Primary-User Mobility Impact on Spectrum Sensing in Cognitive Radio Networks

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***Abstract*—In this paper, the effects of the primary-user (PU) mobility on spectrum sensing in Cognitive Radio (CR) networks are studied. To this aim, first, the spectrum sensing problem is reformulated to account for the PU mobility. Then, the effects of the PU mobility are studied with the objective to determine the parameters that affect the spectrum sensing functionality. For this, two performance metrics are analytically derived: i) the *detection capability*, which measures the PU mobility impact on the CR user detection probability; ii) the *mobility-enabled sensing capacity*, a new metric that measures the expected transmission capacity achievable by a CR user in the presence of PU mobility. The mathematical analysis is carried out in different scenarios, by using mobility and spectrum occupancy models. The results show that the detection capability is affected by five parameters: the PU protection range, the network region size, the PU mobility model, the CR spatial distribution, and the number of PUs that use the same spectrum band. Moreover, it is shown that the sensing capacity can significantly increase in the presence of PU mobility if the PU protection range is smaller than the network region size. The mathematical results are derived by considering the dynamic PU traffic, and validated through simulations.**

I. Introduction

Spectrum Sensing is a key functionality in Cognitive Radio (CR) Networks [1]. Through spectrum sensing, unlicensed users (CR users) can recognize and dynamically exploit portions of the radio spectrum whenever they are vacated by licensed users, referred to as Primary Users (PUs). The interference on the PU transmissions depends on the accuracy of the spectrum sensing, which can be affected by the wireless channel impairments, such as multipath fading and/or shadowing. Thus, recently, the research efforts are devoted to improve the accuracy and efficiency of sensing techniques [1], [2].

Despite these efforts, new challenges arise in the spectrum sensing functionality in the presence of PU mobility. Mobility changes dynamically the mutual distances among the PUs and the CR users and, as a consequence, the connectivity between them varies in time. For this, even if at a certain time an arbitrary CR user is inside the *protection range*[1] of a mobile PU, after the PU movement, the CR user can be outside of it, thus becoming unable to sense the possible PU transmissions. Spectrum sensing should be aware of these topology changes, making necessary to revisit the current formulation of the sensing problem.

In this paper, the effects of the PU mobility on spectrum sensing are studied, with the objective to determine the parameters that affect the sensing performance and design. To the best of our knowledge, this is the first work that addresses this problem.

Specifically, first, the spectrum sensing problem is reformulated to account for the PU mobility. Then two performance metrics are analytically derived: i) the *detection capability*, i.e., the probability of a CR user being inside the protection range of a PU, which measures the mobility impact on the CR detection probability; ii) the *mobility-enabled sensing capacity*, a new metric that measures the expected transmission capacity achievable by a CR user in the presence of PU mobility.

The mathematical analysis is carried out by utilizing two popular mobility models [4], i.e., Radom Walk mobility Model with reflection (RWM) and Random WayPoint mobility Model (RWPM). Moreover, we consider two different PU spectrum occupancy models. In the first model called *Single PU for Band* (SPB), the PUs roaming within the network region use different bands. In the second model called *Multiple PUs for Band* (MPB), different mobile PUs can use the same band. For both mobility and spectrum occupancy models, we derive closed-form expressions for both the detection capability and the mobility-enabled sensing capacity.

The detection capability results show that the detection capability is affected by five different parameters. For the SPB scenario, the detection capability depends on the PU protection range, the extension of the network region, the PU mobility model, and the CR spatial distribution. For the MPB scenario, the detection capability depends on all the above parameters, but also on the number of PUs that use the same band. Hence, the MPB analysis reveals that, from a CR user perspective, the total number of PUs roaming within the network region is not important but the number of PUs that use the same band.

The derived mobility-enabled sensing capacity shows that, when the PU protection range is not comparable with the

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[1] To avoid harmful interference against the PUs, the CR users should be able to detect active PUs within a range, referred to as protection range, determined by the PU transmission range and by the CR interference range [3].

network region size, the PU mobility increases the sensing capacity achievable by the CR users. All results are derived by considering the dynamic PU traffic.

The rest of the paper is organized as follows. In Section II we explain the research problem. In Section III we derive the detection capability and the mobility-enabled sensing capacity for the SPB model, while in Section IV we derive these performance measures for the MPB model. We validate the analytical results by simulation in Section V. In Section VI we conclude the paper. In the Appendix we provide the proofs.

II. Problem Statement

Here we present the models capturing the PU mobility, the PU traffic and the CR network. We then formulate the spectrum sensing problem under PU mobility.

