MIMO Communications Based on Molecular Diffusion

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***Abstract*—Diffusion-based communication refers to the transfer of information using molecules as message carriers whose propagation is based on the law of molecular diffusion. Path loss can have a major impact on the link quality in molecular communication as the signal strength is shown inversely proportional to the cube of the communication distance. In this paper, various diversity techniques for Multi-Input Multi-Output (MIMO) transmissions based on molecular diffusion are proposed to improve the communication performance in nanonetworks in the presence of Multi-User Interference (MUI). Analogous to radio communication, the concept of diversity and Spatial Multiplexing (SM) can be successfully applied in molecular communication. To the best of our knowledge, our paper is the first which investigates the aspects of MIMO transmissions for molecular communication. Numerical results show that the proposed diversity techniques can successfully lower the error rate. Further performance improvement can be obtained by properly allocating molecules among the transmission nodes if the Channel State Information (CSI) is available at the transmitter end. To optimize the system throughput, a dynamic switching mechanism between the diversity mode and the Spatial Multiplexing (SM) mode can be employed.**

***Index Terms*—Molecular communication, diffusion process, MIMO, diversity technique, spatial multiplexing**

I. Introduction

In recent years, there has been dramatic progress made in the development of nanotechnology, which is defined as the technology involving components in a scale from 1 nm to 100 nm [1]. The most basic unit in the nanotechnology is referred to as a nanomachine which performs a specific task, e.g., processing, sensing and actuation [2]. The functionality and capability of one nanomachine alone are quite limited. The idea of forming a *nanonetwork* by interconnecting several nanomachines has been proposed and recently studied in [1], [3].

Several communication mechanisms for nanomachines have been considered and proposed so far, including mechanical, acoustic, electromagnetic, and molecular [1]. However, many of the options listed above have been identified as not directly applicable due to the constraints of size, power, and complexity associated with the nanoscale regime. By using molecules as message carriers for information transportation, the molecular communication technology is considered to be one of the most promising solutions. In molecular communication, the molecules can either follow a specific path or be guided by a fluidic medium to reach the destination [1]. *Diffusion-based communication* refers to the situation where molecules reach the destination relying solely on the laws of molecule diffusion, e.g., pheromone propagation in the air between insects [4] or calcium signaling among living cells [5].

Being a new frontier in the communication technology, diffusion-based communication has been receiving great attention and interests during the past few years. Research efforts are seen in the areas including modeling [6], [7], information theoretic analysis [8], [9], and laboratory experiments [10]–[12]. Different characteristics of the diffusion channel from the traditional ElectroMagnetic (EM) communication have been identified. The phenomenon of signal attenuation analogous to the concept of *path loss* is shown to be inversely proportional to the cube of the communication distance [13]. The fact that nanomachines can move at a speed of several micro-meters per second while the communication range is typically within tens of micro-meters depending on the scenario [7], [14], coupled with the long signaling period up to several seconds [6], [15], makes the diffusion channel exhibit a similar effect of *fast fading*. It is undesirable to improve the performance by simply injecting more molecules into the network as nanomachines are in general limited in resources. Furthermore, such a solution has a major impact on the issue of Multi-User Interference (MUI) in nanonetworks.

In either case of nano-communication system design and interpretation of molecular biological mechanisms, multiple transmission nodes from the same information source or multiple reception nodes for the same information sink would be of great interests in improving the communication performance in nanonetworks in the presence of MUI. This can be generally considered as the basis of Multi-Input Multi-Output (MIMO) molecular communication. In this paper, we investigate the diffusion channel from the perspective of wireless communication and propose various diversity techniques for MIMO

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transmissions. We borrow the paradigms from the traditional EM communication and explore how the concept of diversity and Spatial Multiplexing (SM) can be applied in molecular communication. Specifically, the transmit diversity, selection combining, Maximum-Ratio Combining (MRC), and decision fusion for MIMO molecular communication are proposed and analyzed to obtain the bit error probability. To the best of our knowledge, in this work we first investigate the aspects of MIMO transmissions for molecular communication. Numerical results show that the proposed diversity techniques can lower the error probability significantly. Further performance improvement can be obtained by properly allocating the molecules among the transmission nodes if the Channel State Information (CSI) is available at the transmitter end. To optimize the system throughput, a dynamic switching mechanism between the diversity mode and the SM mode can be employed.

