Intersymbol and Co-channel Interference in Diffusion-based Molecular Communication

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Abstract—**Molecular Communication (MC) is a bio-inspired paradigm where information is exchanged by the release, the propagation and the reception of molecules. The objective of this paper is to analyze the effects of interference in the most general type of MC system, i.e., the diffusion of molecules in a fluidic medium. The study of the InterSymbol Interference (ISI) and Co-Channel Interference (CCI) is conducted through the analysis of the propagation of signals in a diffusion-based channel. An in-depth analysis of the attenuation and the dispersion of signals due to molecule diffusion allows to derive simple closed-form formulas for both ISI and CCI. In this paper, two different modulation schemes, namely, the baseband modulation and the diffusion wave modulation are considered for the release of molecules in the diffusion-based MC and are compared in terms of interference. It is determined that the diffusion wave modulation scheme shows lower interference values than the baseband modulation scheme. Moreover, it is revealed that the higher is the frequency of the modulating diffusion wave, the lower are the effects of the ISI and the CCI on the communication channel. The obtained analytical results are compared and validated by numerical simulation results.**

I. Introduction

Molecular Communication (MC) is increasingly attracting the interest of the research community working in the field of nanonetworking [1]. MC is a bio-inspired paradigm that, amongst others, has been developed by nature for communication among living organisms, such as cells for intracellular and intercellular signaling [12]. In MC, information is exchanged by the release, the propagation and the reception of molecules. Due to its inherent bio-compatibility, MC is a competitive solution to the problem of communication in nanonetworks, especially for bio-nano-medical applications. Many different types of MC have been studied so far, which involve either passive transport of molecules (diffusion-based architectures [13]) or active transport (molecular motors [10], bacteria chemotaxis [6]). The focus of this paper is on diffusion-based passive architectures, as they are the most general case and, moreover, they can be easily tailored/expanded to cover the other alternatives.

One of the challenges in MC is the proper study and characterization of the diffusion channel as a communication medium. Up to date, contributions from the literature in this field fail to show a unified and general approach to this problem for diffusion-based MC in nanonetworks. Many diverse MC architectures have been proposed so far on the basis of the technique used to encode the information in the diffusing molecules. While in [7] the information is encoded in the time of arrival of molecules at the receiver, in [11] each molecule carries a piece of information according to its type. In both cases, the communication performance evaluation is characterized by very low values. In contrast, in [2,15,17], a third architecture is proposed, in which information is encoded in the variations of the concentration of molecules in the space. The high similarity of this architecture to the cellular biological systems, which are characterized by much higher performance than the other aforementioned architectures, encourages the investigation in this direction. Theoretical results from [16] confirm the high potential in terms of achievable information rates for this type of diffusion-based molecular communication system.

The objective of this paper is to analyze the effects of the interference on the diffusion-based MC, with reference to the third architecture discussed above. To the best of our knowledge, our paper is the first where the InterSymbol Interference (ISI) and the Co-Channel Interference (CCI) are jointly analyzed for diffusion-based MC. We consider the ISI as the overlap between two consecutively received signals in molecule concentration which were transmitted from a single molecular transmitter. Differently, the CCI is considered here as the overlap between a received molecule concentration signal which was transmitted by a single transmitter and all the received molecule concentration signals which were transmitted by the other concurrent transmitters. Both ISI and CCI depend greatly on how the signals propagate through the channel from the moment they are transmitted until they combine at the receiver side. In most of the classical communication channels, this propagation is expressed through the so-called wave equation, while in diffusion-based MC it is expressed through the fundamentally different diffusion equation.

In this paper, we provide an in-depth analysis of the propagation of signals though a diffusion-based channel by studying two main parameters, namely, the attenuation and the dispersion. For this, we focus on an interpretation of the diffusion equation in terms of diffusion wave propagation, which allows to apply the wave theory to the realm of the diffusion-based MC and to find mathematical expressions for the attenuation and the dispersion in a diffusion-based channel. From these, we derive simple closed-form formulas for both the ISI and the CCI. In this paper, two different modulation schemes, namely, the baseband modulation and the diffusion

