Spatio-Temporal Estimation for Interference Management in Femtocell Networks

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***Abstract*—Two-tier femtocell-based networks have been proposed as an economic solution to improve coverage and capacity in wireless cellular systems. Although widely studied in the literature, interference management in these networks remains as a technical challenge in need of effective solutions. In particular, the interference estimation is a relevant portion of the problem that enables correct operation of interference management schemes relying on this information. In this paper, a novel downlink cross-tier interference estimation approach is proposed based on spatio-temporal correlation techniques. An ordinary Kriging interpolator using Semivariogram Analysis is applied to the interfering signal followed by an autorregressive model. The signal estimation at the interfered users' location is exploited at the radio resource manager of the femtocell or macrocell base station by formulating a resource allocation problem that is solved by means of a heuristic algorithm. A practical procedure implementing this scheme in the network is also proposed. Numerical results show how performance of cross-tier interference management approaches can be optimized by implementing this idea.**

I. Introduction

Emerging and future cellular wireless systems like 3GPP's LTE and LTE-Advanced or IEEE 802.16e (WiMAX) are being designed with the objective of satisfying an extraordinary increase in the demand for data rates in wireless networks. Recent studies have shown that a great amount of voice and data services take place or are orginated indoors [1], and offloading techniques have started being proposed. An inexpensive solution that has emerged to increase both network capacity and indoor coverage are femtocells [2]. Strictly speaking, a femtocell is the coverage area created by a femtocell access point (FAP) although the term femtocell is typically used for the hadware device as well. A FAP is a short-range low-power device owned and installed by the subscriber that aims to achieve better service at places like a home, an office, a supermarket, etc. by transmitting in licensed spectrum. The traffic is sent over the IP (Internet Protocol) backhaul, which also allows operators to offload macrocell traffic and release resources for other macrocell users.

The major technical challenge and strongest performance limiting factor concerning femtocells is the interference, classified as cross-tier if it occurs among macrocell and femtocell elements (FAPs or users) or co-tier if the interference happens among different femtocell elements. Numerous approaches have been proposed in the literature to cope with cross-tier interference, usually related to power control [3], [4], spectrum management techniques [5], [6], and open/closed/hybrid access modes of operation [7], [8]. An additional critical issue in all of them is the interference estimation, which triggers all the above procedures but may undermine performance if not correctly tackled. There is not much research done especifically addressing this problem, although most of the works assume that interference can be simply calculated based on distance estimations on the uplink or downlink signal measurements at the FAPs. None of these approaches provides accurate interference estimations.

In this paper, we propose a novel downlink cross-tier interference estimation approach for femtocell networks based on spatio-temporal correlation techniques. The scheme can be applied both at the femtocell and macrocell networks depending on the specific scenario needs for downlink interference management. For instance, downlink interference at the femtocell can be severe if the femtocell is located close to the Macrocell Base Station (MBS). Similarly, downlink interference at the macrocell is a serious issue for cell-edge macrocell users near a femtocell. Without loss of generality, we focus on the former scenario. Therefore, we perform an estimation of the macrocell interference at the femtocell users' location. The Kriging Interpolator using Semivariogram Analysis is adopted for the spatial estimation of the macrocell signal. More specifically, the Kriging interpolation is an optimal prediction method which estimates the unknown values from the data observed at known locations. This method uses the Semivariogram Analysis to express the spatial variations of the predicted values. The Semivariogram analysis can be also used to analyze the spatial behavior of the users in cellular networks as in [9]. At the MBS, macrocell users' locations and signal attributes are utilized as inputs to the estimation algorithm. The temporal estimation of the signal is subsequently performed using autorregressive models of different orders. With this estimation, the FAP can perform accurate resource allocation functionalities by utilizing the right amount of power so that femtocell users satisfy their service requirements while the caused cross-tier interference is minimized. We also present an analytical formulation of the resource allocation problem and a suboptimal heuristic algorithm. Finally, we propose a practical network procedure enabling this scheme in the cellular network.

