Can Dynamic Spectrum Access Induce Heavy Tailed Delay?

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***Abstract*—Dynamic Spectrum Access (DSA) allows secondary users (SUs) to opportunistically transmit when the channels belonging to primary users (PUs) are idle. This paper provides an asymptotic analysis of the transmission delay experienced by SUs for DSA networks. It is shown that DSA induces only light-tailed delay as long as both the busy time of PU channels and the message size of SUs are light-tailed. On the contrary, if either the busy time or the message size is heavy tailed, then the SUs' transmission delay is heavy tailed. For this latter case, it is proven that if one of either the busy time or the message size is light-tailed and the other is regularly varying with index α, then the transmission delay is regularly varying with the same index α. As a consequence, the delay has an infinite mean provided $\alpha < 1$ and an infinite variance provided $\alpha < 2$. Furthermore, if both the busy time and the message size are regularly varying with index α and β, respectively, then the tail distribution of the delay is as heavy as the one with the smaller index. Moreover, the impact of spectrum mobility and multi-radio diversity on the delay performance of SUs is studied. It is shown that the spectrum mobility can mitigate the heavy tailed delay of SUs, while the use of multiple radio interfaces may aggravate it.**

I. Introduction

Dynamic spectrum access (DSA) enables the secondary users (SUs) to use or share the spectrum in an opportunistic manner [1]. Under such scheme, SUs access the spectrum during idle periods of the primary users (PUs), and cease transmissions when the PU channels become occupied by the PUs. Apparently, the dynamically changing PU activity has a non-negligible impact on the QoS performance of SUs. This may become more evident when SUs demand for real-time services in order to support multimedia applications, such as voice over IP and online gaming.

The delay, as one of the key QoS metrics, has been widely studied for classical communication network paradigms the last several decades. So far, the transmission delay with heavy tailed distributions has drawn significantly high attentions in the research community due to its significantly different behavior from that of the light tailed (e.g., exponential) distribution [7]. More specifically, the heavy tailed delay can have infinite moments of lower orders, e.g., mean and variance. In this case, the network can exhibit significant performance degradations including the considerably reduced network throughput, queue stability, and system scalability. Despite its importance, the tail behavior of the SUs' transmission delay is still an unexplored area, partially due to the dynamic and complex network environment. In this paper, we analyze the asymptotic tail distribution of the transmission delay experienced by SUs and discover the impact of DSA paradigm on the delay performance.

We consider a cognitive radio network in which an SU can exploit the spectrum holes of multiple stochastically independent channels. A PU channel is modeled by an alternating renewal process, which alternates between busy periods $\{B_i\}_{i\geq 1}$ and idle periods $\{I_i\}_{i\geq 1}$. An SU is only allowed to transmit during the idle periods, and avoid transmissions when the PU channels become busy. Upon the arrival of a message with size $L > 0$, the SU first splits it into multiple packets with constant size $L_p > 0$, which are then sent consecutively over PU channels. Accordingly, the total time an SU takes to complete the transmissions of a message is defined as the transmission delay. Apparently, under such generic settings, the transmission delay has a close relationship with the message size as well as the PU channel availability. For the detailed description of this model, see Section II.

In this paper, we first investigate the delay performance when only a single PU channel is utilized. Specifically, it is shown that DSA induces only light-tailed delay as long as both the busy time of PU channels and the message size of SUs are light-tailed. On the contrary, if either the busy time or the message size is heavy tailed, then the SUs' transmission delay is heavy tailed. For this case, we prove that if one of the busy time or the message size is light-tailed and the other is regularly varying with index α, then the transmission delay is regularly varying with the same index α. As a consequence, the delay has an infinite variance provided $\alpha < 2$ and an infinite mean provided $\alpha < 1$. This implies that SUs can experience extremely high delay variations and even stochastically zero throughout when transmitting messages with finite mean size. Furthermore, if both the busy time and the message size are regularly varying with index α and β, respectively, then the tail distribution of the delay is as heavy as the one with the smaller index.

Moreover, we investigate the benefits of exploiting the transmission opportunities on multiple PU channels. More specifically, we consider two multiple channel access schemes, namely, the spectrum mobility and the multi-radio diversity.

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Under the spectrum mobility, if a PU appears in a channel currently used by an SU, the SU vacates the channel immediately and continues its transmission in another idle channel [1]. Under the multi-radio diversity, an SU is equipped with multiple radio interfaces so that it can simultaneously access multiple channels. We show that compared with the case in which only a single channel is used, the spectrum mobility can mitigate the degree of heavy tailed delay, while the use of multiple radio interfaces may aggravate it.

The rest of this paper is organized as follows. Section II summarizes the related work. Section III introduces system model and preliminaries. Section IV presents the main results regarding the delay performance of SUs. The impact of spectrum mobility and multi-radio diversity is studied in Section V. The simulation results are presented in Section VI. Finally, Section VII concludes this paper.

II. Related Work

Although the delay is an important QoS metric in wireless networks, the delay analysis for cognitive radio networks is still scarce to the best of our knowledge. In [5] and [8], the queuing delay of SUs in a multi-channel cognitive network was investigated with different objectives. Specifically, using large deviation approximation, [5] aimed to analyze the stationary queue distribution of SUs under the Markov chain based PU traffic model. On the contrary, [8] studied the moments of the SUs' queue length under the PU traffic modeled as an alternating ON/OFF process, where the ON periods follow a general distribution and the OFF periods are exponentially distributed. Instead of studying the queuing delay as [5] and [8], we aim to investigate the transmission delay of SUs. To the best of our knowledge, little work on the analysis of such delay has been done for cognitive radio networks. Besides the above mentioned work, a different application that is related to our work is file fragmentation [6]. In this problem, files are partioned into fragments and transferred over wireless channels. The objective is to find the optimal fragmentation policies that minimize the mean transmission time. Different from the file fragmentation application, in which only one file fragment is sent each time the wireless channel is available, SUs will keep sending packets back-to-back as long as the PU channel is detected as idle. Moreover, in the file fragmentation problem, the channel busy time is assumed to be zero [6]. This assumption is not valid in cognitive radio networks due to the existence of PU activities. In particular, recent work, which is based on real-life measurement data, has identified the heavy tailed behavior in the busy periods of PU channels [9]. This behavior was further shown to have a significant impact on the sensing performance of SUs. However, [9] did not answer how this heavy-tailed behavior of PU channels affects the delay performance of SUs, which is one of the key research problems addressed in this paper.

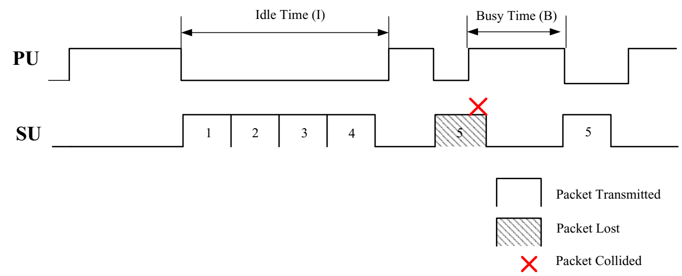

Fig. 1. System model.