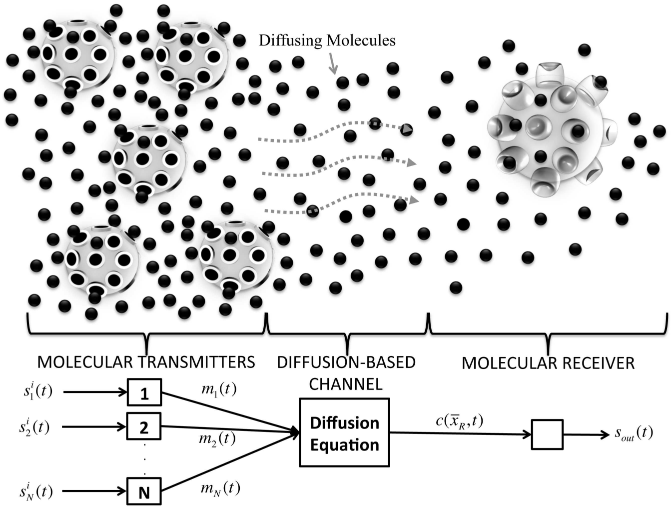

Fig. 1. Block scheme of the diffusion-based MC system considered in this paper.

wave modulation are considered for the release of molecules in the diffusion-based MC and are compared in terms of interference. The obtained analytical results for both ISI and CCI are compared and validated by simulation results. This ultimately allows to assess the validity of the simple closed-form formulas for the evaluation of the interference in a diffusion-based MC system.

The remainder of the paper is organized as follows. In Sec. II, the model of the diffusion-based MC system analyzed in this paper is detailed. General formulas to evaluate the ISI and the CCI are introduced in Sec. III. The analysis of the attenuation and the dispersion which affect the propagation of diffusion waves is treated in Sec. IV, while the definition of simple closed-form formulas for the ISI and CCI is detailed in Sec. V. Numerical results are provided in Sec. VI. Finally, in Sec. VII, we conclude the paper.

II. MC System Model for Interference Analysis

Molecular Communication (MC) realizes the exchange of information through the main elements of an end-to-end communication system [15], namely, the transmitter, the channel and the receiver, which are detailed next.

The MC system model considered in this paper includes N **MOLECULAR TRANSMITTERS**. Each transmitter, denoted by n and located at $\bar{x}_n$, is responsible for the modulation of the number of molecules $m_n(t)$ emitted into the space as a function of the time t according to input information signals, denoted as $s_n^i(t)$, where $i = 1, 2, ...$ is a sequential index. We assume the emitted molecules are identical and undistinguishable between each other. Two different modulation schemes that can be adopted by the molecular transmitters are studied and compared in this paper from the point of view of interference, namely, the *baseband modulation* and the *diffusion wave modulation*. For both modulation schemes, the transmitters produce a number of molecules $m_n(t)$ emitted at location $\bar{x}_n$ and time t corresponding to the amplitude modulation of an oscillation with angular frequency ω_0:

$$m_n(t) = \sum_{i=0}^{\infty} s_n^i(t) e^{j\omega_0 t} \ . \tag{1}$$

where $\omega_0 = 0$ in the baseband scheme and $\omega_0 > 0$ in the diffusion wave scheme.

The MC system model includes a **DIFFUSION-BASED CHANNEL** which is based on the free diffusion of molecules between the transmitter and the receiver. Each molecular transmitter n emits a number of molecules $m_n(t)$ in this space at location $\bar{x}_n$ and time t. For this, the total number of emitted molecules $m(\bar{x}, t)$, which is the input of the molecular channel, is expressed as:

$$m(\bar{x}, t) = \sum_{n=1}^{N} m_n(t) \delta(\bar{x} - \bar{x}_n) \ , \tag{2}$$

where $\delta(\bar{x} - \bar{x}_n)$ is a Dirac delta defined in the three dimensional space and centered at the corresponding transmitter location $\bar{x}_n$. Once emitted, every molecule moves independently from the others and according to its Brownian motion in a fluidic medium. The output of the molecular channel is the molecule concentration $c(\bar{x}, t)$ as function of the space location $\bar{x}$ and the time t, whose relation with the input $m(\bar{x}, t)$ is expressed by the **diffusion equation** [3,14]:

$$\frac{\partial c(\bar{x}, t)}{\partial t} = D\nabla^2 c(\bar{x}, t) + m(\bar{x}, t) \ , \tag{3}$$

where D is the diffusion coefficient and it is considered a constant parameter within the scope of this paper.

