^{K} \sum_{i=1}^{N_m^{(j)}(t)} X_i^{(j)} \le \varepsilon t \bigwedge \left\{\bigcup_{l=1}^{K} N_m^{(l)}(t) > \sigma t\right\}\right) \\ &\le P\left(\bigcup_{l=1}^{K}\left\{\sum_{j=1}^{K}\sum_{i=1}^{N_m^{(j)}(t)} X_i^{(j)} \le \varepsilon t \wedge N_m^{(l)}(t) > \sigma t\right\}\right) \\ &\le \sum_{l=1}^{K} P\left(\sum_{i=1}^{\sigma t} X_i^{(l)} \le \varepsilon t\right). \end{aligned}$$

Let $\widetilde{X}_i^{(l)} = E[X_i^{(l)}] - X_i^{(l)}$. By Chernoff bound, we can always find $\lambda > 0$ such that

$$P\left(\sum_{i=1}^{\sigma t} X_i^{(l)} \le \varepsilon t\right) \le P\left(\sum_{i=1}^{\sigma t} \widetilde{X}_i^{(l)} > \frac{\delta \sigma E\left[X_i^{(l)}\right]}{(1+\delta)} t\right) \le e^{-\lambda t}, \tag{72}$$

which, combining with (71), implies from (67) that $P(R(t) < \varepsilon t) = O(t^{-\alpha^*})$. As a consequence, if $\alpha^* > \alpha_l$, we obtain $P(R(t) < \varepsilon t) = o(P(L > t))$ which verifies the second condition of Theorem 8 and completes the proof. □

6 Simulation Results

In this section, we use simulations to illustrate our theoretical results. As presented in the preceding theorems, the SUs' HT delay is attributed to the HT message size as well as the HT PU busy time. To verify this result, we choose Pareto and exponential distributions to represent HT and LT distributions, respectively. We say that a random variable $X \in \mathcal{PAR}(\alpha, x_m)$ if X follows a Pareto

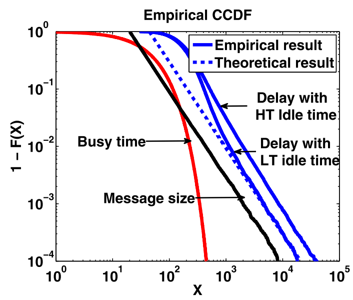

Fig. 3. Delay under HT message size and LT PU busy time.

distribution with parameter α and x_m, i.e., $P(X > t) = (x_m/x)^{\alpha}$. We say that a random variable $X \in \mathcal{EXP}(\lambda)$ if X follows an exponential distribution with parameter λ, i.e., $P(X > t) = e^{-\lambda t}$. Without loss of generality, we let packet size $L_p = 10$.

We first study the delay with both the busy time and the message size being LT. Specifically, we let $\{L, I_i\} \in \mathcal{EXP}(0.02)$ and $B_i \in \mathcal{EXP}(0.01)$. It is shown in Fig. 2 that the delay tail distribution is a straight line on a y-log scale, implying that the delay is LT, specifically, exponentially distributed. Next, we investigate the cases with the HT SU message size and/or the HT PU busy periods. All the following simulation results are plotted on log-log coordinates, by which regularly varying HT distribution can manifest itself as a straight line.

We next investigate the cases where either the message size or the PU busy time is HT. We first let $\{B_i, I_i\} \in \mathcal{EXP}(0.02)$ and $L \in \mathcal{PAR}(1.5, 20)$. It is seen in Fig. 3 that the tail distribution of the transmission delay exhibits itself as a straight line, which is parallel to that of the message size and overlapped with the theoretical delay tail distribution indicated by Theorem 5. This means that the transmission delay is HT and its tail distribution is as heavy as that of the message size. On the contrary, if busy time is HT while message size is LT, as indicated by Theorem 6, SUs can experience the transmission delay which has a tail distribution as heavy as that of the PU channel busy time. To verify this, we let $\{L, I_i\} \in \mathcal{EXP}(0.02)$ and $B_i \in \mathcal{PAR}(1.2, 10)$. It is seen in Fig. 4 that the straight line that represents the tail distribution of the transmission delay is parallel to that of the PU busy time and coincident with the theoretical one stated by Theorem 6. This indicates that the delay tail distribution is as heavy as that of the PU busy time. In sum, Figs. 3 and 4 verify Theorems 5 and 6 by showing that if one of the busy time or message size is light tailed and the other is regularly

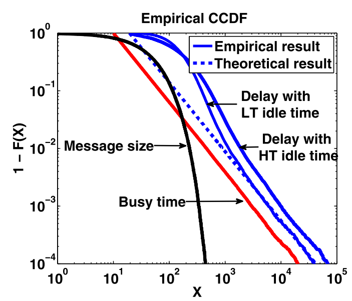

Fig. 4. Delay under LT message size and HT PU busy time.