A. Models and Assumptions

PU Mobility Models: Both the RWM (Random Walk Model) and the RWPM (Random WayPoint Model) initially place the PUs randomly according to a uniform distribution in a network region $\mathbf{A}$, which is assumed very often as a line or as a square. Under the RWM, each PU randomly chooses a direction according to a uniform distribution in the range $[0, 2\pi]$ and a speed in the range $[v_{\text{min}}, v_{\text{max}}]$ m/s. At the end of each movement period, a new direction and speed are calculated. When the edge of the network region $\mathbf{A}$ is reached, a PU is bounced back to the region $\mathbf{A}$. This model produces a uniform steady-state spatial distribution regardless of the average PU speed [5]. Under the RWPM, each PU randomly chooses a destination point inside $\mathbf{A}$ according to a uniform distribution, and it moves towards this destination with a velocity chosen uniformly at random in the interval $[v_{\text{min}}, v_{\text{max}}]$ m/s. When PU reaches its destination, it remains fixed for a certain pause time ("think time"), and then it starts moving again according to the same rule. This model produces a non-uniform steady-state spatial distribution [6]. $f_{\mathbf{X}_{\text{PU}}}(\mathbf{x}_{\text{PU}})$ denotes the probability density function (pdf)[2] of the steady-state PU spatial distribution according to the adopted mobility model, and R is the PU protection radius.

PU Traffic Model: The PU traffic is modeled as a two state birth-death process [2], with death rate α and birth rate β. In the "*on*" state the PU is active, and in the "*off*" state it is inactive. The probabilities of the "*on*" and "*off*" states are:

$$P_{\text{on}} = \frac{\beta}{\alpha + \beta}, \quad P_{\text{off}} = \frac{\alpha}{\alpha + \beta} \tag{1}$$

CR Users Network: The CR users are assumed static and uniformly distributed in the network region $\mathbf{A}$. $f_{\mathbf{X}_{\text{CR}}}(\mathbf{x}_{\text{CR}})$ denotes the pdf of the CR user spatial distribution.

B. Spectrum Sensing Problem Definition under PU Mobility

Consider a typical sensing scenario in which a CR user monitors a certain spectrum band. In static PU Networks (PUNs), the CR users are assumed always inside the PU Protection Range (PrR). Hence, the local sensing for PU signal detection is formulated as a binary hypothesis problem [1], [2]:

$$x(t) = \begin{cases} v(t) & \mathcal{H}_0 \\ g(t)\, s(t) + v(t) & \mathcal{H}_1 \end{cases} \tag{2}$$

where $s(t)$ is the PU signal, $g(t)$ is the sensing channel gain, and $v(t)$ is the additive white Gaussian noise. $\mathcal{H}_0$ and $\mathcal{H}_1$ denote, respectively, the hypotheses of "no PU signal" and "PU signal transmitted". In the presence of PU mobility, the binary problem (2) must be modified, since the common assumption of a CR user being always inside the PU PrR does not hold anymore, because of the dynamic change of the PUN topology. To address the spectrum sensing definition problem under the PU mobility, we introduce the following definitions:

Definition 1: $\mathcal{I}$ denotes the event: "an arbitrary CR user is inside the PU protection range".

Definition 2: $\mathcal{O}$ denotes the event: "an arbitrary CR user is out of the PU protection range".

If the event $\mathcal{O}$ occurs, a CR user cannot listen to the PU transmission; instead, if $\mathcal{I}$ occurs, a CR user can sense the possible PU transmissions. By using the previous considerations, it is possible to reformulate the local sensing for PU detection, by distinguishing between the events $\mathcal{I}$ and $\mathcal{O}$:

Spectrum Sensing Problem under the event $\mathcal{I}$:

$$x(t) = \begin{cases} v(t) & \mathcal{H}_0 \\ g(t)\, s(t) + v(t) & \mathcal{H}_1 \end{cases} \tag{3}$$

Spectrum Sensing Problem under the event $\mathcal{O}$:

$$x(t) = \begin{cases} v(t) & \mathcal{H}_0 \end{cases} \tag{4}$$

The detection performance of an arbitrary CR user are evaluated through the detection P_d and false-alarm P_f probabilities. From the sensing problem definitions (3) and (4) in the presence of PU mobility, these can be expressed as:

$$P_d \triangleq P(Y > \gamma | \mathcal{H}_1) = \underbrace{P(Y > \gamma | \mathcal{H}_1, \mathcal{I})}_{P_{d|\mathcal{I}}} P(\mathcal{I}) + \\ + \overbrace{P(Y > \gamma | \mathcal{H}_1, \mathcal{O})}^{=0} P(\mathcal{O}) = P_{d|\mathcal{I}}\, P(\mathcal{I}) \tag{5}$$