The remainder of this paper is organized as follows. Section II describes the topology as well as the system model where the problem is framed. The proposed scheme consisting of the spatio-temporal estimation, the resource allocation step and the overall network procedure, is presented in section III. Section IV shows performance evaluation results and conclusions are drawn in section V.

II. System Architecture and Model

The considered topology shown in Fig.1 consists of an MBS with several macrocell users in the sourroundings of the femtocell. The femtocell network is located within the coverage region of the macrocell and the FAP serves its femtocell users by using a closed subscriber group approach. The analyzed interference scenario is cross-tier, i.e. the interference between femtocell and macrocell users.

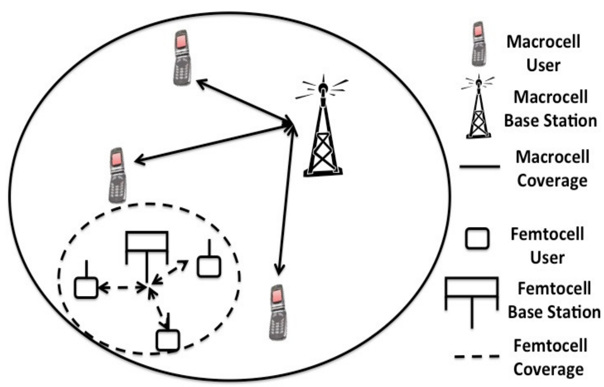

Fig. 1. The Considered Topology

In order to maintain the generality of different types of systems, we consider resources as generic channels that are allocated by both macrocell and femtocell base stations to its users. The Signal-to-Interference-and-Noise Ratio (SINR) per channel is obtained by utilizing the following expression:

$$SINR_n = \frac{P_n}{L(I_n + P_N)} \tag{1}$$

where P_n and I_n are the transmitted power and the received interference power on channel n, respectively, and P_N is the noise power. L is the propagation pathloss between transmitter and receiver. The propagation pathloss both for indoor, outdoor and indoor to outdoor environments follow the channel models suggested by 3GPP [10], as shown by Eqs. (2), (3) and (4).

- Indoors

$$L = 38.46 + 20\log_{10} d_i + 0.7 d_i \quad (dB) \tag{2}$$

- Outdoors

$$L = 15.3 + 37.6\log_{10} d_o \quad (dB) \tag{3}$$

- Indoors-to-outdoors

$$L = 15.3 + 37.6\log_{10} d_i + F \quad (dB) \tag{4}$$

Here, f is the frequency in MHz, d_i is transmitter-receiver distance measured in meters and d_o distance measured in kilometers. F is the log-normal shadow fading random variable with a standard deviation of 8.9 dB.

Finally, the signal power can be obtained from the temporal signal $s(k)$, where K is the number of samples:

$$P_s = \frac{1}{K}\sum_{k=1}^{K} s(k)^2 \tag{5}$$

III. Proposed Scheme

A. Spatio-temporal Estimation

In order to estimate the received signal of the Macrocell Base Station (MBS) at a new unknown location, we propose a spatio-temporal estimation module as shown in Fig. 2 with four basic operations.

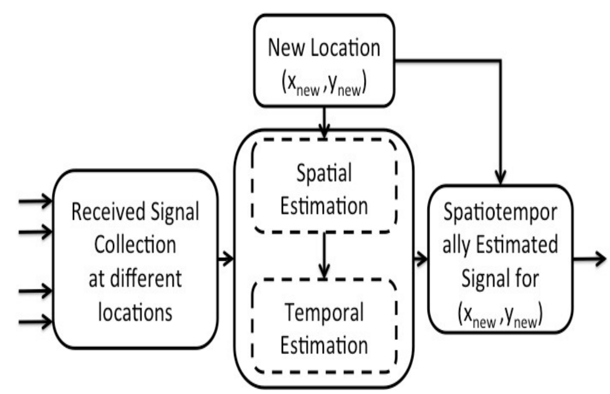

Fig. 2. The Proposed Spatio-Temporal Estimation Module

The module has four basic operations as it can be seen in Fig. 2, which are explained as follows:

- The users located at n different positions estimate their received SINR values $S(t, x_i, y_i)$ (usually via pilot estimation followed by interpolation) for a given time t. These values are sent to the MBS.
- The collected SINR $S(t, x_i, y_i)$ are given as inputs to the spatial estimator as shown in Fig. 2. The spatial estimator uses an ordinary Kriging estimation [11], [12] to predict received signal of the MBS $S(t, x_{new}, y_{new})$ for a new location with coordinates (x_{new}, y_{new}).
- The spatially estimated SINR $S(t, x_{new}, y_{new})$ is then temporally predicted using autoregressive models with orders 1 and 2, AR(1) and AR(2), for a monitoring period $T = 1, 2, .., t$.
- The module outputs $\hat{S}(t, x_{new}, y_{new})$, which is the spatio-temporally estimated received signal of the MBS, for a new location with coordinates (x_{new}, y_{new}).

In the following two subsections, the spatial and temporal estimations are analytically described.

1) Spatial Estimation: The spatial estimation is performed by applying the ordinary Kriging interpolation method. This method collects $S(t, x_i, y_i)$, the received signal samples from the MBS at different locations with coordinates (x_i, y_i) at a given time t. It then estimates new SINR value for a new location with coordinates (x_{new}, y_{new}), by using the spatial correlation among the collected values. In order to achieve a spatial correlation based estimation, the Kriging interpolator uses the *Semivariogram Analysis* [9], [13], [14] which is the characterization of the spatial correlation of the collected signals in a given random field. The semivariogram $\gamma_{(i,j)}$ of two signals $S(x_i, y_i)$ and $S(x_j, y_j)$ is calculated as:

$$\gamma_{(i,j)} = 0.5 \times E[((S(x_i, y_i) - S(x_j, y_j))^2]. \tag{6}$$

The steps of the Kriging based spatial estimation method are explained in the following.

The spatially estimated SINR $S(t, x_{new}, y_{new})$ for a new location with coordinates $(x_{new}, y_{new})$is given by:

$$S(t, x_{new}, y_{new}) = \sum_{i=1}^{n} [\lambda_i \times S(t, x_i, y_i)] \tag{7}$$

where $S(t, x_i, y_i)$ in Eq. 7 is the received signals from the MBS at different locations with coordinates (x_i, y_i) at a given time t and λ_i is the Kriging coefficient for the i^{th} location with coordinates (x_i, y_i). The λ_i in Eq. 7 is expressed as:

$$\lambda_i = \frac{\gamma_{(new,j)}}{\gamma_{(i,j)}} \ \forall\ i, j \tag{8}$$

where $\gamma_{(new,j)}$ is the semivariogram value of the the signal at position (x_{new}, y_{new}) and the signal with position (x_j, y_j). Note that the signal at (x_{new}, y_{new}) is the new position which we want to estimate and the signal at (x_i, y_i) is one of the collected signal. Moreover, $\gamma_{(i,j)}$ is the semivariogram value of the the signal at position (x_i, y_i) and the signal at position (x_j, y_j).

There exist several analytical semivariogram models in the literature to represent the spatially collected signals in a random field [14], [13]. We consider three semivariogram models in this work: The exponential semivariogram $\lambda_i^{(exp)}$, the gaussian semivariogram $\lambda_i^{(gauss)}$ and the linear semivariogram $\lambda_i^{(lin)}$. These are expressed respectively by the Eqs. (9), (10) and (11) as follows.