The rest of this paper is organized as follows. In Section II, we formulate the Single-Input Single-Output (SISO) molecular communication model to serve as the reference system for the subsequent discussions. In Section III, we propose various diversity techniques and the SM mode for performance improvement in MIMO molecular communication. In Section IV, the numerical results are presented. Finally, conclusions are given in Section V.

II. SISO Communications Based on Diffusion

In this section, we formulate the SISO communication model as well as the decision rule. The SISO communication model serves as the reference system for the subsequent discussions of various MIMO transmission schemes.

A. System Model

A pair of nanoscale transmission node and reception node communicating based on molecular diffusion in a three-dimensional space is considered. Once released into the propagation medium, the molecules are assumed to diffuse freely, and the dynamics is described by the Brownian motion. It can be shown that the function of the molecular concentration at the receiver in response to an impulse of molecular emission from the transmitter with Q molecules is of the form: [16]

$$Q\,h(t) = Q\,\frac{1}{(4\pi Dt)^{\frac{3}{2}}}\exp\left(-\frac{d^2}{4Dt}\right), \tag{1}$$

where d denotes the distance between the receiver and the transmitter, and D is the diffusion constant.

We consider the binary digital signaling using On-Off Keying (OOK) modulation. With *a priori* probability p, a number of molecules is emitted in an instantaneous fashion by the transmitter to signify logical 1; no molecule is emitted to signify 0. The reception node is assumed to be perfectly synchronized with the transmission node. It is also assumed that the receiver senses the concentration at the peak of the molecular at $t = \frac{d^2}{6D}$, which is obtained by solving $\frac{dh(t)}{dt} = 0$ [13]. The corresponding peak concentration can be obtained from (1) as

$$Q\,h_p(d) = Q\,\left(\frac{3}{2\pi e}\right)^{3/2}\frac{1}{d^3}. \tag{2}$$

The expression (2) suggests that the peak of the molecular pulse, i.e., the signal strength, is independent of the diffusion constant and is inversely proportional to the cube of the distance.

We consider the molecules used for communication purpose to be indistinguishable. Due to the residual molecule diffusion from the previous symbol transmissions, the molecular communication suffers from the effect of Inter-Symbol Interference (ISI). In addition, the MUI also arises as the reception node has no knowledge whether the molecules were emitted from the intended transmission node or from other interfering sources. As shown in [17], the effect of ISI can be suppressed by properly choosing the system parameters. Signal processing, coding, and decision feedback equalization techniques are also shown to be effective for further mitigation of ISI [18]. On the other hand, the control and mitigation of the MUI is of critical importance to nanonetworks as we envision such networks to be distributed and uncoordinated. In this paper, we lay our emphasis on the effect of MUI. We assume both the ISI and the reception noise are suppressed and negligible as compared with MUI.

Let Z denote the sensed concentration at the receiver, we have

$$Z = X\,Q\,h_p(d) + I, \tag{3}$$

where X denotes the intended binary information, and I is the component of the MUI. We assume the interference comes from a sufficient number of interfering sources such that I follows a normal distribution as $\mathcal{N}(\mu_I, \sigma_I^2)$ according to the Central Limit Theorem (CLT) [19].

B. Decision Rule

The value of the sensed concentration given in (3) resembles the output of a matched filter in conventional communication over the AWGN channel. We are thus motivated to apply the Minimum Error Probability (MEP) criterion of the standard Bayesian detection framework as [20]

$$Z \underset{0}{\overset{1}{\gtrless}} \frac{\sigma_Z^2}{\mu_{Z^1} - \mu_{Z^0}}\ln\left(\frac{1-p}{p}\right) + \frac{1}{2}(\mu_{Z^1} + \mu_{Z^0}) \;\equiv\; \eta, \tag{4}$$

where we have defined η as the decision threshold, and

$$\begin{aligned} \mu_{Z^0} &= E[Z \mid X = 0] = \mu_I, \\ \mu_{Z^1} &= E[Z \mid X = 1] = Qh_p(d) + \mu_I, \\ \sigma_Z^2 &= \mathrm{Var}[Z \mid X = 0] = \mathrm{Var}[Z \mid X = 1] = \sigma_I^2. \end{aligned} \tag{5}$$

The corresponding average error probability is given by

$$P_e = pP_M + (1-p)P_F, \tag{6}$$

where P_M and P_F denote the mis-detection probability and the false alarm probability, respectively. It follows that [20]

$$P_M = Q\left(\frac{\mu_{Z^1} - \eta}{\sigma_Z}\right), \quad P_F = Q\left(\frac{\eta - \mu_{Z^0}}{\sigma_Z}\right). \tag{7}$$

III. MIMO Communications Based on Diffusion

In this section, by exploiting paradigms from the traditional EM communication, i.e., the concepts of diversity and SM, we propose various MIMO transmission schemes for diffusion-based molecular communication.