$$P_f \triangleq P(Y > \gamma | \mathcal{H}_0) = \underbrace{P(Y > \gamma | \mathcal{H}_0, \mathcal{I})}_{P_{f\,|\mathcal{I}}} P(\mathcal{I}) + \\ + \underbrace{P(Y > \gamma | \mathcal{H}_0, \mathcal{O})}_{P_{f\,|\mathcal{O}}} P(\mathcal{O}) = P_{f\,|\mathcal{I}} = P_{f|\mathcal{O}} \tag{6}$$

where Y is the decision variable, which depends on the adopted sensing strategy, γ is the decision threshold, and $P(\mathcal{I})$ and $P(\mathcal{O})$ are the probabilities of the events $\mathcal{I}$ and $\mathcal{O}$, respectively. $P_{d|\mathcal{I}}$ denotes the detection probability conditioned to the event $\mathcal{I}$ and it depends on the adopted sensing strategy. In (6) the last two equalities are justified by the symmetry of the hypothesis $\mathcal{H}_0$ in both the events $\mathcal{I}$ and $\mathcal{O}$.

From (5), it results that the CR detection probability P_d

[2]Throughout the paper, random variables are denoted with upper case letters; specific outcomes of these variables are denoted with lower case.

depends on $P(\mathcal{I})$, referred to as *detection capability*, which depends on the PU mobility (see Section III). Specifically, since the normalized detection probability, i.e., $P_d/P_{d|\mathcal{I}}$, coincides with the *detection capability* $P(\mathcal{I})$, it is required to study this term for addressing the impact of the PU mobility on P_d. This issue is developed in Section III.

III. Single PU for Band Scenario

In this section, we study the impact of the PU mobility on the spectrum sensing for the SPB scenario.

Definition 3: A CR user is inside the protection range R of a PU if the Euclidean distance $S_{\text{CR}}^{\text{PU}}$ between them is not greater than R, i.e., $S_{\text{CR}}^{\text{PU}} \triangleq ||\mathbf{X}_{\text{CR}} - \mathbf{X}_{\text{PU}}|| \leq R$.

By accounting for Definition 1 and Definition 3, it results that:

$$P(\mathcal{I}) = P[S_{\text{CR}}^{\text{PU}} \leq R] = \int_0^R f_{S_{\text{CR}}^{\text{PU}}}(s)\, ds \tag{7}$$

where $f_{S_{\text{CR}}^{\text{PU}}}(s)$ is the pdf of the random variable $S_{\text{CR}}^{\text{PU}}$.

A. Detection Capability for the Random Walk Model

In this subsection, we derive $P(\mathcal{I})$ for one-dimensional (Theorem 1) and bi-dimensional (Theorem 2) network regions.

Proposition 1: For a one-dimensional network region, i.e., $A = [0, a]$, the pdf of the random variable $S_{\text{CR}}^{\text{PU}}$ representing the Euclidean distance between a CR user and a PU moving according to the RWM is given by:

$$f_{S_{\text{CR}}^{\text{PU}}}^{\text{1D-RWM}}(s) = \frac{2}{a}\left(1 - \frac{s}{a}\right) \text{rect}\left(\frac{s - a/2}{a}\right) \tag{8}$$

where rect(s) denotes the rectangular window.

Proof: See Appendix A. ■

Theorem 1: The probability $P(\mathcal{I})$ that a CR user is inside the PrR (Protection Range) of a PU, roaming within a one-dimensional network region $A = [0, a]$ according to the RWM, is given by

$$P_{\text{1D}}^{\text{RWM}}(\mathcal{I}) = 2\left(\frac{R}{a}\right) - \left(\frac{R}{a}\right)^2 \tag{9}$$

Proof: By substituting (8) in (7), after some algebraic manipulations, the equation (9) is obtained. ■

Proposition 2: For a bi-dimensional network region, i.e., $\mathbf{A} = [0, a] \times [0, a]$, the pdf of the random variable $S_{\text{CR}}^{\text{PU}}$ representing the Euclidean distance between a CR user and a PU moving according to the RWM is given by:

$$f_{S_{\text{CR}}^{\text{PU}}}^{\text{2D-RWM}}(s) = \left(\frac{2\,\pi s}{a^2} - \frac{8s^2}{a^3} + \frac{2s^3}{a^4}\right) \text{rect}\left(\frac{s - \sqrt{2}a/2}{\sqrt{2}a}\right) \tag{10}$$

Proof: See Appendix A. ■

Theorem 2: The probability $P(\mathcal{I})$ that an arbitrary CR user is inside the PrR of a PU, roaming within a bi-dimensional network region $\mathbf{A} = [0, a] \times [0, a]$ according to the RWM, is given by

$$P_{\text{2D}}^{\text{RWM}}(\mathcal{I}) = \pi\left(\frac{R}{a}\right)^2 - \frac{8}{3}\left(\frac{R}{a}\right)^3 + \frac{1}{2}\left(\frac{R}{a}\right)^4 \tag{11}$$

Proof: By substituting (10) in (7), after some algebraic manipulations, the equation (11) is obtained. ■

B. Detection Capability for the Random WayPoint Model

In this subsection, we derive $P(\mathcal{I})$ for one-dimensional (Theorem 3) and bi-dimensional (Theorem 4) network regions, by assuming a RWPM with no thinking times (RWPM-NTT). Then, we generalize the analysis by accounting for the Thinking Times (TTs) in Theorem 5.