$$\lambda_i^{(exp)} = \frac{\gamma_{(new,j)}}{\gamma_{(i,j)}} = \frac{c(1 - e^{[-3 \times \frac{\sqrt{(x_{new}-x_j)^2+(y_{new}-y_j)^2}}{a}]})}{c(1 - e^{[-3 \times \frac{\sqrt{(x_i-x_j)^2+(y_i-y_j)^2}}{a}]})} \tag{9}$$

$$\lambda_i^{(gauss)} = \frac{\gamma_{(new,j)}}{\gamma_{(i,j)}} = \frac{c(1 - e^{[-3 \times \frac{(x_{new}-x_j)^2+(y_{new}-y_j)^2}{a}]})}{c(1 - e^{[-3 \times \frac{(x_i-x_j)^2+(y_i-y_j)^2}{a}]})} \tag{10}$$

$$\lambda_i^{(lin)} = \frac{\gamma_{(new,j)}}{\gamma_{(i,j)}} = \frac{c + a \times \sqrt{(x_{new} - x_j)^2 + (y_{new} - y_j)^2}}{c + a \times \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}. \tag{11}$$

In Eqs. (9) ,(10) and (11), the parameter *c* is the *sill* value which represents the maximum spatial correlation level for any two points and *a* is the *range* value which represents the longest Euclidian distance between two points at which these two points have reached the sill value. In other words, *a* is the lag distance for the two points having the highest spatial correlation value. For further information about these parameters, one can refer to [14].

The spatially estimated SINR, if the received signals in the random field are exponentially distributed, can be expressed by inserting Eq. (9) into Eq. (7) as follows:

$$S^{(exp)}(t, x_{new}, y_{new}) =$$

$$\sum_{i=1}^{n} [\frac{c(1 - e^{[-3 \times \frac{\sqrt{(x_{new}-x_j)^2+(y_{new}-y_j)^2}}{a}]})}{c(1 - e^{[-3 \times \frac{\sqrt{(x_i-x_j)^2+(y_i-y_j)^2}}{a}]})} \times S(t, x_i, y_i)] \ \forall\ j. \tag{12}$$

The spatially estimated SINR, if the received signals in the random field are gaussian distributed, can be expressed by inserting Eq. (10) into Eq. (7) as follows:

$$S^{(gauss)}(t, x_{new}, y_{new}) =$$

$$\sum_{i=1}^{n} [\frac{c(1 - e^{[-3 \times \frac{(x_{new}-x_j)^2+(y_{new}-y_j)^2}{a}]})}{c(1 - e^{[-3 \times \frac{(x_i-x_j)^2+(y_i-y_j)^2}{a}]})} \times S(t, x_i, y_i)] \ \forall\ j. \tag{13}$$

The spatially estimated SINR of the received signal from the MBS at a new location (x_{new}, y_{new}), if the received signals in the random field are linearly distributed, can be expressed by inserting Eq. (11) into Eq. (7) as:

$$S^{(linear)}(t, x_{new}, y_{new}) =$$

$$\sum_{i=1}^{n} [\frac{c + a \times \sqrt{(x_{new} - x_j)^2 + (y_{new} - y_j)^2}}{c + a \times \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}} \times S(t, x_i, y_i)] \ \forall\ j. \tag{14}$$

2) Temporal Estimation: The temporal estimation of the spatially estimated signals which are expressed by Eqs. (9), (10) and (11) can be found by using *AutoRegressive Model* with order p (AR(p)). Thus, the spatio-temporal estimation of the received signal from the MBS at a new location with coordinates (x_{new}, y_{new}) is given by $\hat{S}(t, x_{new}, y_{new})$ and expressed as:

$$\hat{S}(t, x_{new}, y_{new}) = \beta_0 + [\sum_{n=1}^{p} \beta_n \times S(t-n, x_{new}, y_{new})] \tag{15}$$

where β_0 and β_n are the coefficients of the autoregressive model which are calculated using Ordinary Least Square (OLS) method as:

$$\beta_n = \frac{\sum_{i=1}^{n}(S(t-i)S(t) - nE[S(t-i)S(t)])}{\sum_{i=1} n(S^2(t-i) - nE[S_2(t-i)])} \tag{16}$$