The *linearity* of (3) gives the following results:

- Given a modulated number of emitted molecules $m_n(t)$ from a single transmitter n, the output molecule concentration $c(\bar{x}, t)$ at any location $\bar{x}$ and time t is computed through the convolution integral with the Green's function [9] $g(\bar{x}, t)$ (*linear channel*):

$$c(\bar{x}, t) = m(\bar{x}, t) * g(\bar{x}, t) = \int_0^{\infty} m_n(t) g(\bar{x}_n - \bar{x}, t' - t) dt' \ , \tag{4}$$

  where $(. * .)$ denotes the convolution integral between the two arguments. The Green's function [9] is the solution of the diffusion equation (3) when the input $m(\bar{x}, t)$ is a Dirac delta and it is expressed as follows:

$$g(\bar{x}, t) = \frac{1}{\sqrt{(4\pi D t)^3}} e^{-\frac{|\bar{x}|^2}{4Dt}} \ . \tag{5}$$

- Given the modulated number of molecules $m_n(t)$ emitted simultaneously from multiple transmitters, where $n = 1, ..., N$, the output molecule concentration is the sum of the outputs of the diffusion-based channel applied independently to each single molecule concentration rate input (*additive channel*):

$$c(\bar{x}, t) = \sum_{n=1}^{N} \left(m_n(t) \delta(\bar{x} - \bar{x}_n) * g(\bar{x}, t) \right) \ . \tag{6}$$

The MC system model includes a single **MOLECULAR RECEIVER**, whose task is to read the incoming molecular concentration $c(\bar{x}_R, t)$ at its location $\bar{x}_R$ and to demodulate the output information signal $s_{out}(t)$. For this, the molecular receiver produces an output information signal $s_{out}(t)$ equal to the real part of the molecule concentration signal $c(\bar{x}_R, t)$ at the receiver location $\bar{x}_R$, multiplied by an oscillation with angular frequency $-\omega_0$:

$$s_{out}(t) = \Re \left\{ c(\bar{x}_R, t) e^{-j\omega_0 t} \right\} \ , \tag{7}$$

where $\omega_0 = 0$ or $\omega_0 > 0$ in case the transmitter adopted the baseband or the diffusion wave modulation scheme, respectively. $\Re\{.\}$ denotes the operator which extracts the real part from the complex operand.

III. Interference Formulas

The ISI is quantified as the time integral of the product of two output information signals which derive from two input information signals sent from a transmitter n:

$$ISI = \int_{-\infty}^{\infty} s_{n,\text{out}}^{i}(t) s_{n,\text{out}}^{i+1}(t) dt \ , \tag{8}$$

where $s_{n,\text{out}}^{i}(t)$ is the output information signal of the MC system when the input information signal $s_n^i(t)$ is sent by the transmitter n.

The CCI is quantified as the time integral of the product of an output information signal which is sent in a modulated number of emitted molecules by a transmitter n with all the other received output information signals which are sent as modulated number of molecules by all the other $N-1$ transmitters:

$$CCI = \int_{-\infty}^{\infty} s_{n,\text{out}}^{i}(t) \sum_{k=1}^{N,\, k\neq n} \sum_{l=0}^{\infty} s_{k,\text{out}}^{l}(t) dt \ . \tag{9}$$

In case of *baseband modulation* scheme, the output information signal $s_{n,\text{out}}^{i}(t)$, which derives from the input information signal $s_n^i(t)$ sent by the transmitter n, has the following expression:

$$s_{n,\text{out}}^{i}(t) = s_n^i(t) * g(\bar{x}, t) \ , \tag{10}$$

where $(. * .)$ denotes the convolution integral between the two arguments and $g(\bar{x}, t)$ has the expression from (5). In case of *diffusion wave modulation* scheme the same output information signal $s_{n,\text{out}}^{i}(t)$ has the following expression:

$$s_{n,\text{out}}^{i}(t) = \Re\left\{\left[\left(s_n^i(t)e^{j\omega_0 t}\right) * g(\bar{x}, t)\right] e^{-j\omega_0 t}\right\} \ . \tag{11}$$

In order to evaluate the ISI through (8) and the CCI through (9), it is necessary to analyze how the shape of an information signal changes from its transmission as $s_n^i(t)$ until its reception as $s_{n,\text{out}}^{i}(t)$. For this, we decompose an input information signal into its frequency components $S_n^i(\omega)$ by applying the Fourier transform [4]:

$$s_n^i(t) = \int_{0}^{\infty} S_n^i(\omega) e^{j\omega t} d\omega \ . \tag{12}$$

Each frequency component $S_n^i(\omega)e^{j\omega t}$, as it propagates in the diffusion-based channel defined by (3), is in general attenuated and it has a finite propagation velocity. As will be proved in the following section, this attenuation and velocity are functions of the angular frequency ω of the frequency component $S_n^i(\omega)$ itself. As a consequence, the output information signal $s_{n,\text{out}}^{i}(t)$ will be composed by the same frequency components as the transmitted input information signal, each one attenuated by a different value and propagated with a different velocity. These two effects, identified as the **attenuation** and the **dispersion** of a signal, are at the basis of the changes in the information signal shape as it propagates through the diffusion-based channel. For this, in the following section we analyze these two parameters by using the wave theory [8].