Proposition 3: For a one-dimensional network region, i.e., $A = [0, a]$, the pdf of the random variable $S_{\text{CR}}^{\text{PU}}$ representing the Euclidean distance between a CR user and a PU moving according to the RWPM-NTT is given by:

$$f_{S_{\text{CR}}^{\text{PU}}}^{\text{1D-RWP-NTT}}(s) = \left(\frac{2}{a} + \frac{4s^3}{a^4} - \frac{6s^2}{a^3}\right) \text{rect}\left(\frac{s - a/2}{a}\right) \tag{12}$$

Proof: See Appendix A. ■

Theorem 3: The probability $P(\mathcal{I})$ that a CR user is inside the PrR of a PU, roaming within a one-dimensional network region $A = [0, a]$ according to the RWPM-NTT, is given by

$$P_{\text{1D}}^{\text{RWPM-NTT}}(\mathcal{I}) = 2\left(\frac{R}{a}\right) - 2\left(\frac{R}{a}\right)^3 + \left(\frac{R}{a}\right)^4 \tag{13}$$

Proof: By substituting (12) in (7), after some algebraic manipulations, the equation (13) is obtained. ■

Proposition 4: For a bi-dimensional network region, i.e., $\mathbf{A} = [0, a] \times [0, a]$, the pdf of the random variable $S_{\text{CR}}^{\text{PU}}$ representing the Euclidean distance between a CR user and a PU moving according to the RWPM-NTT is given by:

$$f_{S_{\text{CR}}^{\text{PU}}}^{\text{2D-RWP-NTT}}(s) = \left(\frac{2\,\pi}{a^2}s - \frac{6\,\pi}{a^4}s^3 + \frac{32}{3\,a^5}s^4 + \frac{9\,\pi}{4\,a^6}s^5 + \right.$$
$$\left. - \frac{32}{5\,a^7}s^6 + \frac{4}{3\,a^8}s^7\right) \text{rect}\left(\frac{s - \sqrt{2}a/2}{\sqrt{2}a}\right) \tag{14}$$

Proof: See Appendix A. ■

Theorem 4: The probability $P(\mathcal{I})$ that a CR user is inside the PrR of a PU, roaming within a bi-dimensional network region $\mathbf{A} = [0, a] \times [0, a]$ according to the RWPM-NTT is given by

$$P_{\text{2D}}^{\text{RWPM-NTT}}(\mathcal{I}) = \pi\left(\frac{R}{a}\right)^2 - \frac{3\,\pi}{2}\left(\frac{R}{a}\right)^4 + \frac{32}{15}\left(\frac{R}{a}\right)^5 +$$
$$+ \frac{3\,\pi}{8}\left(\frac{R}{a}\right)^6 - \frac{32}{35}\left(\frac{R}{a}\right)^7 + \frac{1}{6}\left(\frac{R}{a}\right)^8 \tag{15}$$

Proof: By substituting (14) in (7), after some algebraic manipulations, the equation (15) is obtained. ■

When TTs are considered, the resulting spatial PU distribution is given by a linear combination of the thinking component and the mobile component. By denoting with p_p the probability that a PU pauses at a randomly chosen time, the PU spatial distribution can be written as [6] $f_{\mathbf{X}_{\text{PU}}}(\mathbf{x}_{\text{PU}}) = p_p f_{\mathbf{X}_{\text{PU}},\text{T}}(\mathbf{x}_{\text{PU}}) + (1 - p_p) f_{\mathbf{X}_{\text{PU}},\text{m}}(\mathbf{x}_{\text{PU}})$, where $f_{\mathbf{X}_{\text{PU}},\text{T}}(\mathbf{x}_{\text{PU}})$ is the pdf of the thinking component.

Theorem 5: The detection capability $P(\mathcal{I})$ that an arbitrary CR user is inside the PrR of a PU roaming within a network region $\mathbf{A}$ according to the RWPM with TTs (RWPM-TT) is equal to

$$P^{\text{RWPM-TT}}(\mathcal{I}) = p_p\, P^{\text{RWM}}(\mathcal{I}) + (1 - p_p) P^{\text{RWPM-NTT}} \quad (16)$$

where $P^{\text{RWM}}(\mathcal{I})$ is given by (9) and (11) for one- and bi-dimensional network region, respectively. $P^{\text{RWPM-NTT}}$ is given by (13) and (15) for one- and bi-dimensional network region.