and

$$\beta_0 = E[S(t)] - \sum_{i=1}^{n} \beta_i \times E[S(t-i)] \tag{17}$$

B. Resource Allocation

In this section, we formulate the problem of allocating resources at the FAP as an optimization problem exploiting the previously estimated interference at the femtocell users. Let $\mathcal{F}$ and $\mathcal{M}$ be the number of femtocell and macrocell users respectively, and N the total number of channels available at the femtocell. Let $\alpha(u,n)$ be a binary indicator that contains 1 if channel n is assigned to user u or 0 otherwise. We define a matrix P^F of $\mathcal{F}$ rows and N columns that contains the femtocell power allocation. If the user i is assigned the channel j with transmission power p, then the ij^{th} element of P^F will contain the value p. Let I_{un} be the interference power estimated at femtocell user u on channel n. Given that we want to minimize the cross-tier interference, the objective function to be minimized is the total emplyed power, i.e. the sum of all the elements of P^F, as shown by Eq. (18).

The optimization problem can be formulated as follows:

$$\min_{P,\alpha} \sum_{u=1}^{\mathcal{F}} \sum_{n=1}^{N} P_{un}^F \cdot \alpha(u,n) \tag{18}$$

Subject to:

$$C1: \frac{P_{un}^F}{L_u^F(I_{un} + \sigma^2)} \geq \beta^F \alpha(u,n) \quad \forall n, \forall u = 1, ..., \mathcal{F}$$

$$C2: \frac{(\sum\limits_{u=1}^{\mathcal{F}} \sum\limits_{n=1}^{N} P_{un}^F)^2}{M(\sum\limits_{n=1}^{N} P_{un})^2} = 1$$

$$C3: \sum_{u=1}^{\mathcal{F}} \alpha(u,n) = 1 \quad \forall n$$

$$C4: \frac{S_{un}^M}{\frac{1}{L_u^M} \sum\limits_{u=1}^{M} P_{un}^F + \sigma^2} \geq \beta^M \gamma(u,n) \quad \forall n, \forall u = 1, ..., \mathcal{M}$$

$$P_{un}^F \geq 0, \quad \alpha(u,n), \gamma(u,n) \in \{0,1\} \quad \forall n, \forall u = 1, ..., \mathcal{F}$$

where C1 sets the SINR constraint to β^F on every channel n allocated to femtocell user u; C2 is the fairness constraint among femtocell users implemented by a Jain's fairness index of 1 [15] guaranteeing that all users will receive same power; C3 ensures that no interference will appear among femtocell users in any channel; and C4 sets the SINR contraint for the macrocell users, where $\gamma(u,n)$ is a known function containing the macrocell channel allocation.

The problem specified by Eq. (18) and constraints C1 to C4 is not a convex optimization problem and cannot be solved by standard methods in optimization theory. Therefore, we propose an alternative simple heuristic algorithm as shown in Algorithm 1. This algorithm assumes interference minimization is effectively achieved by minimizing femtocell transmit power. The idea is summarized as follows: Channels are assigned to users based on their requested power to achieve the target SINR. Users with channels requesting a lesser amount of power are served first and fairness is guaranteed by trying to assign the same number of channels to each user.

Algorithm 1

Maximum number of channels per user
max_channels $\leftarrow \lceil \frac{N}{M} \rceil$
Power requested for each user in each channel
for $u \leftarrow 1, M$ **do**
  for $n \leftarrow 1, N$ **do**
    $P_{un}^{req} = \beta^F \left(I_{un} + \sigma^2\right) L_u^F$
  end for
end for
Channel and power allocation
while $P^{req} \neq \emptyset$ **do**
  $[p^*, u^*, n^*] \leftarrow min P^{req}$
  if *Channels assigned to* $u^* <$ *max_channels* **then**
    Remove column n^* *from* P^{req}
    $P(u^*) \leftarrow P(u^*) + p^*$
  else
    Remove row u^* *from* P^{req}
  end if
end while