III. System Model and Preliminaries

A. System Model

Consider a PU channel and an SU which transmits when the PU channel is idle. Without loss of generality, we assume that the PU channel is of unit capacity. This channel is modeled by an alternating renewal process, which alternates between busy periods $\{B_i\}_{i\geq 1}$ and idle periods $\{I_i\}_{i\geq 1}$. $\{B_i\}_{i\geq 1}$ and $\{I_i\}_{i\geq 1}$ are mutually independent random sequences of i.i.d. random variables with distribution F_B and F_I, respectively. Let $L > 0$ denote the size of the messages generated by the SU, and L is a random variable (r.v.) independent of $\{B_i\}_{i\geq 1}$ and $\{I_i\}_{i\geq 1}$. For each message, the SU divides it into packets with constant size $L_p > 0$, which are then sent over the PU channel. In each idle period I_i, the SU attempts to transmit, and if $I_i > L_p$, the SU sends packets consecutively until the remaining time of the idle period I_i is less than the packet size L_p. Otherwise, if $I_i < L_p$, the SU transmits unsuccessfully and waits for the next idle period for retransmission. An illustration of this model is given in Fig. 1.

Definition 1 *During an idle period I_i, the transmission time X_i of the SU is defined as*

$$X_i := \sup\{nL_p : nL_p \leq I_i\}, \tag{1}$$

the total number of idle periods the SU occupies for transmitting a message of size L is defined as

$$M := \inf\left\{m : \sum_{i=1}^{m} X_i \geq L\right\}, \tag{2}$$

and the total delay T of the SU transmitting a message of size L is defined as

$$T(L) := \sum_{i=1}^{M} \{I_i + B_i\}. \tag{3}$$

B. Preliminaries

In this paper we use the following notations. For any two real functions $a(t)$ and $b(t)$, we let $a(t) \sim b(t)$ denote $\lim_{t\to\infty} a(t)/b(t) = 1$. We say that $a(t) \lesssim b(t)$ if $\limsup_{t\to\infty} a(t)/b(t) \leq 1$, and $a(t) \gtrsim b(t)$ if $\liminf_{t\to\infty} a(t)/b(t) \geq 1$. Furthermore, we say that $a(t) = o(b(t))$ if $\lim_{t\to\infty} a(t)/b(t) = 0$. In addition, for any two non-negative r.v.s X and Y, we say that $X \leq_{a.s.} Y$ if $X \leq Y$

almost surely, and $X \leq_{s.t.} Y$ if X is stochastically dominated by Y, i.e., $P(X > t) \leq P(Y > t)$ for all $t \geq 0$. Also, let $F(x) = P(X \leq x)$ denote the cumulative distribution function (cdf) of a non-negative r.v. X. Let $\overline{F}(x) = P(X > x)$ denote its tail distribution function.

Definition 2 *A r.v. X is heavy tailed (HT) if for all $\theta > 0$*

$$\lim_{x\to\infty} e^{\theta x}\overline{F}(x) = \infty, \tag{4}$$

or, equivalently, if for all $z > 0$

$$E[e^{zX}] = \infty. \tag{5}$$

Definition 3 *A r.v. X is light tailed (LT) if there exists $\theta > 0$ such that*

$$\lim_{x\to\infty} e^{\theta x}\overline{F}(x) = 0, \tag{6}$$

or, equivalently, if there exists $z > 0$ such that

$$E[e^{zX}] < \infty. \tag{7}$$

Remark 1 Generally speaking, a r.v. is HT if its tail distribution decreases slower than exponentially. Some typical HT distributions include Pareto, log-normal, Bur, and Weibull (with shape parameter less than 1) distributions. On the contrary, a r.v. is LT if its tail distribution decreases exponentially or faster. Some typical LT distributions cover exponential, Gamma, and Weibull (with shape parameter larger than 1) distributions. A key characteristic that distinguishes HT r.v.s from LT ones is that the moment generating function of any HT r.v. X is infinite, i.e., $E(e^{zX}) = \infty, \forall\ z > 0$.

An important subclass of HT distributions is the class of regularly varying distributions [2]. Its definition involves the slowly varying function which is defined as follows.

Definition 4 *A measurable positive function $\mathcal{L}(x)$ defined in some interval $[a, \infty)$ is called slowly varying if for all $y > 0$*

$$\lim_{x\to\infty} \frac{\mathcal{L}(yx)}{\mathcal{L}(x)} = 1 \tag{8}$$

For example, a constant and a logarithmic function are both slowly varying functions.

Lemma 1 *(Properties of slowly varying function [2])*

1) *If $\mathcal{L}(x)$ varies slowly, then $\lim_{x\to 0} \log(\mathcal{L}(x))/x = 0$.*
2) *If $\mathcal{L}(x)$ varies slowly, so does $(\mathcal{L}(x))^a$ for every $a \in \mathbb{R}$.*
3) *If $\mathcal{L}_1(x)$ and $\mathcal{L}_2(x)$ vary slowly, so do $\mathcal{L}_1(x) + \mathcal{L}_2(x)$ and $\mathcal{L}_1(x)\mathcal{L}_2(x)$.*

Definition 5 *A r.v. X is called regularly varying with index $\alpha > 0$, denoted by $X \in \mathcal{RV}(\alpha)$, if*

$$\overline{F}(x) \sim x^{-\alpha}\mathcal{L}(x), \tag{9}$$

where $L(x)$ is a slowly varying function.

Remark 2 Regularly varying distributions are a generalization of power law distributions, which satisfy $\overline{F}(x) \sim cX^{-b}$ for $c > 0$ and $b > 0$. The index α indicates how heavy the tail distribution is, where smaller values of α imply heavier tail. Moreover, for a r.v. $X \in \mathcal{RV}(\alpha)$, the exact values of α determine whether the moments of X are bounded or not. This is explained in the following lemma.

Lemma 2 *For any r.v. $X \in \mathcal{RV}(\alpha)$, the moments of order $m > \alpha$ is unbounded, i.e.,*

$$E[X^m] = \infty, \quad \forall m > \alpha. \tag{10}$$

In particular, for any r.v. $X \in \mathcal{RV}(\alpha)$, if $\alpha < 1$, X has an infinite mean. If $1 < \alpha < 2$, X has a finite mean but an infinite variance. The following Lemmas regarding regularly varying distributions are also useful in this paper.

Lemma 3 *Let $X \in \mathcal{RV}(\alpha)$ and $Y \in \mathcal{RV}(\beta)$. If $\alpha > \beta$, then*

$$P(X > at) = o(P(Y > bt))$$

with $a > 0$ and $b > 0$.

Proof: See Appendix ∎

Lemma 4 *Let X be LT and $Y \in \mathcal{RV}(\alpha)$. Then,*

$$P(X > at) = o(P(Y > bt))$$

with $a > 0$ and $b > 0$.

Proof: See Appendix ∎

Lemma 5 *Let X and Y be non-negative random variables. If $X \in \mathcal{RV}(\alpha)$ and $P(Y > t) = P(X > bt)$ with $b > 0$, then $Y \in \mathcal{RV}(\alpha)$*

Proof: See Appendix ∎

Let $\{Y_i\}_{i\geq 1}$ be non-negative i.i.d. random variables independent of the non-negative random variable N. Define $S_N := \sum_{i=1}^{N} Y_i$. We have following Lemma 6 [4] and 7.

Lemma 6

1) *Assume $Y_1 \in \mathcal{RV}(\alpha)$, $E[N] < \infty$ and $P(N > t) = o(P(Y_1 > t))$. Then,*

$$P(S_N > t) \sim E[N]P(Y_1 > t).$$

2) *Assume $N \in \mathcal{RV}(\alpha)$, $E[Y_1] < \infty$, and $P(Y_1 > t) = o(P(N > t))$. Moreover, assume that $E[N] < \infty$ if $\alpha = 1$. Then,*

$$P(S_N > t) \sim P(N > (E[Y_1])^{-1}t).$$

Lemma 7 *Assume $N, Y_1 \in \mathcal{RV}(\alpha)$ with $E[N] < \infty$. Let $P(N > t) = t^{-\alpha}\mathcal{L}_1(t)$ and $P(Y_1 > t) = t^{-\alpha}\mathcal{L}_2(t)$. Then,*

$$P(S_N > t) \sim E[N]P(Y_1 > t) + (E[Y_1])^{\alpha}P(N > t). \tag{11}$$

Proof: See Appendix. ∎

IV. Asymptotic Analysis of the Transmission Delay

In this section, we study the tail asymptotics for the transmission delay experienced by SUs.