A. Transmit Diversity

We first consider the case where the information source has control over M physically separated nano-transmission nodes, e.g., M nanomachines attached to the same cell membrane for releasing information molecules, while there is only one reception node at the information sink. An illustration is provided in Fig. 1 by setting $N = 1$. This is analogous to the Multi-Input Single-Output (MISO) configuration in EM communication. We assume the information source is capable of coordinating the transmission nodes such that signals can be synchronized and coherently summed together at the reception node. The resulting net equivalent signal strength at the reception node is therefore

$$Z = X \sum_{m=1}^{M} Q_m h_p(d_m) + I, \tag{8}$$

where d_m denotes the distance between transmission node m and the reception node, and Q_m is the number of molecules emitted by transmission node m for signifying logical 1. It follows that

$$\begin{aligned} \mu_{Z^0} &= \mu_I, \\ \mu_{Z^1} &= \sum_{m=1}^{M} Q_m h_p(d_m) + \mu_I, \\ \sigma_Z^2 &= \sigma_I^2. \end{aligned} \tag{9}$$

By replacing (5) with (9), we can obtain the corresponding decision rule and the average error probability. Note that the probability of error P_e in (6) now becomes a function of $Q_1, Q_2, \ldots,$ and Q_M, denoted as $P_e(Q_1, Q_2, \ldots, Q_M)$.

As mentioned previously, the distance separations between the transmission nodes and the reception node can have a major impact on the link performance. We define the CSI in MIMO molecular communication as the distance information between the transmission nodes and the reception nodes. Such information can be estimated at the receiver end, e.g., pilot molecular impulses, and fed back to the transmitters. In the case that the perfect CSI is available at the information source, the following optimization problem is formulated

$$\min\ P_e(Q_1, Q_2, \ldots, Q_M) \tag{10}$$

subject to

$$\sum_{m=1}^{M} Q_m = Q. \tag{11}$$

It is clear that for the MISO configuration, the optimal solution is trivial and is achieved by allocating all the molecules to the transmission node with the minimum distance separation. In the case that the CSI is unavailable, we propose a *uniform molecule allocation* scheme such that $Q_m = \frac{1}{M} Q$.

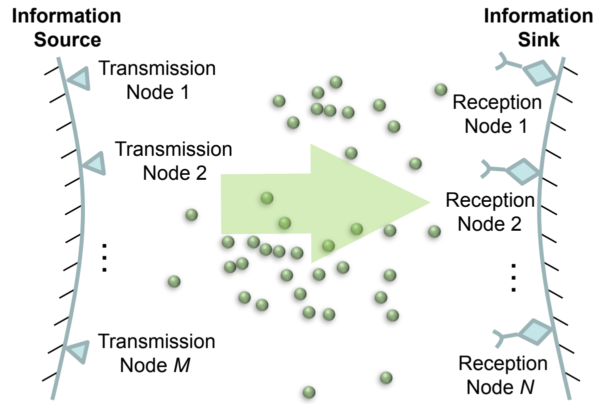

Fig. 1. Illustration of the MIMO molecular communication.

B. Diversity Combining

We next consider the case where the information sink has access to N physically separated reception nodes, while there is only one transmission node, i.e., the Single-Input Multiple-Output (SIMO) configuration. An illustration is provided in Fig. 1 by setting $M = 1$. Let d_n denote the distance between the transmission node and reception node n. We assume that each reception node is separated from one another by a large enough distance such that the MUI experienced at each reception node is statistically independent. The MUI component at reception node n is denoted by I_n, which follows a normal distribution as $\mathcal{N}(\mu_{I_n}, \sigma_{I_n}^2)$. The perfect CSI, i.e., the distance information, is assumed to be available at each reception node.