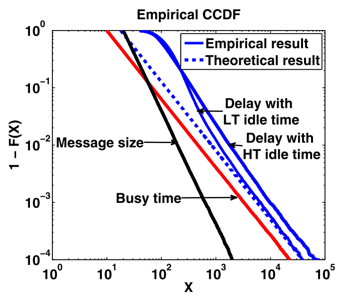

Fig. 5. Delay under HT message size and HT PU busy time with $\alpha_b < \alpha_l$.

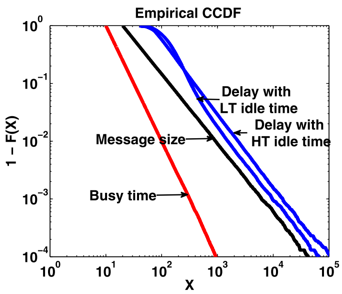

Fig. 6. Delay under HT message size and HT PU busy time with $\alpha_b \geq \alpha_l$.

varying, then the tail of the transmission delay is asymptotically proportional to the one with regularly varying distribution.

We now study the case where both message size and PU busy time are HT. In this case, Theorem 7 states that the delay performance is determined by either the busy time or the message size whichever has the heavier tail. Fig. 5 shows the case, where $\alpha_l > \alpha_b$ by letting $L \in \mathcal{PAR}(2, 20)$, $B_i \in \mathcal{PAR}(1.2, 10)$, and $I_i \in \mathcal{EXP}(0.02)$, while Fig. 6 illustrates the case, where $\alpha_l \leq \alpha_b$ by letting $L \in \mathcal{PAR}(1.2, 20)$, $B_i \in \mathcal{PAR}(2, 10)$, and $I_i \in \mathcal{EXP}(0.02)$. It is shown in Figs. 5 and 6 that the tail distribution of the delay is parallel to that of either the message size or the busy time whichever has the heavier tail or smaller index, which is consistent with Theorem 7. Moreover, Fig. 5 also verifies the exact asymptotic result stated in Theorem 7(1) by showing its consistence with the empirical one.

To show the impact of the HT idle time on the delay performance, we also plot the delay tail distribution with $I_i \in \mathcal{PAR}(1.2, 10)$ in Figs. 3, 4, 5, and 6, respectively. It can be seen that the delay tail distribution with HT idle time is parallel to the one with LT idle time in each figure. This is as expected since as indicated by Theorem 9, HT idle time has no impact on the tail heaviness of the delay.

We now evaluate the impact of spectrum mobility and static multiradio diversity on the delay performance of cognitive radio users. As indicated by Theorem 10(1) and 10(2), the delay under spectrum mobility is determined by the best channel which has the busy time with the lightest tail, while the delay under static multiradio diversity is determined by the worst channel with the busy time having the heaviest tail. To verify this, we consider the scenario where there exists three PU channels with LT idle times, i.e., $\{I_i^{(1)}, I_i^{(2)}, I_i^{(3)}\} \in \mathcal{EXP}(0.01)$, and HT busy times, i.e., $B_i^{(1)} \in \mathcal{PAR}(1, 10)$, $B_i^{(2)} \in \mathcal{PAR}(0.6, 10)$, and $B_i^{(3)} \in \mathcal{PAR}(0.4, 10)$. We evaluate the delay under the case with HT message size as well as with LT message size by letting

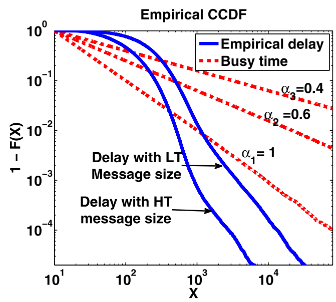

Fig. 7. Delay under spectrum mobility.

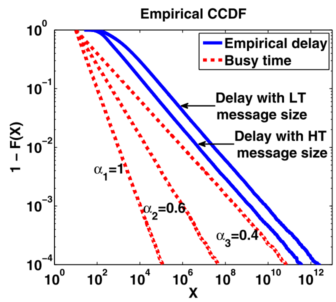

Fig. 8. Delay under static multiradio diversity.