Theorem 1 *If the message size L is heavy tailed, then the number M of idle periods for sending such file is heavy tailed.*

Theorem 2 *If either the busy period B_i or the message size L is heavy tailed, then the transmission delay $T(L)$ is heavy tailed.*

Theorem 3 *If both the busy period B_i and the message size L are light tailed, then the transmission delay $T(L)$ is light tailed.*

Remark 3 From these results, we see that under the DSA paradigm, SUs can experience light tailed transmission delay if and only if both message size of SUs and busy time of PUs are light tailed. In other words, the heavy tailed delay originates not only from the heavy tailed file size but also from the heavy tailed busy time. In this case, the SUs' transmission delay probably has infinite moments of certain orders, e.g., mean and variance, and definitely has an infinite moment generating function, i.e., infinite exponential moments of all orders.

Proof of Theorem 1: From (2), we have

$$P(M > t) = P(L > \sum_{i=1}^{t} X_i). \tag{12}$$

Let $\mu := E[X_1]$. For $\varepsilon \in (0, \mu)$, by the law of large numbers, we obtain

$$\begin{aligned}
& P(M > t) = P(L > \sum_{i=1}^{t} X_i) \\
\geq \; & P(L > \sum_{i=1}^{t} X_i \wedge t(\mu - \varepsilon) < \sum_{i=1}^{t} X_i < t(\mu + \varepsilon)) \\
\geq \; & P(L > t(\mu + \varepsilon))P(t(\mu - \varepsilon) < \sum_{i=1}^{t} X_i < t(\mu + \varepsilon)) \\
\sim \; & P(L > t(\mu + \varepsilon))
\end{aligned}$$

Letting $\varepsilon \downarrow 0$ yields $P(M > t) \gtrsim P(L > \mu t)$. Let $t' = \mu t$. For any $\theta > 0$,

$$\begin{aligned}
\lim_{t\to\infty} e^{\theta t} P(M > t) & \geq \lim_{t\to\infty} e^{\theta t} P(L > \mu t) \\
& = \lim_{t'\to\infty} e^{\frac{\theta}{\mu} t'} P(L > t') \\
& = \infty.
\end{aligned}$$

The last equation holds since L is HT. Thus, M is HT by the Definition 2.

■

Proof of Theorem 2: **Case 1: L is HT**

For any $\delta > 0$, we have

$$\begin{aligned}
P(T(L) > t) & = P(\sum_{i=1}^{M} I_i + B_i > t) \\
& \geq P(\sum_{i=1}^{M} I_i > t \wedge M \geq \frac{t(1+\delta)}{E[X_1]}) \\
& = P(M \geq \frac{t(1+\delta)}{E[X_1]}) \\
& \quad - P(\sum_{i=1}^{M} I_i < t \wedge M \geq \frac{t(1+\delta)}{E[X_1]}) \\
& \geq P(M \geq \frac{t(1+\delta)}{E[X_1]}) - P(\sum_{i=1}^{t(1+\delta)/E[X_1]} I_i < t)
\end{aligned}$$

Let $\widetilde{I_i} := E[I_1] - I_i$. Since I_i is LT, then $\widetilde{I_i}$ is LT. Thus, by applying Chernoff bound, we can argue that there exists a positive constant λ such that for large enough t

$$P(\sum_{i=1}^{t(1+\delta)/E[X_1]} I_i < t) = P(\sum_{i=1}^{t(1+\delta)/E[X_1]} \widetilde{I_i} > \delta t) < e^{-\lambda t}.$$

Since L is HT, then M is HT by Theorem 2. Thus, for any $0 < \theta < \lambda$

$$\lim_{t\to\infty} e^{\theta t} P(T(t) > t) = \infty. \tag{13}$$

For any $\theta > \lambda$, there always exists a constant $0 < \widetilde{\theta} < \lambda$ such that

$$\lim_{t\to\infty} e^{\theta t} P(T(t) > t) > \lim_{t\to\infty} e^{\widetilde{\theta} t} P(T(t) > t) = \infty. \tag{14}$$

Combining (13) and (14), we have for any $\theta > 0$,

$$\lim_{t\to\infty} e^{\theta t} P(T(t) > t) = \infty. \tag{15}$$

This implies that $T(L)$ is HT by the Definition.

Case 2: B_i is HT

Case 1 proves if L is HT, then $T(L)$ is HT. Thus, in case 2, we assume that L is LT. It is easy to see

$$P(T(L) > t) = P(\sum_{i=1}^{M} I_i + B_i > t) \geq P(\sum_{i=1}^{M} B_i > t). \tag{16}$$

which implies $T(L)$ is HT provided one can prove $Z := \sum_{i=1}^{M} B_i$ is HT. Towards this, by the independence between M and B_i, we obtain the moment generating function $M_Z(x)$ of Z, i.e.,

$$M_Z(x) = E[e^{x \sum_{i=1}^{M} B_i}] = E[(E[e^{B_1}])^{xM}]$$

Since function $f(y) = a^y$ is convex and B_1 is HT, by Jensen's inequality, for all $x > 0$

$$M_Z(x) = E[(E[e^{B_1}])^{xM}] \geq (E[e^{B_1}])^{xE[M]} = \infty$$

Thus, it follows that $T(L)$ is HT by the Definition.

■

The proof of Theorem 3 relies on Lemma 8 and 9, which we state first.

Lemma 8 *Properties of LT Distributions [6]*

1) *If X and Y are non-negative LT random variables, then $X + Y$ is LT.*
2) *Let $\{X_i\}_{i\geq 1}$ be i.i.d. LT random variables, and N be integer LT random variable. Then, the random sum $\sum_{i=1}^{N} X_i$ is LT.*
3) *Let L be a non-negative random variable and $\{X_i\}_{i\geq 1}$ be non-negative i.i.d. random variables independent of L and satisfying $P(X_i) > 0$. If L is LT, so is $\inf\{n : \sum_{i=1}^{n} X_i \geq L\}$.*

Lemma 9 *Let X and Y be non-negative random variables. If $P(Y > t) = P(X > a(t + b))$ with $0 < a < \infty$ and $0 < b < \infty$, then Y is LT provided X is LT.*

Proof of Lemma 9: See Appendix. ■

Proof of Theorem 3: By Definition 1, we have $X_i = N_i L_p$ with N as a positive integer random variable, where

$$N_i = \sup\{n : \sum_{i=1}^{n} nL_p \leq I_i\}. \tag{17}$$

It is easy to see

$$P(N_i > n) = P(I_i > (n + 1)L_p) \tag{18}$$

This implies that N_i is LT by Lemma 9. Accordingly, it follows easily from the definition that $X_i = N_i L_p$ is LT. Therefore, invoking Lemma 8(3), we obtain that M is LT. Since $I_i + B_i$ is LT by Lemma 8(1), we finally obtain that $T(L)$ is LT using Lemma 8(3). ■

The above Theorems state the conditions under which the SUs' transmission delay exhibits heavy tailed behavior. The following Theorems present the exact asymptotic results for this delay under the regularly varying busy time of PUs and message size of SUs.