Proof: Since $f_{\mathbf{X}_{\text{PU}},\text{T}}(\mathbf{x}_{\text{PU}})$ is uniform [6] as the distribution produced by the RWM, $P(\mathcal{I})$ for a PU moving according to the RWPM-TT is a linear combination of the results obtained for the RWM and the RWPM without TTs. ■

Remark: The results derived above for both the adopted mobility models show that the detection capability depends on the normalized PU PrR, i.e., R/a, on the PU mobility model, and on the CR spatial distribution.

C. Mobility-Enabled Sensing Capacity

By using the previous analysis, and by following the approach adopted in [2] for static PUNs, we introduce the new notion of *mobility-enabled sensing capacity* as follows:

Definition 4: The *mobility-enabled sensing capacity* C_i^{mob} is the expected transmission capacity on the spectrum band i that a CR user can achieve in the presence of PU mobility:

$$C_i^{\text{mob}} = \eta_i\, \rho_i\, W_i\, \left[(1 - P(\mathcal{I})) + P_{\text{off},i}\, P(\mathcal{I})\right] \quad (17)$$

where η_i, W_i, and $P_{\text{off},i}$ represent the sensing efficiency, the bandwidth, and the off state probability (1) of the spectrum band i. ρ_i is the spectral efficiency of the band i (bit/sec/Hz) [2], and $P(\mathcal{I})$ is the detection capability evaluated before.

Remark: (17) states that a CR user can use a certain band if it is outside of the PU PrR or if it is inside but the PU is in the off state. C_i^{mob} reflects the dynamic nature of both the PU topology through $P(\mathcal{I})$, and the PU traffic through $P_{\text{off},i}$.

Remark: In static PU network, the sensing capacity was derived in [2] and it is equal to $C_i^{\text{static}} = \eta_i\, \rho_i\, W_i\, P_{\text{off},i}$. By comparing such an equation with (17), it results that

$$C_i^{\text{mob}} > C_i^{\text{static}} \text{ if } P(\mathcal{I}) < 1 \text{ and } P_{\text{off},i} < 1 \quad (18)$$
$$\lim_{P(\mathcal{I})\to 1} C_i^{\text{mob}} = C_i^{\text{static}}$$

Hence, if $P(\mathcal{I})$ is small, e.g., if the PU normalized PrR, R/a, is small, the mobility-enabled capacity C_i^{mob} can be significantly greater than C_i^{static}, with a gain constituted by the term related to $P(\mathcal{O}) = 1 - P(\mathcal{I})$. In fact, thanks to the PU mobility, a CR user has more chances to use the band, since it can be outside of the PU PrR with high probability. Clearly, if the PU never transmits, i.e., $P_{\text{off},i} = 1$, $C_i^{\text{mob}} = C_i^{\text{static}}$.

IV. Multiple PU for Band Scenario

In this section, we study the impact of the PU mobility on the spectrum sensing for the MPB scenario. If in $\mathbf{A}$ there are n mobile PUs that use the same band, the previous expressions are not valid anymore, since here $P(\mathcal{I})$ represents the probability of a CR user being inside the PrR of at least one PU. By denoting with $P(\mathcal{O})$ the probability that a CR is not inside the PrR of any PU that use the band, $P(\mathcal{I})$ is:

$$P(\mathcal{I}) = 1 - P(\mathcal{O}) \quad (19)$$

To evaluate (19), let us consider a CR user at a certain location $\mathbf{x}_{\text{CR}} \in \mathbf{A}$. The CR user is inside the PU PrR if the mobile PU is placed within a disk $\mathbf{C}(\mathbf{x}_{\text{CR}})$ of radius R around $\mathbf{x}_{\text{CR}}$. The probability of this event is:

$$P_{\mathbf{C}}(\mathbf{x}_{\text{CR}}) = \int\!\!\int_{\mathbf{C}(\mathbf{x}_{\text{CR}})} f_{\mathbf{X}_{\text{PU}}}(\mathbf{x}_{\text{PU}})\, d\mathbf{x}_{\text{PU}} \quad (20)$$

Since the PUs move independently of each other in both the considered mobility models [5], [6], the number K of PUs within $\mathbf{C}(\mathbf{x}_{\text{CR}})$ obeys a binomial distribution:

$$P(K = k|\mathbf{x}_{\text{CR}}) = \binom{n}{k} \left(P_{\mathbf{C}}(\mathbf{x}_{\text{CR}})\right)^k \left(1 - P_{\mathbf{C}}(\mathbf{x}_{\text{CR}})\right)^{n-k} \quad (21)$$