1) Selection Combining: The concept of selection combining can be applied in a straightforward way by switching to the reception node with the shortest distance from the transmitter. The formulation of the decision criterion and the average error probability in this case is trivial as it is equivalent to the SISO communications after the selection process.

2) Maximum-ratio Combining: The technique of soft combining can also be applied to improve the detection performance. Let Z_n denote the sensed molecular concentration at reception node n. We assume that the capacity of the backhaul link connecting the reception nodes is large enough to collect Z_n for soft combining. By applying the principle of MRC, we have

$$Z = \sum_{n=1}^{N} a_n Z_n, \tag{12}$$

where we have defined $a_n = h_p(d_n) \frac{\prod_{i=1}^{N} \sigma_{I_i}^2}{\sigma_{I_n}^2}$. It can then be shown that

$$\begin{aligned} \mu_{Z^0} &= \sum_{n=1}^{N} a_n \mu_{I_n}, \\ \mu_{Z^1} &= \sum_{n=1}^{N} a_n (Q h_p(d_n) + \mu_{I_n}), \\ \sigma_Z^2 &= \sum_{n=1}^{N} a_n^2 \sigma_{I_n}^2, \end{aligned} \tag{13}$$

where the assumption of each Z_n being independently distributed has been utilized. By replacing (5) with (13), we can obtain the corresponding decision rule and the average error probability.

3) Decision Fusion: In the case that the capacity of the backhaul link is low such that the transportation of soft values, or the computational complexity associated with soft combining is impractical, we propose to apply the principle of decision fusion for diversity combining. Let Y_n denote the local binary decision on X made by reception node n, and P_{M_n} and P_{F_n} denote the corresponding mis-detection and false alarm probabilities, respectively. The optimal rule for decision fusion based on the MEP criterion is given by [21]

$$\sum_{n=1}^{N} \ln \frac{P_{M_n} + Y_n(1-2P_{M_n})}{1-P_{F_n}+Y_n(2P_{F_n}-1)} \underset{0}{\overset{1}{\gtrless}} \ln\left(\frac{1-p}{p}\right). \quad (14)$$

The associated average error probability in this case can be numerically evaluated.

Note that the proposed techniques of diversity combining and transmit diversity can be combined directly if both the information source and the information sink have access to multiple nodes for communication purpose. The simplest case would be transmissions with the uniform molecule allocation scheme coupled with the selection combining. It is noted that the optimization of molecule allocation with perfect or imperfect CSI in the case of MIMO transmissions is not as straightforward as in the case of MISO. This work is currently in progress.

C. Spatial Multiplexing

In addition to the diversity schemes proposed previously, the SM mode can also be applied for boosting the system throughput. In the SM mode, the information source divides the original bit stream into several independent streams, each fed to a particular transmission pair. Each transmission pair acts as the interference to the other. Ideally, the timing of the molecular emissions across all the transmission nodes can be arranged to keep the mutual interference to a minimum. Here we analyze the performance bound by considering the worst-case interference. Consider an $M \times M$ MIMO configuration. From the perspective of transmission pair n (transmission node n to reception node n), we have

$$Z = X_n Q_n h_p(d_{nn}) + I_n + \sum_{m=1, m\neq n}^{M} X_m Q_m h_p(d_{mn}), \quad (15)$$

where X_i denotes the binary information sent from transmission node i, and d_{ij} stands for the distance between transmission node i and reception node j. It then follows that

$$\begin{aligned} \mu_{Z^0} &= \mu_{I_n} + \sum_{m=1, m\neq n}^{M} p_m Q_m h_p(d_{mn}), \\ \mu_{Z^1} &= Q_n h_p(d_{nn}) + \mu_{Z^0}, \\ \sigma_Z^2 &= \sigma_{I_n}^2 + \sum_{m=1, m\neq n}^{M} (p_m - p_m^2) Q_m^2 h_p(d_{mn})^2, \quad (16) \end{aligned}$$

where p_m stands for the *a priori* probability of transmission node m. By replacing (5) with (16), we can obtain the corresponding decision rule and the average error probability P_{e_n} for transmission pair n.

For a fair comparison with the diversity mode, we define the total throughput under the SM mode as

$$\sum_{n=1}^{M} (1 - P_{e_n})^L, \quad (17)$$

where L denotes the number of bits a packet contains. We remark that the issue of molecule allocation for throughput optimization can be similarly formulated here, which is also currently in progress.