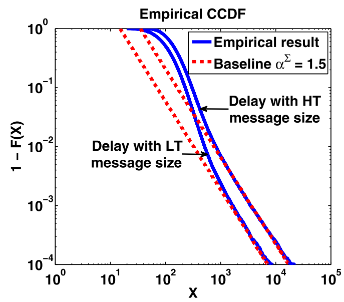

Fig. 9. Delay under dynamic multiradio diversity with $\alpha_l > \alpha^{\Sigma}$.

$L \in \mathcal{PAR}(2, 10)$ and $L \in \mathcal{EXP}(0.01)$, respectively. As shown in Fig. 7, by taking advantage of spectrum mobility, the delay tail distribution decays faster than that of the best channel, which has the lightest tail or largest index $\alpha_1 = 1$. This implies the existence of bounded average delay. This is in sharp contrast to the delay performance of static multiradio diversity illustrated in Fig. 8, where the delay tail distribution decays as fast as the worst channel with the heaviest tail or smallest index $\alpha_3 = 0.4$. This implies that the SU will experience unbounded delay even when transmitting messages with finite mean.

We finally investigate the delay performance under dynamic multiradio diversity. We first consider the case where there exists three PU channels with LT idle times, i.e., $\{I_i^{(1)}, I_i^{(2)}, I_i^{(3)}\} \in \mathcal{EXP}(0.01)$, and HT busy times, i.e., $B_i^{(1)} \in \mathcal{PAR}(0.4, 10)$, $B_i^{(2)} \in \mathcal{PAR}(0.5, 10)$, and $B_i^{(3)} \in \mathcal{PAR}(0.6, 10)$. Fig. 9 shows the delay of sending messages with LT size, i.e., $L \in \mathcal{EXP}(0.05)$, and messages with HT size, i.e., $L \in \mathcal{PAR}(2, 30)$, respectively. It can be seen that the delay tail distribution, as expected from Theorem 10(3), matches the baseline one which has the index of 1.5, i.e., the sum of the indices ($\alpha_1 = 0.4, \alpha_2 = 0.5, \alpha_3 = 0.6$) of the three channels. This implies that the SU will have finite average delay, even through the average delay is unbounded if the message is transmitted on each individual channel alone.

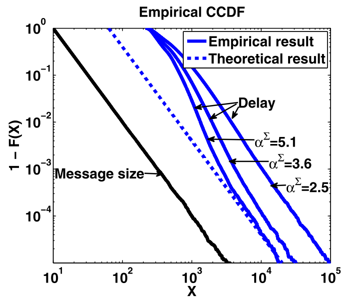

Fig. 10. Delay under dynamic multiradio diversity with $\alpha_l \leq \alpha^{\Sigma}$.

Moreover, Fig. 10 shows that as the sum of indices increases and becomes larger than 2, which is the index of message size, the tail heaviness of the delay is asymptotically equivalent to that of the message size. In addition, when the sum of the indices satisfies the condition $\sum_{i=1}^{3}(\alpha_i - 1) > \alpha_l$, e.g., $\sum_{i=1}^{3}\alpha_i = 5.1$, by Theorem 11, we can obtain the exact asymptotic result of the delay tail distribution, which, as shown in Fig. 10, is consistent with the empirical one.

7 Conclusions

This paper provides an asymptotic analysis of the transmission delay experienced by SUs. It is shown that SUs can have light-tailed delay if and only if both the busy time of PU channels and the message size of SUs are light tailed. In other words, the heavy-tailed transmission delay can originate from either the heavy-tailed busy time or the heavy-tailed message size. In this case, it is proven that if one of the busy time or the message size is light tailed and the other is regularly varying, then the transmission delay is regularly varying with the same index. Furthermore, if both the busy time and the message size are regularly varying with different indices, then the tail distribution of the delay is as heavy as the one with the smaller index. Moreover, to exploit benefits of multiple PU channels, spectrum mobility and multiradio diversity are considered. It is shown that both spectrum mobility and dynamic multiradio diversity can greatly mitigate the heavy-tailed delay by maximizing the orders of its finite moments, while by doing the opposite, static multiradio diversity can aggravate the heavy-tailed delay.

Acknowledgments

This work was supported by the US Army Research Office (ARO) as a Multidisciplinary University Research Initiative (MURI) under Grant No. W911NF-08-1-0238. The preliminary version was presented at IEEE International Dynamic Spectrum Access Networks Symposium (DySPAN 2011) [12].

References

[1] I.F. Akyildiz, W.Y. Lee, and K. Chowdhury, "CRAHNs: Cognitive Radio Ad Hoc Networks," *J. Ad Hoc Networks,* vol. 7, no. 5, pp. 810-836, July 2009.

[2] N.H. Bingham, C.M. Goldie, and J.L. Teugels, *Regular Variation.* Cambridge Univ., 1989.