Theorem 4 *If $L \in \mathcal{RV}(\alpha)$, then $M \in \mathcal{RV}(\alpha)$ and*

$$P(M > t) \sim P(L > E[X_1]t). \tag{19}$$

Remark 4 Comparing with Theorem 1, the Theorem 4 provides more refined results regarding the total number of idle periods a SU occupies for transmitting a message. Specifically, if the message size is regularly varying, then the number of idle periods for transmitting such message is also regularly varying with the same index. This implies that if the message size has infinite mean and variance, so does the number of idle periods occupied by SUs.

Theorem 5 *If $L \in \mathcal{RV}(\alpha)$ and B_i is LT, then $T(L) \in \mathcal{RV}(\alpha)$ and*

$$P(T(L) > t) \sim P(L > \frac{E[X_1]}{E[I_1] + E[B_1]}t). \tag{20}$$

Theorem 6 *If $B_i \in \mathcal{RV}(\alpha)$ and L is LT, then $T(L) \in \mathcal{RV}(\alpha)$ and*

$$P(T(L) > t) \sim E[M]P(B_1 > t). \tag{21}$$

Remark 5 The preceding results establish the relationship between the tail asymptotics of L, B_i, and $T(L)$. Specifically, if one of the busy time or message size is light tailed and the other is regularly varying, then the tail of the transmission delay is asymptotically proportional to the one with regularly varying distribution. This result implies that SUs can experience extremely high delay variance and stochastically zero throughput even when the transmitting messages are of finite mean size. For example, if the message size is LT, then its mean is finite. In this case, by Theorem 6, when $2 > \alpha > 1$, the transmission delay does not have finite variance, and when $1 > \alpha > 0$, it does not have finite mean, which implies approximately zero throughput on the average.

Theorem 7 *Assume that $B \in \mathcal{RV}(\alpha_b)$, $L \in \mathcal{RV}(\alpha_l)$, and $E[L] < \infty$. Then, we have*

1) *If $\alpha_b < \alpha_l$, then $T(L) \in \mathcal{RV}(\alpha_b)$ and*

$$P(T(L) > t) \sim E[M]P(B_1 > t) \tag{22}$$

.

2) *If $\alpha_b \geq \alpha_l$,*

$$\lim_{t\to\infty} \frac{\log\left[P(T(L) > t)\right]}{\log t} = -\alpha_l. \tag{23}$$

Corollary 1 *If $B \in \mathcal{RV}(\alpha_b)$, $L \in \mathcal{RV}(\alpha_l)$, and $E[L] < \infty$, then*

$$\lim_{t\to\infty} \frac{\log\left[P(T(L) > t)\right]}{\log t} = -\min(\alpha_b, \alpha_l),$$

and accordingly, the moments of orders $m > \min(\alpha_b, \alpha_l)$ is unbounded, i.e.,

$$E[T(L)^m] = \infty$$

Remark 6 Comparing the above Theorem and Theorem 4 - 6, we observe that the exact asymptotic tail for the transmission delay is not available in the case of $\alpha_b \geq \alpha_l$. Instead, Corollary 1 states that if both busy time and message size are regularly varying, then the tail of the transmission delay is asymptotically equivalent to the one with smaller index on the logarithmic scale. In this case, it follows directly from Theorem 2 in [3] that the transmission delay still has infinite moments of orders larger than the index $\min(\alpha_b, \alpha_l)$, even through this delay does not strictly follow regular varying distributions.

The proof of Theorem 4 relies on Lemma 10, which we state and prove first.

Lemma 10 *Let $\widetilde{T}(L) = \sum_{i=1}^{M} I_i$. If $L \in \mathcal{RV}(\alpha)$, then*

$$P(\widetilde{T}(L) > t) \sim P(L > \delta t), \tag{24}$$

where $\delta = E[X_1]/E[I_1]$.

Proof of Lemma 10: See Appendix. ■

Proof of Theorem 4: Let $\widetilde{T}(L) = \sum_{i=1}^{M} I_i$. By Lemma 10, (19) follows provided one can show that

$$P(\widetilde{T}(L) > t) \sim P(M > \frac{t}{E[I_1]}) \tag{25}$$

For all $1 > \delta > 0$, we obtain

$$\begin{aligned} P(T(L) > t) &\leq P(M \geq \frac{t(1-\delta)}{E[X_1]}) \\ &\quad + P(\sum_{i=1}^{M} I_i > t \wedge M \leq \frac{t(1-\delta)}{E[I_1]}) \\ &\leq P(M \geq \frac{t(1-\delta)}{E[X_1]}) + P(\sum_{i=1}^{t(1-\delta)/E[X_1]} I_i > t) \\ &\sim P(M \geq \frac{t(1-\delta)}{E[X_1]}) \end{aligned} \tag{26}$$

The last step follows the Chernoff bounds. Letting $\delta \downarrow 0$, this proves the upper bound in (25). As to the lower bound, for all $\delta > 0$,

$$\begin{aligned} P(T(L) > t) &\geq P(M \geq \frac{t(1+\delta)}{E[X_1]}) \\ &\quad - P(\sum_{i=1}^{M} I_i < t \wedge M \geq \frac{t(1+\delta)}{E[I_1]}) \\ &\geq P(M \geq \frac{t(1+\delta)}{E[X_1]}) - P(\sum_{i=1}^{t(1+\delta)/E[X_1]} I_i > t) \\ &\sim P(M \geq \frac{t(1+\delta)}{E[X_1]}) \end{aligned} \tag{27}$$

Letting $\delta \downarrow 0$, this proves the lower bound in (25). By (26) and (27), we obtain

$$P(M > \frac{t}{E[I_1]}) \sim P(\widetilde{T}(L) > t) \sim P(L > \frac{E[X_1]}{E[I_1]} t),$$

which implies $P(M > t) \sim P(L > E[X_1]t)$. This completes the proof of (19). From (19), it follows that $M \in \mathcal{RV}(\alpha)$ using Lemma 5. ■

Proof of Theorem 5: We use the same techniques used in proving Lemma 10. See appendix for a proof. ■

To facilitate the proofs of Theorem 6 - 7, we define $T_I := \sum_{i=1}^{M} I_i$ and $T_B := \sum_{i=1}^{M} B_i$. This implies $T(L) = T_I + T_B$.

Proof of Theorem 6: To prove the Theorem, we first show that T_I is LT and $P(T_B > t) \sim E[M]P(L > t)$. First, we argue that T_I is LT. Since L is LT, from Lemma 8(3), we conclude that M is LT. This implies that $T_I := \sum_{i=1}^{M} I_i$ is LT using Lemma 8(2).

We now show that $P(T_B > t) \sim E[M]P(L > t)$. Since M is independent of B_i and $B_i \in \mathcal{RV}(\alpha)$, it follows that $P(M > t) = o(P(B_i > t))$ invoking Lemma 4. From Lemma 6(1), we see that

$$P(T_B > t) \sim E[M]P(L > t) \tag{28}$$

which, in turn, implies $T_B \in \mathcal{RV}(\alpha)$ by invoking Lemma 1(3).