IV. Attenuation and Dispersion of Diffusion-Waves

This section deals with the analysis of the attenuation and the dispersion which affect any modulated total number of emitted molecules $m(\bar{x}, t)$ in the diffusion-based channel defined in Sec. II as it propagates from the transmitter to the receiver. For this, as suggested in [8,9], we apply the wave theory to the diffusion equation from (3).

According to the wave theory, given an oscillatory input $q(t)$ with angular frequency ω of the following type:

$$q(\bar{x}, t) = Q(\bar{x}, \omega) e^{j\omega t} \ , \tag{13}$$

the propagation of a wave defined by the following expression

$$u(\bar{x}, t) = U(\bar{x}, \omega) e^{j\omega t} \tag{14}$$

stems from a differential equation that can be defined in the space $\bar{x}$ and angular frequency ω as follows:

$$\nabla^2 U(\bar{x}, \omega) - k^2(\omega) U(\bar{x}, \omega) = Q(\bar{x}, t) \ , \tag{15}$$

where $Q(\bar{x}, t)$ and $U(\bar{x}, \omega)$ are the input and the output respectively, as function of the space $\bar{x}$ and the input angular frequency ω. $k(\omega)$ is the wavenumber, which is in general a function of ω. We have the following definitions based on $k(\omega)$:

- The attenuation of a wave $\alpha(\omega)$ is the imaginary part of the wavenumber $k(\omega)$:
$$\alpha(\omega) = \Im\{k(\omega)\} \ , \tag{16}$$
where $\Im\{.\}$ denotes the operator which extracts the imaginary part from the complex operand.
- The phase velocity v_p is equal to the angular frequency ω divided by the real part of the wavenumber $k(\omega)$ and it is defined as the propagation velocity of a point of constant phase (wavefront velocity):
$$v_p = \frac{\omega}{\Re\{k(\omega)\}} \ . \tag{17}$$
- The group velocity v_g is the time first derivative of the angular frequency ω with respect to the real part of the wavenumber $k(\omega)$ and it is defined as the propagation velocity of a group of waves having a narrow frequency range around ω (wave-packet velocity):
$$v_g = \frac{\partial\omega}{\partial\Re\{k(\omega)\}} \ . \tag{18}$$

The wave propagation expressed through (15) is subject to dispersion if the expressions of the phase velocity (17) and the group velocity (18) are different. The resulting propagating wave from (14) can be written as function of the oscillatory input $q(\bar{x}, t)$, the attenuation $\alpha(\omega)$ and the phase velocity v_p as follows:

$$u(\bar{x}, t) = q(\bar{x}, t) e^{-\alpha(\omega)|\bar{x}|} e^{j\frac{\omega}{v_p}|\bar{x}|} \ . \tag{19}$$

By taking the Fourier transform [4] of the diffusion equation from (3) and by rearranging the terms we obtain an expression of the same type as (15) defined in the space $\bar{x}$ and angular frequency ω:

$$\nabla^2 C(\bar{x}, \omega) - \frac{j\omega}{D} C(\bar{x}, \omega) = M(\bar{x}, \omega) \ , \tag{20}$$

where $M(\bar{x}, \omega)$ and $C(\bar{x}, \omega)$ are the Fourier transforms [4] of the modulated total number of emitted molecules $m(\bar{x}, t)$ and the output molecule concentration signal $c(\bar{x}, t)$, respectively.

The similarity with the wave equation in (15) suggests an interpretation of the diffusion equation in terms of waves, thus identifying the so-called diffusion waves [8]. Although the diffusion waves have different properties [9] if compared to the waves generated by the wave equation, also for the diffusion waves we can identify a wavenumber $k(\omega)$, this time equal to:

$$k(\omega) = \sqrt{\frac{j\omega}{D}} = (1+j)\sqrt{\frac{\omega}{2D}} \, . \tag{21}$$

As a consequence, the attenuation of a diffusion wave $\alpha(\omega)$ is given by (16):

$$\alpha(\omega) = \sqrt{\frac{\omega}{2D}} \, . \tag{22}$$

The phase velocity v_p is given by applying (21) to (17):

$$v_p = \sqrt{2D\omega} \tag{23}$$

and the group velocity v_g is computed through (21) and (18):

$$v_g = 2\sqrt{2D\omega} \, . \tag{24}$$

Since the phase velocity in (23) and the group velocity in (24) are different, the wave propagation in the diffusion-based channel is affected by dispersion. This is a consequence of the frequency dependency of the phase velocity and the group velocity of the diffusion waves.