[3] D.B.H. Cline and T. Hsing, "Large Deviation Probabilities for Sums of Random Variables with Heavy or Subexponential Tails," technical report, Texas A&M Univ., 1998.

[4] D.J. Daley, "The Moment Index of Minima 2," *J. Applied Probability,* vol. 38A, pp. 33-36, 2001.

[5] G. Fay, B. Gonzalez-Arevalo, T. Mikosch, and G. Samorodnitsky, "Modeling Teletraffic Arrivals by a Poisson Cluster Process," *Queueing Systems: Theory and Applications,* vol. 54, no. 2, pp. 121-140, 2006.

[6] A. Laourine, S. Chen, and L. Tong, "Queuing Analysis in Multichannel Cognitive Spectrum Access: A Large Deviation Approach," *Proc. IEEE INFOCOM,* Mar. 2010.

[7] S.V. Nagaev, "Large Deviations of Sums of Independent Random Variables," *Annals Probability,* vol. 7, pp. 745-789, 1979.

[8] J. Nair, M. Andreasson, L.L.H. Andrew, S.H. Low, and J.C. Doyle, "File Fragmentation over an Unreliable Channel," *Proc. IEEE INFOCOM,* Mar. 2010.

[9] K. Park and W. Willinger, *Self-Similar Network Traffic and Performance Evaluation.* Wiley, 2000.

[10] W. Rudin, *Real and Complex Analysis.* McGraw-Hill, 1987.

[11] K. Sriram and W. Whitt, "Characterizing Superposition Arrival Processes in Packet Multiplexers for Voice and Data," *IEEE J. Selected Areas Comm.,* vol. 4, no. 6, pp. 833-846, Sept. 1986.

[12] P. Wang and I.F. Akyildiz, "Can Dynamic Spectrum Access Induce Heavy Tailed Delay?" *Proc. IEEE Symp. New Frontiers in Dynamic Spectrum Access Networks (DySPAN '11),* May 2011.

[13] S. Wang, J. Zhang, and L. Tong, "Delay Analysis for Cognitive Radio Networks with Random Access: A Fluid Queue View," *Proc. IEEE INFOCOM,* Mar. 2010.

[14] M. Wellens and P. Mahonen, "Lessons Learned from an Extensive Spectrum Occupancy Measurement Campaign and a Stochastic Duty Cycle Model," *J. Mobile Networks and Applications,* vol. 15, no. 3, pp. 461-474, 2010.

[15] M. Wellens, J. Riihijarvi, and P. Mahonen, "Empirical Time and Frequency Domain Models of Spectrum Use," *J. Physical Comm.,* vol. 2, nos. 1/2, pp. 10-32, 2009.

Pu Wang received the BS degree in electrical engineering from the Beijing Institute of Technology, China, in 2003 and the MEng degree in computer engineering from the Memorial University of Newfoundland, St. Johns, Canada, in 2008. He is a graduate research assistant in the Broadband Wireless Networking Laboratory (BWN Lab), School of Electrical and Computer Engineering, Georgia Institute of Technology. Currently, he is working toward the PhD degree in electrical and computer engineering under the supervision of professor Ian F. Akyildiz. His research interests include wireless sensor networks, dynamic spectrum access networks, and multimedia communications. He is a student member of the IEEE.

Ian F. Akyildiz received the BS, MS, and PhD degrees in computer engineering from the University of Erlangen-Nuernberg, Germany, in 1978, 1981, and 1984, respectively. Currently, he is the Ken Byers chair professor with the School of Electrical and Computer Engineering, Georgia Institute of Technology (Georgia Tech), Atlanta, the director of the Broadband Wireless Networking Laboratory, and chair of the Telecommunication Group at Georgia Tech. In June 2008, he became an honorary professor with the School of Electrical Engineering, Universitat Politècnica de Catalunya (UPC), Barcelona, Spain. He is also the director of the newly founded N3Cat (NaNoNetworking Center in Catalunya). He has also been an honorary professor with the University of Pretoria, South Africa, since March 2009. His research interests include nanonetworks, cognitive radio networks, and wireless sensor networks. He is the editor-in-chief of the Elsevier's *Computer Networks* and the founding editor-in-chief of the *Ad Hoc Networks Journal*, the *Physical Communication Journal*, and the *Nano Communication Networks Journal*. He serves on the advisory boards of several research centers, journals, conferences, and publication companies. He has received numerous awards from the IEEE and the ACM. He has been an ACM fellow since 1997 and is a fellow of the IEEE and the IEEE Computer Society.