By using (21), the probability that a CR user located in $\mathbf{x}_{\text{CR}}$ is not inside the PrR of any PUs using the same band is:

$$P(\mathcal{O}|\mathbf{x}_{\text{CR}}) = P(K = 0|\mathbf{x}_{\text{CR}}) = \left(1 - P_{\mathbf{C}}(\mathbf{x}_{\text{CR}})\right)^n \quad (22)$$

Hence, by accounting for (21) and (22), the probability that a CR user located in $\mathbf{x}_{\text{CR}}$ is inside the PrR of at least one PU using the same band is given by:

$$P(\mathcal{I}|\mathbf{x}_{\text{CR}}) = 1 - P(\mathcal{O}|\mathbf{x}_{\text{CR}}) = 1 - \left(1 - P_{\mathbf{C}}(\mathbf{x}_{\text{CR}})\right)^n = \\ = \sum_{k=1}^{n} \binom{n}{k} \left(P_{\mathbf{C}}(\mathbf{x}_{\text{CR}})\right)^k \left(1 - P_{\mathbf{C}}(\mathbf{x}_{\text{CR}})\right)^{n-k} \quad (23)$$

By integrating $P(\mathcal{I}|\mathbf{x}_{\text{CR}})$ over all the CR locations, we have:

$$P(\mathcal{I}) = \int\!\!\int_{\mathbf{A}} P(\mathcal{I}|\mathbf{x}_{\text{CR}}) f_{\mathbf{X}_{\text{CR}}}(\mathbf{x}_{\text{CR}})\, d\mathbf{x}_{\text{CR}} = \quad (24)$$
$$1 - \int\!\!\int_{\mathbf{A}} \left(1 - P_{\mathbf{C}}(\mathbf{x}_{\text{CR}})\right)^n f_{\mathbf{X}_{\text{CR}}}(\mathbf{x}_{\text{CR}})\, d\mathbf{x}_{\text{CR}} =$$
$$\sum_{k=1}^{n} \binom{n}{k} \int\!\!\int_{\mathbf{A}} P_{\mathbf{C}}(\mathbf{x}_{\text{CR}})^k \left(1 - P_{\mathbf{C}}(\mathbf{x}_{\text{CR}})\right)^{n-k} f_{\mathbf{X}_{\text{CR}}}(\mathbf{x}_{\text{CR}})\, d\mathbf{x}_{\text{CR}}$$

From (24), to evaluate $P(\mathcal{I})$ we need to derive $P_{\mathbf{C}}(\mathbf{x}_{\text{CR}})$.

Proposition 5: In a one-dimensional network region $A = [0, a]$, the probability that a CR user located in x_{CR} is inside a PU PrR is equal to for the RWM and RWPM, respectively:

$$P_{\mathbf{C}}^{\text{RWM}}(x_{\text{CR}}) = \begin{cases} \frac{2R}{a} & R \le x_{\text{CR}} \le a - R \\ \frac{x_{\text{CR}}+R}{a} & 0 \le x_{\text{CR}} \le R \\ \frac{a - x_{\text{CR}}+R}{a} & a - R \le x_{\text{CR}} \le a \\ 0 & \text{otherwise} \end{cases} \quad (25)$$

$$P_{\mathbf{C}}^{\text{RWPM}}(x_{\text{CR}}) = \quad (26)$$
$$\begin{cases} \frac{12R\, x_{\text{CR}}}{a^2} - \frac{4\,R}{a^3}(R^2 + 3\, x_{\text{CR}}^2) & R \le x_{\text{CR}} \le a - R \\ \left(\frac{x_{\text{CR}}+R}{a}\right)^2 \left[3 - \frac{2}{a}(x_{\text{CR}} + R)\right] & 0 \le x_{\text{CR}} \le R \\ 1 + \left(\frac{x_{\text{CR}}-R}{a}\right)^2 \left[\frac{2}{a}(x_{\text{CR}} - R) - 3\right] & a - R \le x_{\text{CR}} \le a \\ 0 & \text{otherwise} \end{cases}$$