IV. Numerical Results

In this section we present the numerical results for the error probability and the throughput of the proposed MIMO transmission schemes over the diffusion channels. We consider the short-range molecular communication where communication distances are typically within tens of micrometers. The *a priori* probability p and the distribution of the MUI, i.e., μ_I and σ_I, are set to be identical for all reception nodes as $p = 1/2$, $D = 10^{-6}$ cm^2/s, and $\mu_I = 2 \times 10^{16}$ molecules·cm^{-3}. Such magnitude for the MUI, as an illustration, is equivalent to the received signal strength of a single interfering transmission node with $Q \approx 2 \times 10^9$ molecules and a distance separation of 20 μm. For the variance of the MUI, we assume a medium coefficient of variation (CV) of 0.3 such that $\sigma_I = 0.3\,\mu_I$.

In Fig. 2, we plot the error probability as a function of the total number of molecules Q for the SISO molecular communication system. Four curves corresponding to various distance separations d between the transmission node and the reception nodes are plotted. We observe that a lower P_e can be achieved with a higher Q and a smaller d since the effect of MUI is equivalently lower. Furthermore, we observe that the communication performance in terms of P_e is critically determined by the distance separation as an increase from 20 μm to 35 μm entails a huge performance loss.

In Fig. 3, we plot the error probability against Q for the 4×1 MISO molecular communication system with transmit diversity. We consider a configuration where four transmission nodes are available at the information source and are at distance separations of $20, 20, 25, 30$ μm from the receiver, respectively. In this case, if perfect CSI is available at the information source, the best policy would be allocating all the molecules to the transmission node with minimum distance separation ($d = 20$ μm), while the equal molecule allocation scheme without CSI entails a performance loss in the form of an additional number of molecules of $\approx 2.5 \times 10^9$ at $P_e = 10^{-5}$. On the other hand, if the technique of transmit diversity is not employed, the worst-case performance is given by the SISO communication with the maximum distance separation. The performance degradation in this case then becomes unacceptable.

In Fig. 4, we plot the error probability against Q for the 1×4 SIMO molecular communication system using the techniques

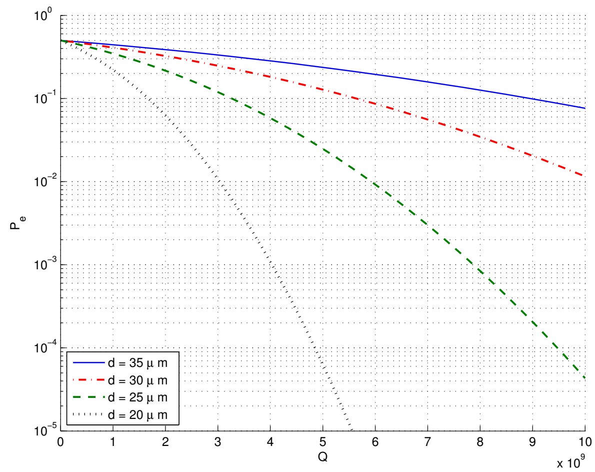

Fig. 2. The error probability P_e versus the total number of molecules Q for the SISO molecular communication system under different distances between the Tx and the Rx nodes.

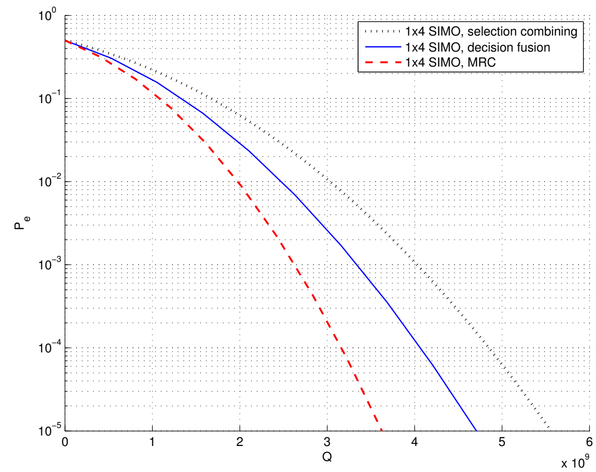

Fig. 4. The error probability P_e versus the total number of molecules Q for the 1×4 SIMO molecular communication system under different techniques of diversity combining.