C. Network Implementation Procedure

The above resource allocation procedure must be performed at the FAP. For this we need a network procedure that allows the exchange of signalling information between the macrocell and the femtocell taking place over the air or via the backhaul network. It is not realistic to assume known at the MBS the position of the macrocell users; however, the FAP can perform estimations on the location of both the femtocell and close-by macrocell users since the femtocell itself will be aware of its location via e.g. a GPS receiver. Techniques based on signal strength or more sophisticated MIMO approaches based on direction-of-arrival estimation could be employed. The procedure enabling this exchange of information is described as follows:

1) The FAP estimates the users' locations and sends them to the MBS along with their identification. This information is available at the FAP since both femtocell

subscribers and non-subscribers can attempt to camp on the femtocell.

2) Macrocell users provide SINR feedback to the MBS, and users interfering with the femtocell will be identified.
3) The MBS, using its own users' location information and their received signal attributes, runs the spatio-temporal framework to estimate the interference at the femtocell users' location for a certain period of time.
4) Interference power values are sent back from the MBS to the femtocell.
5) The FAP runs a resource allocation algorithm exploiting the estimated interference values.
6) The procedure is repeated once the estimation period expires or the femtocell users substantially change locations.

Overall, only some signalling information need to be exchanged between macrocell and femtocell. The necessary amount of such information is another interesting research question, since spatial correlation properties of both the femtocell and macrocell users could be exploited to reduce the amount of exchanged data.

In order to guarantee certain quality of service at the macrocell users, this scheme could be reciprocally triggered at the macrocell to estimate and optimize the amount of interference that femtocell users are causing to macrocell users. In addition, this interference estimation approach can be utilized to improve other power control based interference management schemes (such as the one in [4]) whose performance may be undermined by inaccurate interference estimation.

IV. Performance Evaluation

We perform several experiments with the objective of evaluating the performance of the different proposed spatio-temporal estimators. The scenario is similar to the one depicted in Fig. 1, with five macrocell users (mUEs) radomly deployed in the surroundings of the femtocell (up to 40 meters). We perform simulations for one single femtocell with a varying number of users suffering from cross-tier interference and randomly deployed in a coverage radius of 20 meters. The relevant simulation parameters common to all the experiments are shown in Table I.

TABLE I
SIMULATION PARAMETERS

Carrier frequency (f_c)	$2\ GHz$
Simulation time	$500\ ms$
Noise power (σ^2)	$-115\ dBm/channel$
MBS coverage distance	$1km$
MBS transmission power	$42dBm$
Number of macrocell users	5
Femtocell range	$20m$
FAP maximum transmission power	$0.375mW/channel$
Femto user SINR requirements	$15\ dB$

The first experiment investigates how the different estimation approaches affect the total transmit power of the femtocell, i.e. the interference caused to the macrocell. A total number of six spatio-temporal techniques are to be simulated, combining the three spatial schemes (linear, exponential and gaussian) and two temporal autorregresive models of order one and two, as shown in Fig. 3. The number of femtocell users is fixed to four and the y axis represents the number of available channels at the femtocell. The results show significant differences among the spatial approaches, being the linear estimator the lowest-energy interference estimator and therefore allowing the FAP to satisfy users' requirements causing the lowest cross-tier interference.

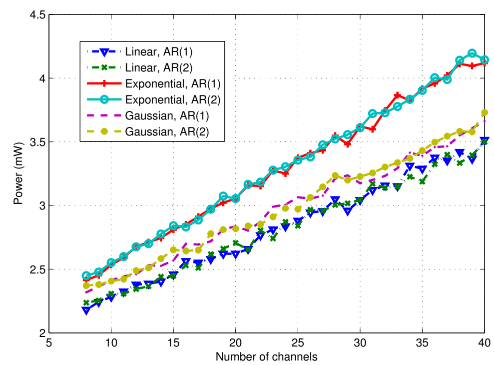

Fig. 3. Femtocell transmit power with number of available channels.