Proof: See Appendix A. ■

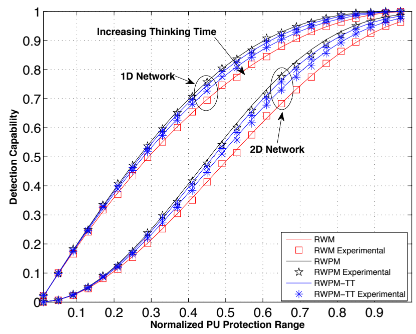

Fig. 1. Detection Capability vs Normalized PU PrR

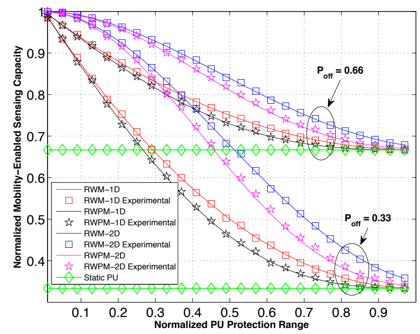

Fig. 2. Normalized Mobility-Enabled Sensing Capacity vs Normalized PU PrR

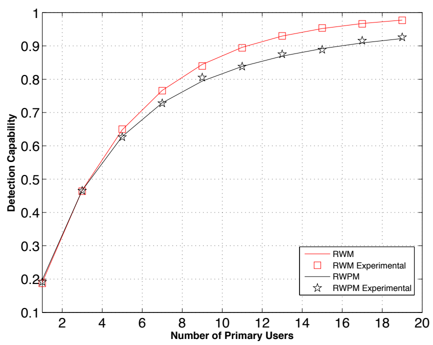

Fig. 3. Detection Capability vs Number of PUs

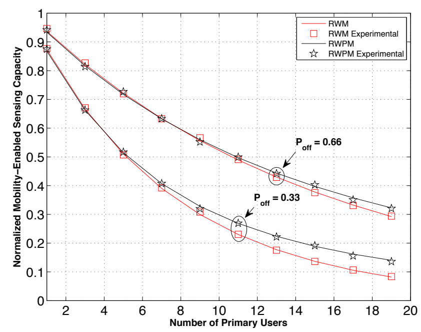

Fig. 4. Normalized Mobility-Enabled Sensing Capacity vs Number of PUs

By substituting (25) and (26) in (24), and by using $f_{\mathbf{X}_{\text{CR}}}(\mathbf{x}_{\text{CR}}) = 1/||\mathbf{A}||$, for $\mathbf{x}_{\text{CR}} \in \mathbf{A}$, as a consequence of the CR user network model, $P(\mathcal{I})$ can be calculated.

Remark: The previous equations show that $P(\mathcal{I})$ depends not only on the mobility model, the CR spatial distribution, the normalized PU PrR, but also on the number of PUs that use the band of interest. Hence, in the MPB scenario, from a CR user perspective, the total number of PUs roaming within the network region is not important but the number of PUs that use the same band. As in Section III, to account for TTs in the RWPM it is enough to linearly combine the results obtained for the RWM and RWPM-NTT.

By using Definition 4 and the previous results for $P(\mathcal{I})$ in the MPB scenario, the expression of the *mobility-enabled sensing capacity* C_i^{mob} has to be modified. In fact, a CR user can use the band of interest i if it is not inside the PrR of any PU that can use that band, or if it is inside but no PU transmits. By assuming that the PUs activities are independent among each other, and by denoting with $P_{i,l}^{\text{off}}$ the off state probability of the l-th PU, C_i^{mob} can be expressed as:

$$C_i^{mob} = \eta_i \, \rho_i \, W_i \left(\overline{P(\mathcal{I})} + \sum_{k=1}^{n} \binom{n}{k} \prod_{l=1}^{k} P_{i,l}^{\text{off}} \int\!\!\int_{\mathbf{A}} P_{\mathbf{C}}(\mathbf{x}_{\text{CR}})^k \, (1 - P_{\mathbf{C}}(\mathbf{x}_{\text{CR}}))^{n-k} f_{\mathbf{X}_{\text{CR}}}(\mathbf{x}_{\text{CR}}) \, d\mathbf{x}_{\text{CR}} \right) \quad (27)$$

From (27), it results that when the number n of PUs that use the band i increases, C_i^{mob} decreases as well, approaching to zero when $n \to +\infty$.

V. Validation of the Theoretical Results

In this section we validate the mathematical results with Monte Carlo simulations. We generate 10^4 topologies by placing both the PUs and the CR users randomly in a line/squared network region $\mathbf{A}$. Then, for each topology, we let the PUs move according to the adopted mobility model (RWM or RWPM) for enough time to reach a steady-state distribution (10^4 seconds). Then we calculate the average $P(\mathcal{I})$.

SPB scenario: In Fig. 1 we show the detection capability $P(\mathcal{I})$ versus the normalized PU PrR, R/a, where the analytical expressions (9), (11), (13) and (15) for both the adopted PU mobility models match well the simulation results. We observe that, when R/a increases, $P(\mathcal{I})$ increases as well, since the probability that a CR user is inside the PU PrR increases. More specifically, $P^{\text{RWPM}}(\mathcal{I}) \geq P^{\text{RWM}}(\mathcal{I})$. In 2D networks, $P(\mathcal{I})$ has smaller values with respect to 1D networks due to the higher degree of freedom given by the additional dimension. Moreover, in Fig. 1 we also depict $P(\mathcal{I})$ values when the TTs in the RWPM are considered, for two values of p_p, i.e., $p_p = 0.2$ and $p_p = 0.5$. Also in this case there is a very good agreement between the mathematical and the simulation results. Introducing TTs decreases $P(\mathcal{I})$.