We are now ready to prove the upper bound in (21). For any $0 < \delta < 1$

$$\begin{aligned} P(T(L) > t) &= P(T_I + T_B > t) \\ &= P(T_I + T_B > t \wedge T_I > \delta t) \\ &\quad + P(T_I + T_B > t \wedge T_I < \delta t) \\ &\leq P(T_B > (1-\delta)t) + P(T_I > \delta t) \\ &\sim P(T_B > (1-\delta)t) \end{aligned}$$

The last step follows since $P(T_I > \delta t) = o(P(T_B > (1-\delta)t))$ using Lemma 4. Letting $\delta \downarrow 0$, this proves the upper bound in (21). As to the lower bound, it is easy to see

$$P(T(L) > t) = P(T_I + T_B > t) \geq P(T_B > t)$$

which in conjunction with the upper bound, completes the proof of (21). Moreover, (21) implies $T(L) \in \mathcal{RV}(\alpha)$ using Lemma 1(3). This completes the proof. ■

Proof of Theorem 7: **Case 1:** $\alpha_b < \alpha_l$

Since $L \in \mathcal{RV}(\alpha_l)$ and $E[M] < \infty$, using Theorem 4, we obtain that $M \in \mathcal{RV}(\alpha_l)$ and $E[M] < \infty$. This, in conjunction with $\alpha_b < \alpha_l$, implies that $P(M > t) = o(P(B_1 > t))$ using Lemma 3. Invoking Lemma 6(1), we conclude that

$$T_B = \sum_{i=1}^{M} B_i \sim E[M]P(B_1 > t).$$

which in turn implies that $T_B \in \mathcal{RV}(\alpha_b)$ by Lemma 5. By Lemma 10, we can see that $T_I \in \mathcal{RV}(\alpha_l)$ since $L \in \mathcal{RV}(\alpha_l)$.

We are now ready to prove the upper bound in (22). For any $1 > \delta > 0$, we obtain that

$$\begin{aligned} P(T(L) > t) &= P(T_I + T_B > t) \\ &\leq P(T_B > (1-\delta)t) + P(T_I > \delta t) \end{aligned} \tag{29}$$

Since $T_I \in \mathcal{RV}(\alpha_l)$, $T_B \in \mathcal{RV}(\alpha_b)$, and $\alpha_b < \alpha_l$, using Lemma 3, we obtain that $P(T_I > \delta t) = o(P(T_B > (1-\delta)t))$. This implies that $P(T(L) > t) \lesssim P(T_B > (1-\delta)t)$ from (29). Letting $\delta \downarrow 0$, we verify the upper bound in (22). For the lower bound, it is easy to see that $P(T(L) > t) \geq P(T_B > t)$. Since the lower and upper bounds coincide, this completes the proof of (22).

Case 2: $\alpha_b \geq \alpha_l$

Since $L \in \mathcal{RV}(\alpha_l)$, from Lemma 10 and regular variations, we obtain that

$$P(T_I > t) \sim P(L > \frac{E[X_1]}{E[I_1]} t) \sim (\frac{E[I_1]}{E[X_1]})^{\alpha_l} P(L > t). \tag{30}$$

From Theorem 4, we conclude that $M \in \mathcal{RV}(\alpha_l)$ and

$$P(M > t) = P(L > E[X_1]t).$$

If $\alpha_b > \alpha_l$, it follows that $P(M > t) = o(P(B_i > t))$. This implies, using Lemma 6(2), that

$$P(T_B > t) \sim P(L > \frac{E[X_1]}{E[B_1]} t) \sim (\frac{E[B_1]}{E[X_1]})^{\alpha_l} P(L > t). \tag{31}$$

If $\alpha_b = \alpha_l$, using Lemma 7, we obtain that

$$P(T_B > t) \sim E[M]P(B_1 > t) + P(L > t). \tag{32}$$

Combining (29), (30), (31) and (32), we obtain that

$$\limsup_{t\to\infty} \frac{\log T(L)}{\log t} \leq -\alpha_l$$

which, in conjunction with

$$\liminf_{t\to\infty} \frac{\log T(L)}{\log t} \geq \liminf_{t\to\infty} \frac{\log T_I}{\log t} \geq -\alpha_l$$

completes the proof. ■

V. Impact of Spectrum Mobility and Multi-radio Diversity

In this section, we study the impact of spectrum mobility and multi-radio diversity on the delay performance of SUs. By spectrum mobility, we mean that if a PU appears in a channel currently used by an SU, the SU should vacate the channel immediately and continue its transmission in another idle channel. By multi-radio diversity, we mean that an SU is equipped with multiple radio interfaces. In this case, an SU can simultaneously access multiple channels.

A. System Model

We first introduce the system model. Assume that there exist $K \geq 1$ PU channels, which are modeled by K independent alternating renewal processes as defined in section II. Each channel $K \geq j \geq 1$ is denoted by $CH^j = \{(B_i^j, I_i^j)\}_{i\geq 1}$ and channels $\{CH^j\}_{K\geq j\geq 1}$ are heterogenous, i.e., $\{B_1^j\}_{K\geq j\geq 1}$ (or/and $\{I_1^j\}_{K\geq j\geq 1}$) are not identically distributed. To simplify the analysis, we assume that the idle and busy periods are much larger than the packet size L_p.

B. Spectrum mobility

By spectrum mobility, we mean that an SU can switch to the idle channels when its current operating channel is occupied by a PU. As a consequence, the SU sees K channels as a single virtual channel, which stays idle if one of K channels is idle and stays busy if all K channels are busy. Apparently, this virtual channel can be modeled by a random process that alternates between busy $\{B_i^s\}_{i\geq 1}$ and idle $\{I_i^s\}_{i\geq 1}$ periods. (Note that neither $\{B_i^{sm}\}_{i\geq 1}$ nor idle $\{I_i^s\}_{i\geq 1}$ are necessarily i.i.d. random sequences.) Therefore, the transmission delay under spectrum mobility is defined similarly as the one in Definition 1, i.e.,

Definition 6 *Let X_i^s be the transmission time of a SU during an idle period I_i^s and M_s be the number of idle periods for the SU transmitting a message of size L. Then, the total delay under spectrum mobility is defined as*

$$T_s(L) := \sum_{i=1}^{M_s-1} (I_i^s + B_i^s) + t_{M_s} \tag{33}$$

where t_{M_s} is the time for last transmission, i.e., $t_n = L - \sum_{i=1}^{M_s-1} X_i^s$.

By multi-radio diversity we mean that an SU is equipped with $N < K$ radio interfaces with each one operating on a different channel. For each message, the SU divides it into N fragments and sends each fragment over a different interface. The total transmission delay is the time for the SU to finish sending all fragments.

Definition 7 *Consider a message of size L, which is divided into fragments of sizes $\{r_i L\}_{N\geq i\geq 1}$ such that $\sum_{i=1}^N r_i = 1$. Let $T_i(r_i L)$ be the delay of sending a fragment of size $r_i L$ over interface i. Then, the total delay under multi-radio diversity is defined as*

$$T_m(L) := \max_{N\geq i\geq 1} T_i(r_i L) \tag{34}$$

Theorem 8 *Assume $\{B_1^j\}_{K\geq j\geq 1}$ are regularly varying random variables with indices $\alpha_1, \alpha_2, ..., \alpha_K$, respectively. Define $\alpha_s := \max_{K\geq j\geq 1} \alpha_j$ and $\alpha_m := \min_{K\geq j\geq 1} \alpha_j$.*

1) *Under spectrum mobility, If L is LT, then*

$$\lim_{t\to\infty} \frac{\log[P(T_s(L) > t)]}{\log t} \leq -\alpha_s. \tag{35}$$

If $L \in \mathcal{RV}(\alpha_l)$, then

$$\lim_{t\to\infty} \frac{\log[P(T_s(L) > t)]}{\log t} \leq -\min(\alpha_s, \alpha_l) \tag{36}$$

2) *Under multi-radio diversity, If L is LT, then*

$$\lim_{t\to\infty} \frac{\log[P(T_m(L) > t)]}{\log t} = -\alpha_m \tag{37}$$

If $L \in \mathcal{RV}(\alpha_l)$, then

$$\lim_{t\to\infty} \frac{\log[P(T_m(L) > t)]}{\log t} = -\min(\alpha_m, \alpha_l) \tag{38}$$

Remark 7 In the above results, Theorem 8(1) implies that the delay performance under spectrum mobility is upper bounded by that of the best channels, which have the largest index α_j. On the contrary, Theorem 8(2) implies that the transmission delay under multi-radio diversity is determined by the worst channels, which have the smallest index α_j. As a consequence, compared with the case in which only a single channel is used, spectrum mobility can mitigate the heavy tailed delay by increasing the orders of its finite moments as least up to $\max_{K\geq j\geq 1} \alpha_j$, while multi-radio diversity can aggravate it by decreasing the orders of its finite moments to $\min_{N\geq j\geq 1} \alpha_j$. Apparently, spectrum mobility is more beneficial than the use of multiple radio interfaces.