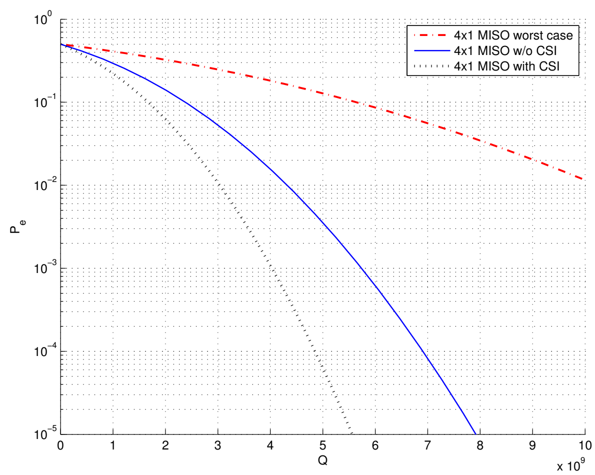

Fig. 3. The error probability P_e versus the total number of molecules Q for the 4×1 MISO molecular communication system under different techniques of transmit diversity.

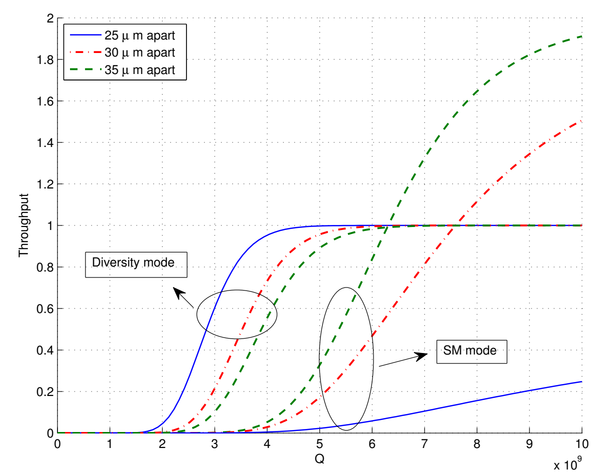

Fig. 5. The throughput versus the total number of molecules Q for the 2×2 MIMO molecular communication system under different transmission modes and distances between the Tx and the Rx nodes.

of selection combining, MRC, and decision fusion. A configuration where four reception nodes with distance separations of $20, 20, 25, 30$ μm from the transmitter is considered. We observe that the decision fusion achieves a performance gain of $\approx 1 \times 10^9$ molecules at $P_e = 10^{-5}$ when compared with the selection combining, which is effectively equivalent to the case of SISO transmissions with $d = 20$ μm here. If the MRC is allowed, we observe that further performance gain of another $\approx 1 \times 10^9$ molecules can be achieved at $P_e = 10^{-5}$.

In Fig. 5, we plot the throughput against Q for the 2×2 MIMO molecular communication system using the SM mode and the diversity mode. We set the packet length L to 50 bits. The transmission nodes and the reception nodes are put on the four corners of a rectangle with a separation of 20 μm between the transmission nodes and the reception nodes. Three sets of results corresponding to different distance separations between the two transmission pairs are provided. For the diversity mode, molecules are all allocated to a particular transmission node with MRC at the receiver end. We observe that when the two transmission pairs are very close to each other, i.e., 25 μm apart, the performance yielded by the SM mode is unacceptable due to the severe mutual interference, while the diversity mode works best amongst the three configurations. In this case one should always operate using the diversity mode. For larger distance separations, hence lower interference, we observe that the SM mode outperforms the diversity mode in an increasing range of Q. A throughput of 2 bits can be ultimately achieved. The results suggest that a dynamic switching algorithm is desirable for MIMO molecular transmissions to achieve the optimal throughput.

V. Conclusions

In this paper, the diffusion channels have been investigated from the perspective of wireless communication. Various diversity techniques for MIMO transmissions have been proposed to improve the communication performance in nanonetworks in the presence of MUI. The transmit diversity, selection combining, MRC, and decision fusion for MIMO molecular communication have been proposed and analyzed to obtain the error probability. Numerical results show that the distance between the transmitter and the receiver plays the key role in determining the link performance, and the proposed diversity techniques can successfully lower the error probability. Further performance improvement can be obtained by properly allocating the molecules among the transmission nodes if the CSI is available at the transmitter end. A dynamic switching mechanism between the diversity mode and the SM mode is desirable for optimizing the system throughput.

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