The resulting propagating diffusion wave can be written as function of an oscillatory total number of emitted molecules $M(\bar{x},\omega)e^{j\omega t}$, the attenuation $\alpha(\omega)$ and the phase velocity v_p as follows:

$$c(\bar{x},t) = M(\bar{x},\omega)e^{j\omega t}e^{-\sqrt{\frac{\omega}{2D}}|\bar{x}|}e^{j\sqrt{\frac{\omega}{2D}}|\bar{x}|} \, . \tag{25}$$

V. Interference Analysis

In this section, the i-th input information signal $s_n^i(t)$ for the transmitter n is modeled as a Gaussian-shaped pulse with standard deviation σ, which is a user-defined parameter:

$$s_n^i(t) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(t-t_n-i\Delta t)^2}{2\sigma^2}} \, , \tag{26}$$

where $t_n + i\Delta t$ is the time instant at which the receiver n transmits the maximum of the i-th pulse. Equation (26) allows to simplify the following interference analysis and to find closed-form expressions for the interference. Although these expressions depend on (26), this does not prevent from considering the general conclusions of this paper valid for any other input signal shape.

We describe the changes in the shape of the pulse sent by the transmitter n from its transmission as $s_n^i(t)$ until its reception as $s_{n,\mathrm{out}}^i(t)$ by using two parameters, namely, the amplitude A_n at the peak maximum and the broadening factor B_n:

$$s_{n,\mathrm{out}}^i(t) = A_n s_n^i\left(\frac{t-t_d}{B_n}\right) = A_n\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(t-t_n-i\Delta t-t_d)^2}{2(B_n\sigma)^2}} \, , \tag{27}$$

where t_d is the pulse propagation delay and its value is not relevant for the following interference analysis since we account only for the time of pulse reception. The relation in (27) is a first approximation of the changes in the pulse shape and it allows a simplification of the expressions of the ISI and the CCI in (8) and (9), respectively. In light of (27) the ISI becomes:

$$ISI \simeq 2A_n^2 erfc\left(\frac{\Delta t/2}{\sqrt{2}B_n\sigma}\right)\left[1 - erfc\left(\frac{\Delta t/2}{\sqrt{2}B_n\sigma}\right)\right] \, . \tag{28}$$

Similarly, the CCI becomes:

$$CCI \simeq \sum_{k=1}^{N,\,k\neq n} 2A_nA_k erfc\left(\frac{|t_n-t_k|/2}{\sqrt{2}B_n\sigma}\right) \cdot \cdot\left[1 - erfc\left(\frac{|t_n-t_k|/2}{\sqrt{2}B_k\sigma}\right)\right] \, , \tag{29}$$

where t_n and t_k are the time instants of transmission of the pulses and B_n and B_k are the broadening factors for the pulses transmitted by the transmitter n and the transmitter k, respectively. $erfc(x/\sqrt{2}B_n\sigma)$ denotes the complementary error function, which corresponds to the integral of the Gaussian pulse dilated by a factor B_n between x and ∞.

By stemming from the formulas discussed in Sec. III concerning the attenuation and the dispersion of the diffusion waves, we can derive closed-form formulas for the amplitude A_n at the peak maximum and the broadening factor B_n:

- The pulse *amplitude A_n at the peak maximum* after propagation from the transmitter n located at $\bar{x}_n$ to the receiver located at $\bar{x}_R$ is given by the double of the integral of the attenuation contribution $e^{-\sqrt{\frac{\omega}{2D}}|\bar{x}_R-\bar{x}_n|}$ of each frequency component $M(\bar{x}_n,\omega)$ of the transmitted signal:

$$A_n = 2\int_0^\infty M(\bar{x}_n,\omega)e^{-\sqrt{\frac{\omega}{2D}}|\bar{x}_R-\bar{x}_n|}d\omega \, , \tag{30}$$

  where $M(\bar{x}_n,\omega)$ is the Fourier transform [4] of the total number of emitted molecules $m(\bar{x}_n,t)$ when only a single pulse is transmitted from (7), having $\omega_0 = 0$ in the case of baseband scheme and $\omega_0 > 0$ in the case of diffusion wave modulation.
- The pulse *broadening* factor B_n is computed as the squared root of the sum of 1 with the squared integral of the delay contribution $\frac{\partial}{\partial\omega}\left(\frac{1}{v_g}\right)|\bar{x}_R-\bar{x}_n|$ of each frequency component $M(\bar{x}_n,\omega)$ of the total number of emitted molecules:

$$B_n = \sqrt{1+\left(\int_0^\infty \frac{\partial}{\partial\omega}\left(\frac{1}{v_g}\right)|\bar{x}_R-\bar{x}_n|M(\bar{x}_n,\omega)d\omega\right)^2} \, , \tag{31}$$

  where v_g is the group velocity expressed in (24). The first derivative of the inverse of the group velocity $1/v_g$ with respect to the angular frequency ω has the following expression:

$$\frac{\partial}{\partial\omega}\left(\frac{1}{v_g}\right) = \sqrt{\frac{1}{2D\omega^3}} \, . \tag{32}$$

In order to compare the ISI and CCI results for the two modulation schemes, we simplify further (30) and (31) by approximation. For the **baseband modulation** scheme, the amplitude A_n^{base} at the peak maximum becomes:

$$A_n^{base} = \frac{2}{\sigma^2}e^{-\sqrt{\frac{\omega_c}{2D}}|\bar{x}_R-\bar{x}_n|} \, , \tag{33}$$

where ω_c is the cut-off frequency of the Gaussian pulse (26). The cut-off frequency is the angular frequency of the pulse spectrum component whose amplitude value is half the amplitude of the maximum. The pulse broadening factor B_n^{base} for the baseband modulation scheme can be approximated with

$$B_n^{base} = \sqrt{1 + \left(\frac{1}{\sigma^2} \sqrt{\frac{1}{2D\omega_c^3}} |\bar{x}_R - \bar{x}_n| \right)^2} \ . \quad (34)$$

In case of **diffusion wave modulation** scheme, the amplitude A_n^{wave} at the peak maximum becomes:

$$A_n^{wave} = \frac{2}{\sigma^2} e^{-\sqrt{\frac{\omega_0}{2D}} |\bar{x}_R - \bar{x}_n|} , \quad (35)$$

where ω_0 is the frequency of the modulating oscillation, as expressed in (1). The pulse broadening factor B_n^{wave} for the diffusion wave modulation scheme can be approximated with:

$$B_n^{wave} = \sqrt{1 + \left(\frac{1}{\sigma^2} \sqrt{\frac{1}{2D\omega_0^3}} |\bar{x}_R - \bar{x}_n| \right)^2} , \quad (36)$$

where for both (35) and (36) we assumed to have a frequency ω_0 much higher than the the cut-off frequency ω_c of the Gaussian pulse (26).

The amplitude at the peak maximum for the baseband modulation scheme A_n^{base} and for the diffusion wave modulation scheme A_n^{wave}, if compared, guide to the following result:

$$A_n^{base} > A_n^{wave} \ \ \forall \omega_0 > \omega_c \ . \quad (37)$$

When comparing the pulse broadening in case of baseband modulation scheme B_n^{base} and in case of diffusion wave modulation scheme B_n^{wave}, we can conclude the following result:

$$B_n^{base} > B_n^{wave} \ \ \forall \omega_0 > \omega_c \ . \quad (38)$$

As a conclusion, for a diffusion-based channel model as defined by (3), the intersymbol interference $ISI_{\bar{x}_R}^{wave}$ in the case of diffusion wave modulation scheme is lower with respect to the intersymbol interference $ISI_{\bar{x}_R}^{base}$ in the case of baseband modulation scheme:

$$ISI_{wave} < ISI_{base} \ \ \forall \omega_0 > \omega_c \ . \quad (39)$$

In addition, we deduce that the higher is the wave modulation frequency ω_0, the lower is the value of the intersymbol interference ISI_{wave}:

$$ISI_{wave}|_{\omega_0=\omega_1} < ISI_{wave}|_{\omega_0=\omega_2} \ \ \forall \omega_1 > \omega_2 \ . \quad (40)$$

Similarly, we compare the co-channel interference for the two modulation schemes applied to systems having the same values for the locations $\bar{x}_n$ and the time instants t_n, which correspond to the maximum of the transmitted Gaussian pulses for all the N transmitters. We deduce that also for the CCI:

$$CCI_{wave} < CCI_{base} \ \ \forall \omega_0 > \omega_c \quad (41)$$

and:

$$CCI_{wave}|_{\omega_0=\omega_1} < CCI_{wave}|_{\omega_0=\omega_2} \ \ \forall \omega_1 > \omega_2 \ . \quad (42)$$

VI. Numerical Results

In this section we simulate the system detailed in Sec. II and we compare the results in terms of ISI and CCI with the simple formulas resulting from the interference analysis of Sec. V, which stems from the diffusion-wave attenuation and dispersion studied in Sec. IV. The goal of this comparison is to prove that the simple formulas for the ISI (28) and for the CCI (29) constitute a valid approximation for the evaluation of the interference in a diffusion-based MC system.