Proof: Define M_j and $T_j(L)$ as the total number of idle periods and total delay, respectively, when a message of size L is sent over a single channel $K \geq j \geq 1$. Under spectrum mobility, K channels act as an virtual channel which stays idle as long as one of K channels is idle. Thus, it is easy to see that during any time interval, the virtual channel has longer idle time than any single channel $K \geq j \geq 1$. Thus, if the transmission of a message is completed on any single channel, it is completed on the virtual channel. This implies that

$$M_s \leq_{a.s.} M_j \quad \text{and} \quad T_s(L) \leq_{a.s.} T_j(L) \quad \forall K \geq j \geq 1 \tag{39}$$

If L is LT, then from Theorem 6, we conclude that $T_j(L) \in \mathcal{RV}(\alpha_j)$. This implies from (39) that

$$\lim_{t\to\infty} \frac{\log[P(T_s(L) > t)]}{\log t} \le -\max_{K\ge j\ge 1} \alpha_j.$$

which completes the proof of (35).

If $L \in \mathcal{RV}(\alpha_l)$, then from Corollary 1, we conclude that

$$\lim_{t\to\infty} \frac{\log[P(T_i(L) > t)]}{\log t} = -\min(\alpha_l, \alpha_i),$$

which implies from (39) that

$$\lim_{t\to\infty} \frac{\log[P(T_s(L) > t)]}{\log t} \le -\max_{K\ge i\ge 1} \min(\alpha_l, \alpha_i).$$

This completes the proof of (36).

By the Definition of T_m, we obtain

$$P(T_m > t) = P\left(T_1(r_1 L) > t \ \cup \ldots \cup \ T_N(r_N L) > t\right)$$

This, using the union bound, implies

$$P(T_m > t) \le \sum_{j=1}^{N} P(T_j(r_j L) > t). \qquad (40)$$

and

$$P(T_m > t) \ge P(T_j(r_j L) > t) \ \ \forall N \ge j \ge 1. \qquad (41)$$

If L is LT, then from Theorem 6, we conclude that $T_j(r_j L) \in \mathcal{RV}(\alpha_j)$. This, using Lemma 3, implies that $P(T_j(r_j L) > t) = o(P(T_i(r_i L) > t)$ if $\alpha_j > \alpha_i$. This in turn implies from (40) and (41) that

$$-\min_{N\ge j\ge 1} \alpha_j \le \lim_{t\to\infty} \frac{\log[P(T_m(L) > t)]}{\log t} \le -\min_{N\ge j\ge 1} \alpha_j,$$

which completes the proof of (37).

If $L \in \mathcal{RV}(\alpha_l)$, then from Corollary 1, we conclude

$$\lim_{t\to\infty} \frac{\log[P(T_j(r_j L) > t)]}{\log t} = -\min(\alpha_l, \alpha_i),$$

which implies from (40) and (41) that

$$\lim_{t\to\infty} \frac{\log[P(T_s(L) > t)]}{\log t} = -\min_{N\ge i\ge 1} \min(\alpha_l, \alpha_i).$$

This completes the proof of (38). ■

VI. Simulation Results

In this section, we use simulations to illustrate our theoretical results. In particular, we focus on the relationship between the tail asymptotics of message size L, PU busy time B_i, and transmission delay $T(L)$. As presented in the preceding Theorems, the SUs' HT delay is attributed to the HT message size as well as the HT PU busy time. To verify this result, we choose Pareto and exponential distributions to represent HT and LT distributions, respectively. We say that a random variable $X \in \mathcal{PAR}(\alpha, x_m)$ if X follows a Pareto distribution with parameter α and x_m, i.e., $P(X > t) = (x_m/x)^{\alpha}$. We say that a random variable $X \in \mathcal{EXP}(\lambda)$ if X follows an exponential distribution with parameter λ, i.e., $P(X > t) = e^{-\lambda t}$.

Figure 2 illustrates the asymptotic results in Theorem 5. We let $B_i, I_i \in \mathcal{EXP}(0.01)$, $L \in \mathcal{PAR}(1.2, 50)$, and $L_p = 10$. The simulation results for the delay, PU busy time, and message size are plotted on log-log scale, respectively. We see that the tail distribution of the transmission delay exhibits itself as a straight line, which is parallel to that of the message size. This indicates that the tail of the transmission delay is as heavy as that of the message size. This is consistent with the Theorem 5.

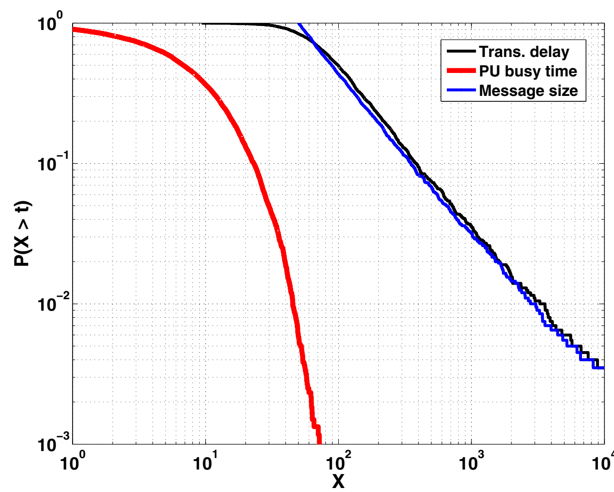

Fig. 2. Impact of the regularly varying message size and the light tail PU busy time on the transmission delay.

Nowadays, the multimedia and Internet traffic has become dominant in current wireless and cellular networks. Accordingly, the PU channels trend to exhibit HT busy time. In this case, Theorem 6 states that SUs can experience the transmission delay which has a tail distribution as heavy as that of the PU channel busy time. To verify this theoretical result, we let $\{L, I_i\} \in \mathcal{EXP}(0.01)$, $B_i \in \mathcal{PAR}(0.8, 5)$, and $L_p = 10$. We can see in Figure 3 that the straight line that represents the tail distribution of the transmission delay is parallel to that of the message size. This indicates that the delay tail distribution is as heavy as that of the PU busy time. Since $B_i \in \mathcal{PAR}(0.8, 5)$, this implies that the transmission delay has the infinite mean.