Fig. 4 shows the results of the next experiment, where we compare the quality of the spatio-temporal linear estimator (of AR order 1) with other interference estimation approaches. The interference signal is generated at the MBS, delivered to the macrocell users and measured at the femtocell users. The optimal femtocell power corresponds to the minimum value that is needed to meet SINR constraints at the femtocell users. We seek the closest scheme to this curve, knowing that excessive power will cause more interference and not enough power will not allow femtocell users achieve their requirements. We compare linear estimation with several schemes: An estimation-free approach where power is allocated based on distance and interference is neglected, a maximum power approach delivering the maximum allowable power in each channel, and a conventional FAP interference estimation scheme where macrocell signal is estimated directly at the FAP from the received signal strength and the pathloss formula. The spatio-temporal estimation is the closest curve to the optimal one, and the conventional FAP approach happens to deliver a much larger power value than the spatio-temporal one.

The last two experiments are depicted in Fig. 5 and Fig. 6, respectively. The first one shows the total femtocell power allocation for an increasing number of femtocell users in the three spatial estimation cases. Since requirements are set in terms of SINR per assigned channel and fairness must be satisfied, the larger the number of users the larger the flexibility to minimize total power. Fig. 6 explores the inaccuracy of the power assignment when the femtocell users move away from the locations utilized for the spatial estimation. The X-axis

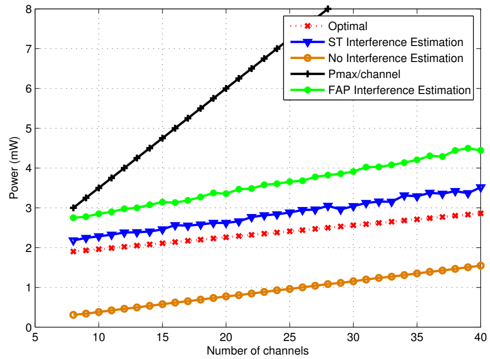

Fig. 4. Femtocell transmit power for different interference estimation methods.

shows the magnitude of the displacement although the distance to the FAP is kept constant. The Y-axis shows the difference of the *old-coordinates* and the *new-coordinates* power allocations. Clearly the gaussian estimator is the most sensitive one to the changes in the users' location, an interesting result since the location information exchange is a signalling overhead that should be kept as low as possible.

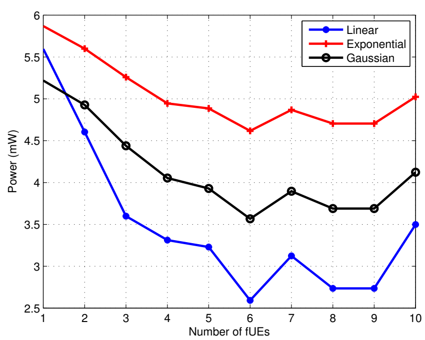

Fig. 5. Femtocell transmit power with number of femtocell users.

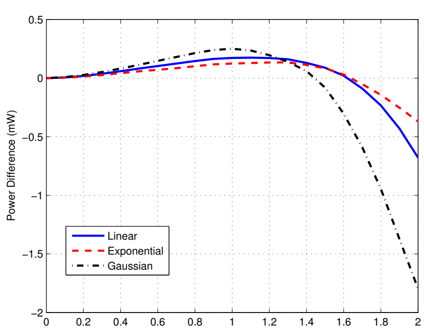

Fig. 6. Power mismatch with fUEs movement.

V. Conclusion

This paper presents a novel spatio-temporal approach to estimate the downlink cross-tier interference in femtocell networks, an important part of the interference management problem that challenges two-tier networks. The scheme is composed of a spatial estimation step that makes use of an ordinary Kriging interpolator using Semivariogram Analysis followed by a temporal estimation step implemented by an autorregressive model. This estimation is exploited via a resource allocation algorithm, and the whole scheme is enabled through a network procedure. Simulation results show the validity and potential of this approach, establishing the linear estimator followed by an autorregressive model as the most suitable option to solve this problem.

Acknowledgements

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