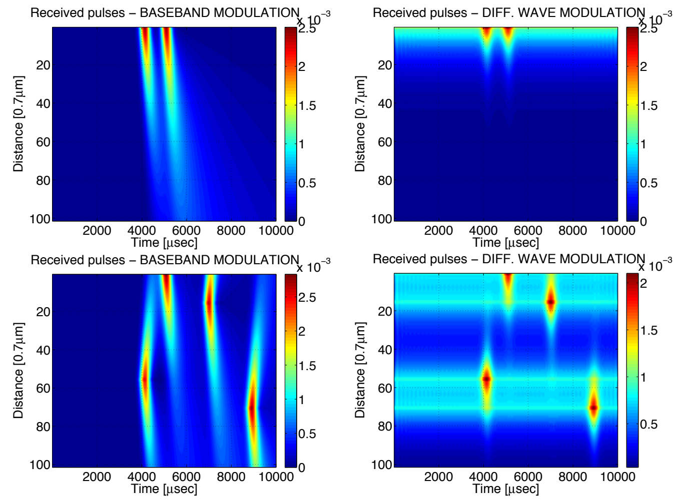

Fig. 2. The received pulses as function of the time and the distance in the case of baseband modulation (left) and diffusion wave modulation (right) for the simulation-based for the simulation-based ISI evaluation (upper) and CCI evaluation (lower).

The simulations are based on (1), (4) and (7), $\omega_0 = 0$ in the baseband scheme and $\omega_0 > 0$ in the diffusion wave scheme. The diffusion coefficient D in (5) is set to $\sim 10^{-6} cm^2 sec^{-1}$ of calcium molecules diffusing in a biological environment (cellular cytoplasm, [5]) and the distance $\bar{x} - \bar{x}_n$ is varied from 0 to $70\mu m$.

For the evaluation of the ISI, the simulations are performed by sending two Gaussian pulses of the type in (26) where σ is set to $0.32sec$, Δt is set to $0.96sec$ and $i = 1, 2$. The parameter t_n is set to $4.14sec$ from the starting time of the simulation. The resulting amplitude of the received pulses as function of the time t ranging from 0 to $10sec$ and the distance $\bar{x} - \bar{x}_n$ ranging from 0 to $70\mu m$ are shown in Fig. 2 (upper), (left) for the baseband modulation and (right) for the diffusion wave modulation. The ISI is evaluated by applying (8) with the computed values of $s_{n,\text{out}}^1(t)$ and $s_{n,\text{out}}^2(t)$.

In Fig. 3 (upper-right) and Fig. 3 (lower-right) we show the results of the numerical evaluation of the formula (28) for the baseband modulation and the diffusion wave modulation, respectively. In case of baseband modulation, we apply (33) and (34), while for the diffusion wave modulation we used (35) and (36). The comparison of the ISI simulation results shown in Fig. 3 (upper-left) and Fig. 3 (lower-left) with the results of the simple formulas in Fig. 3 (upper-right) and Fig. 3 (lower-right) reveals strong similarities between the results of (8) and (28) in the case of baseband modulation and diffusion wave modulation and confirms the validity of the simple formulas for the ISI developed in Sec. V. Moreover, both the results in terms of ISI in the simulation and in the numerical evaluation confirm the relation in (39).

For the evaluation of the CCI, the simulations are performed by sending a Gaussian pulse of the type in (26) from each one of $N = 4$ transmitters placed at distances from the second transmitter $\|\bar{x}_n - \bar{x}_2\|$ equal to $39.2\mu m$, $0\mu m$, $11.2\mu m$ and $49.7\mu m$ respectively. In (26) σ is set to $0.32sec$ and the index i is set to 0. We set $t_1 = 4.14sec, t_2 = 5.09sec, t_3 = 7sec$ and $t_4 = 8.9sec$. The resulting amplitude of the received pulses $s_{n,\text{out}}^i(t)$ as function of the time t ranging from 0 to $10sec$ and the distance $\bar{x}_R - \bar{x}_2$ ranging from 0 to $70\mu m$ are shown in Fig. 2 (lower-left) and Fig. 2 (lower-right) for the baseband