The above simulation results discuss the cases where either the message size or the PU busy time is HT. In practice, both PUs and SUs can demand for real-time transmissions which are initiated by the applications such as video conferencing and on-line gaming. As a consequence, both the message size of SUs and the busy periods of PUs can be HT. In this case, Theorem 7 states that the one with heavier tail distribution determines the delay performance of SUs. To illustrate this asymptotic result, we let $L \in \mathcal{PAR}(1.2, 50)$, $B_i \in \mathcal{PAR}(0.8, 5)$, and $I_i \in \mathcal{EXP}(0.01)$. We see in Figure 4 that the tail distribution of the transmission delay is parallel to that of the busy time, which has a smaller index. This implies that the delay performance in a cognitive radio network can

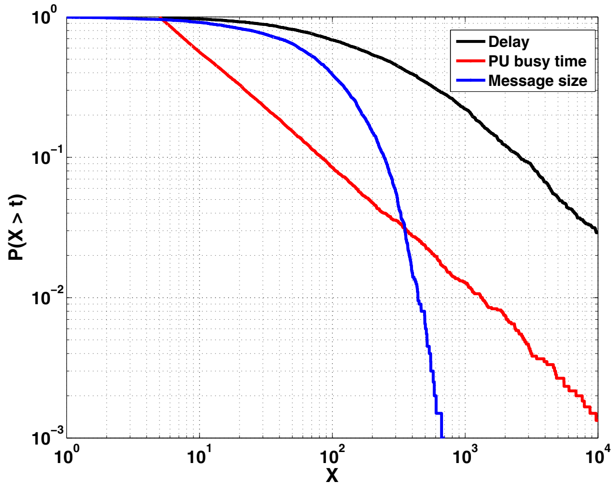

Fig. 3. Impact of the regularly varying PU busy time and the light tail message size on the transmission delay.

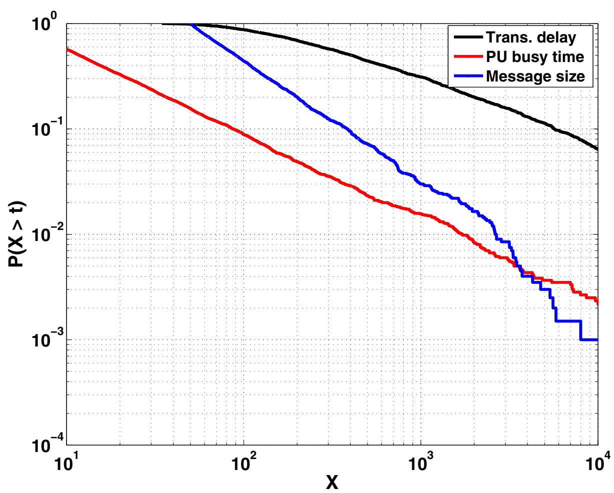

Fig. 4. Impact of the regularly varying PU busy time and the message size on the transmission delay.

be significantly degraded by the traffic conditions of the PUs.

VII. Conclusions

This paper provides an asymptotic analysis of the transmission delay experienced by SUs. It is shown that SUs can have light-tailed delay if and only if both the busy time of PU channels and the message size of SUs are light-tailed. In other words, the heavy tailed transmission delay can originate from either the heavy tailed busy time or the heavy tailed message size. In this case, it is proven that if one of the busy time or the message size is light-tailed and the other is regularly varying, then the transmission delay is regularly varying with the same index. Furthermore, if both the busy time and the message size are regularly varying with different indexes, then the tail distribution of the delay is as heavy as the one with the smaller index. Moreover, to exploit benefits of multiple PU channels, the spectrum mobility and multi-radio diversity are considered. It is shown that spectrum mobility can mitigate the heavy tailed delay of SUs, while the use of multiple radio interfaces may aggravate it.

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Appendix

A. Proof of Lemma 3

Let $P(X > t) = t^{-\alpha}\mathcal{L}_1(t)$ and $P(Y > t) = t^{-\beta}\mathcal{L}_2(t)$. Since $\mathcal{L}_2(t)$ varies slowly, so does $1/\mathcal{L}_2(t)$ from Lemma 1(2). This, in turn, implies that for any $a > 0$ and $b > 0$, $\mathcal{L}_3(t) := a^{-\alpha}\mathcal{L}_1(t)/(b^{-\beta}\mathcal{L}_2(t))$ varies slowly using Lemma 1(3). Since $\alpha > \beta$, then we have

$$\lim_{t\to\infty} \frac{P(X > at)}{P(Y > bt)} = \lim_{t\to\infty} t^{-(\alpha-\beta)}\mathcal{L}_3(t) = 0. \quad (42)$$

which implies that $P(X > at) = o(P(Y > bt))$. This completes the proof.

B. Proof of Lemma 4

Since X is LT and $Y \in \mathcal{RV}(\alpha)$, there exists $\theta > 0$ such that $\lim_{t\to\infty} e^{\theta at}P(X > at) = 0$ and $\lim_{t\to\infty} e^{\theta at}P(Y > bt) = \infty$. Then, we have

$$\lim_{t\to\infty} \frac{P(X > at)}{P(Y > bt)} = \lim_{t\to\infty} \frac{e^{\theta at}P(X > at)}{e^{\theta at}P(Y > bt)} = 0. \quad (43)$$

This implies that $P(X > at) = o(P(Y > bt))$. This completes the proof.

C. Proof of Lemma 5

Let $P(X>t)=t^{-\alpha}\mathcal{L}_1(t)$. By regular variation, for any $b>0$,

$$P(Y>t)\sim P(X>bt)\sim(b)^{-\alpha}P(X>t)=t^{-\alpha}\mathcal{L}_2(t)$$

with $\mathcal{L}_2(t)=(b)^{-\alpha}\mathcal{L}_1(t)$, which is a slowly varying function by Lemma 1(3). This implies that $Y\in\mathcal{RV}(\alpha)$.

D. Proof of Lemma 7

We use techniques similar to those used in [4] to prove that the lower and upper bounds in (11) asymptotically coincide. For every fixed n_0 we obtain

$$\begin{aligned} P(S_N>t) &= \sum_{n=1}^{n_0}P(N=n)P(S_n>t)\\ &\quad+\sum_{n=n_0}^{\infty}P(N=n)P(S_n>t)\end{aligned}$$

Since $Y_1\in\mathcal{RV}(\alpha)$, Y_1 is subexponentially distributed. By the subexponentiality of Y_1 and the independence of N and Y_1, we obtain

$$\begin{aligned}\sum_{n=1}^{n_0}P(N=n)P(S_n>t) &\sim \sum_{n=1}^{n_0}P(N=n)nP(Y_1>t)\\ &\sim E[Y_1]P(Y_1>t), n_0\to\infty\end{aligned}$$

For any $1>\delta>0$, we obtain for large enough t

$$\begin{aligned}&\sum_{n=n_0+1}^{\infty}P(N=n)P(S_n>t)\\ =&\ (\sum_{n=n_0+1}^{t(1-\delta)/E[Y_1]}+\sum_{n=t(1-\delta)/E[Y_1]}^{\infty})P(N=n)P(S_n>t)\\ :=&\ I+II.\end{aligned}$$

For term II, we obtain

$$\begin{aligned} II &= (\sum_{n=t(1-\delta)/E[Y_1]}^{t(1+\delta)/E[Y_1]}+\sum_{n=t(1+\delta)/E[Y_1]}^{\infty})P(N=n)P(S_n>t)\\ &:= J_1+J_2 \end{aligned} \tag{44}$$

By the law of large numbers and letting $\delta\downarrow 0$, we obtain

$$\begin{aligned} J_1 &\le \sum_{n=t(1-\delta)/E[Y_1]}^{t(1+\delta)/E[Y_1]}\left(P(N=n)P(\sum_{i=1}^{t(1+\delta)/E[Y_1]}Y_i>t)\right)\\ &\sim P(N>\frac{t(1-\delta)}{E[Y_1]})-P(N>\frac{t(1+\delta)}{E[Y_1]})=o(1)\end{aligned}$$