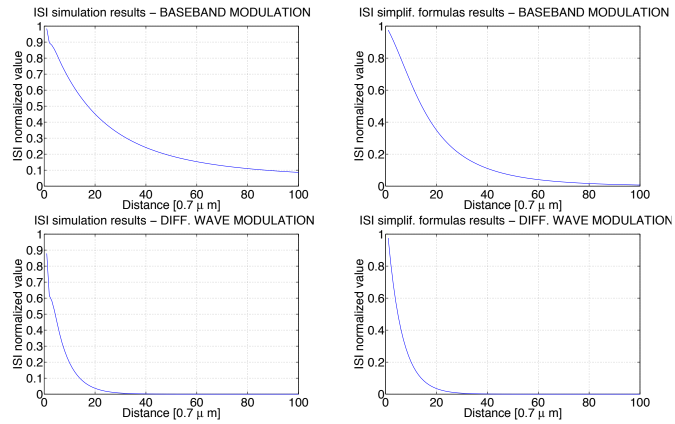

Fig. 3. The ISI values for the baseband modulation scheme (upper) and the diffusion wave modulation scheme (lower) from the simulation (left) and from the simple formulas (right).

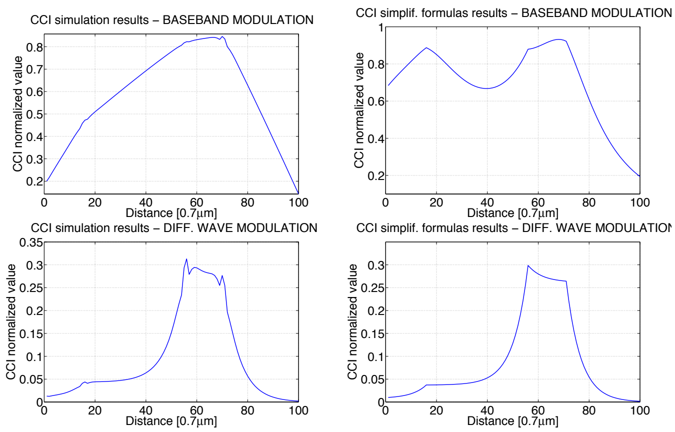

Fig. 4. The CCI values for the the baseband modulation scheme (upper) and the diffusion wave modulation scheme (lower) from the simulation (left) and from the simple formulas (right).

modulation and the diffusion wave modulation, respectively. The CCI is evaluated through (9) with the values of $s^i_{n,\mathrm{out}}(t)$.

In Fig. 4 (upper-right) and Fig. 4 (lower-right) we show the results of the numerical evaluation of the formula (29) for the baseband modulation and the diffusion wave modulation, respectively. In case of baseband modulation, we apply (33) and (34), while for the diffusion wave modulation we used (35) and (36). The comparison of the CCI simulation results shown in Fig. 4 (upper-left) with results of the simple formulas in Fig. 4 (upper-right) clearly show the limit of the simple formulas to properly capture the real CCI curve as function of the distance. This is explained by the fact that the shape of the received pulse in case of baseband modulation is subject to a high value of dispersion and it is distorted with respect to the Gaussian shape assumed for the simple formula (29). This phenomenon is more clearly visible for the CCI computation (9) since its value is the result of the contributions of more than two received pulses, as in the case of the ISI (8). On the contrary, the results in Fig. 4 (lower) guide to the same conclusion as mentioned above for the ISI and confirm the validity of the simple formulas for the CCI developed in Sec. V. Moreover, the result from (41) is supported by both Fig. 4 (upper) and Fig. 4 (lower).

VII. Conclusion

In this paper, we analyze the effects of the InterSymbol Interference (ISI) and the Co-Channel Interference (CCI) in a diffusion-based molecular communication system. For this, we provide a characterization of the diffusion channel in terms of signal propagation by studying two main parameters, namely, the attenuation and the dispersion, and we derive simple closed-form formulas for the evaluation of the ISI and the CCI for the baseband modulation and the diffusion wave modulation schemes. According to the ISI and CCI formulas, the diffusion wave modulation scheme shows lower values of interference with respect to the baseband modulation scheme. This is also confirmed by numerical results obtained through the simulation of the MC system, which also assess the validity of the derived simple closed-form formulas. The interference analysis presented in this paper will contribute to the general understanding of molecular communication systems based on diffusion and it will support the design of nanonetwork architectures based on this paradigm.

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