For J_2, we have

$$J_2\le\sum_{n=t(1+\delta)/E[Y_1]}^{\infty}P(N=n)=P(N>\frac{1+\delta}{E[Y_1]}t) \tag{45}$$

and by the law of large numbers,

$$\begin{aligned} J_2 &\ge \sum_{n=t(1+\delta)/E[Y_1]}^{\infty}\left(P(N=n)P(\sum_{i=1}^{t(1+\delta)/E[Y_1]}Y_i>t)\right)\\ &= P(N>\frac{1+\delta}{E[Y_1]}t)\end{aligned} \tag{46}$$

. Combining (45) and (46) and letting $\delta\downarrow 0$, we have

$$J_2\sim P(N>\frac{t}{E[Y_1]})\sim(E[Y_1])^{\alpha}P(N>t) \tag{47}$$

For term I, we have

$$I=\sum_{n=n_0+1}^{t(1-\delta)/E[Y_1]}P(N=n)P(S_n-nE[Y_1]>t-nE[Y_1])$$

Since $n<t(1-\delta)/E[Y_1]$, we obtain that $t-nE[Y_1]>0$. By large deviations theory, it follows that for any $\varepsilon>0$

$$\lim_{n\to\infty}\sup_{t>\varepsilon n}\left|\frac{P(S_n-nE[Y_1]>t)}{nP(Y_1>t)}-1\right|=0$$

which implies that there exists some positive constant C such that

$$\lim_{n_0\to\infty}\limsup_{x\to\infty}I\le\lim_{n_0\to\infty}C\sum_{n=n_0+1}^{\infty}P(N=n)nP(Y_1>t)=0$$

This, in conjunction with (44), (45) and (47), completes the proof.

E. Proof of Lemma 9

Since X is LT, we can find a positive constant θ such that

$$\lim_{t\to\infty}e^{\theta t}p(X>t)<\infty. \tag{48}$$

Thus, there exists $\widetilde{\theta}=a\theta>0$ such that

$$\begin{aligned}\lim_{t\to\infty}e^{\widetilde{\theta}t}P(Y>t) &= \lim_{t\to\infty}e^{\widetilde{\theta}t}P(X>a(t+b))\\ &\le \lim_{t\to\infty}e^{\theta at}P(X>at)<\infty,\end{aligned}$$

which implies Y is LT by the Definition.

F. Proof of Lemma 10

Proof of Lemma 10 relies on the Theorem 9. This technique is similar to the one used in the proof for optimal file fragmentation [6].

Theorem 9 *[4] Let $L\in\mathcal{RV}(\alpha)$. Let $R(t)$ be a non-negative, almost surely non-decreasing random process independent of L. If $R(t)$ satisfies following conditions:*

1) *$R(t)/t\to\gamma$ almost surely as t goes to infinity, with $0<\gamma<1$.*
2) *There exists a positive and finite constant K such that $P(R(t)/t<K)=o(P(L>t))$.*

Then $P(L>R(t))\sim P(L>\gamma t)$

Proof of Lemma 10: We define

$$N_t:=\sup\left\{n:\sum_{i=1}^{n}I_i<t\right\},$$

and

$$R(t) = \sum_{i=1}^{N_t} X_i.$$

It is easy to see that $P(\widetilde{T}(L) > t) = P(L > R(t))$. Thus, to prove Lemma 10, it is sufficient to prove condition 1 and 2 of Theorem 9 are satisfied. By renewal theory, we have

$$\lim_{t\to\infty} \frac{R(t)}{t} = \frac{E[X_1]}{E[I_1]} = \gamma \tag{49}$$

almost surely. Since $X_1 \leq_{a.s.} I_1$, we conclude $E[X_1] < E[I_1]$ and $0 < \gamma < 1$, implying condition 1 of Theorem 9 is safisfied. Next, we will prove that the condition 2 of Theorem 9 is also satisfied. Let $K = (1-\delta)E[X_1]/((1+\delta)E[I_1])$. Then, for any $1 > \delta > 0$, we have

$$\begin{aligned} P(R(t) < Kt) &= P(\sum_{i=1}^{N_t} X_i < Kt) \\ &\leq P(N_t < \frac{t(1-\delta)}{E[I_1]}) \\ &\quad + P(\sum_{i=1}^{N_t} X_i < Kt \wedge N_t > \frac{t(1-\delta)}{E[I_1]}) \\ &:= J_1 + J_2 \end{aligned}$$

For J_1, since I_i is LT, by Chernoff bound, there exists $\lambda_1 > 0$ such that

$$J_1 \leq P(\sum_{i=1}^{t(1-\delta)/E[I_1]} I_i > t) \leq e^{-\lambda_1 t}. \tag{50}$$

For J_2, let $\widetilde{X}_i = E(X_i) - X_i$, we obtain

$$\begin{aligned} J_2 &< P(\sum_{i=1}^{t(1-\delta)/E[I_1]} X_i < Kt) \\ &= P(\sum_{i=1}^{t(1-\delta)/E[I_1]} \widetilde{X}_i > \frac{\delta(1-\delta)E[X_1]}{(1+\delta)E(I_1)} t) \end{aligned}$$

Since X_i is LT, by Chernoff bound, we can always find $\lambda_2 > 0$ such that

$$J_2 \leq e^{-\lambda_2 t}. \tag{51}$$

By (50) and (51), we conclude

$$P(R(t) < Kt) \leq e^{-\lambda_1 t} + e^{-\lambda_2 t} \tag{52}$$

Since $L \in \mathcal{RV}(\alpha)$, we have

$$\limsup_{t\to\infty} \frac{P(R(t) < Kt)}{P(L > t)} \leq \limsup_{t\to\infty} \frac{e^{-\lambda_1 t} + e^{-\lambda_2 t}}{t^{-\alpha}\mathcal{L}(t)} = 0 \tag{53}$$

The last equality holds since regularly varying distributions are a subclass of HT distributions. Accordingly, for any $\lambda > 0$, $\lim_{t\to\infty}(e^{\lambda t} t^{-\alpha}\mathcal{L}(t)) = \infty$. By (53), we conclude $P(R(t) < Kt) = o(P(L > t))$. Therefore, both condition 1 and 2 of Theorem 9 are satisfied. This completes tbe proof. ■

G. Proof of Theorem 5

We will apply Theorem 9. Define $N_t := \sup\{n : \sum_{i=1}^{n} I_i + B_i < t\}$ and $R(t) := \sum_{i=1}^{N_t} X_i$. It is easy to see $P(T(L) > t) = P(L > R(t))$.

Using renewal theory, we have

$$\gamma = \lim_{t\to\infty} \frac{R(t)}{t} = \frac{E[X_1]}{(E[I_1] + E[B_1])} \tag{54}$$

almost surely. Since $X_1 <_{a.s.} I_1$, we conclude that $E[X_1] < E[I_1]$, which implies that $0 < \gamma < 1$. This verifies the first condition of Theorem 9.

We will now argue that the second condition of Theorem 9 is also satisfied, i.e., $P(R(t) < Kt) = o(P(L > t))$ for some $K > 0$. Since both I_i and B_i are LT, then $I_i + B_i$ is LT. This implies, using Chernoff bounds, that for any $0 < \delta < 1$ there exists $K = (1-\delta)E[X_1]/((1+\delta)(E[I_1] + E[B_1]))$ such that

$$P(R(t) < Kt) = o(P(L > t)). \tag{55}$$

This verifies the second condition of Theorem 9 and thus we conclude that

$$P(T(L) > t) = P(L > R(t)) \sim P(L > \gamma t).$$

This completes the proof.