In Fig. 2, both the normalized mobility-enabled sensing capacity $\overline{C}_i^{\text{mob}} \triangleq C_i^{\text{mob}}/(\eta_i\,\rho_i\,W_i)$ and the normalized static sensing capacity $\overline{C}_i^{\text{static}} = P_{\text{off},i}$ are reported as functions of R/a for two values of P_{off}, i.e., $P_{\text{off}} = 1/3$ and $P_{\text{off}} = 2/3$. For both values, we observe that the PU mobility introduces a significant sensing capacity gain with respect to static PUNs for a small value of R/a. For example, for $P_{\text{off}} = 0.66$ and $R/a = 0.1$, $\overline{C}_i^{\text{mob}}$ is in the worst case (1D network) at least 44% greater than $\overline{C}_i^{\text{static}} = 0.66$. This gain increases if $P_{\text{off}} = 0.33$, since the PU mobility can help to overcome the high PU traffic activity. The capacity gain can be justified by noting that, thanks to the PU mobility, a CR user has more chances to use the band of interest, since it can be outside of the PU PrR. Moreover, for the RWM $\overline{C}_i^{\text{mob}}$ decreases slower than the capacity associated to the RWPM, as a consequence of the results shown in Fig. 1, i.e., $P^{\text{RWPM}}(\mathcal{I}) \geq P^{\text{RWM}}(\mathcal{I})$.

MPB scenario: Fig. 3 shows $P(\mathcal{I})$ versus the number n of PUs that use the same band of interest, for $R/a = 0.1$. The results validate the theoretical analysis for both the PU mobility models, since there is a very good agreement between the theoretical and the experimental results. When n increases, $P(\mathcal{I})$ increases as well. In particular, in the MPB scenario, the RWPM impacts on the detection capability $P(\mathcal{I})$ differently from the SPB scenario: $P^{\text{RWPM}}(\mathcal{I}) \leq P^{\text{RWM}}(\mathcal{I})$, for the non-uniform distribution of the PUs when the RWPM is adopted.

In Fig. 4, we show the normalized mobility-enabled sensing capacity $\overline{C}_i^{\text{mob}}$ versus the number n of PUs that use the same band, for $R/a = 0.1$. We obtain these results by assuming that the PUs have the same $P_{\text{off}} = 1/3$ and $2/3$. It is clear again that the analytical results match very well with the simulation results. As expected, $\overline{C}_i^{\text{mob}}$ decreases when n increases, since the probability that a CR user is outside the PrR decreases. Finally, since $P^{\text{RWPM}}(\mathcal{I}) \leq P^{\text{RWM}}(\mathcal{I})$, for the RWPM $\overline{C}_i^{mob}$ decreases slower than $\overline{C}_i^{\text{mob}}$ associated to the RWM.

VI. Conclusions

In this paper, the effects of the primary-user (PU) mobility on spectrum sensing in Cognitive Radio (CR) networks have been studied. To this aim, two performance metrics have been analytically derived: i) the detection capability, which measures the mobility impact on the CR user detection probability; ii) the mobility-enabled sensing capacity, which measures the expected capacity achievable by a CR user in the presence of PU mobility. The mathematical analysis is carried out in different scenarios, by adopting two mobility and two spectrum occupancy models. It allowed us to determine each parameter that affects the detection capability and the mobility-enabled sensing capacity. Moreover, we show that the sensing capacity increases significantly in the presence of PU mobility if the PU protection range is smaller than the network region size. The analytical results are validated through simulations.

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Appendix

Proof of Proposition 1: For one-dimensional network region $A = [0, a]$, by using the random variable transformation Theorem [7], the pdf of the Euclidean distance $S_{\text{CR}}^{\text{PU}}$ between a CR user and a mobile PU is equal to:

$$f_{S_{\text{CR}}^{\text{PU}}}^{1D}(s) = \left[\left(f_{X_{\text{CR}}} \otimes f_{X_{\text{PU}}} \right)(s) + \left(f_{X_{\text{CR}}} \otimes f_{X_{\text{PU}}} \right)(s) \right] u(s) \quad (28)$$

where $u(s)$ is the step function and $\left(f_{X_{\text{CR}}} \otimes f_{X_{\text{PU}}} \right)(s)$ denotes the convolution between the pdfs of the spatial distributions of the CR users and the PUs. When the RWM is adopted $f_{X_{\text{PU}}}(x)$ is uniform [5], i.e., $f_{X_{\text{PU}}}(x) = 1/a$, for $0 \leq x \leq a$, 0 otherwise, and also $f_{X_{\text{CR}}}(x)$ is uniform for the adopted CR network model. By substituting the expressions of $f_{X_{\text{PU}}}(x)$ and $f_{X_{\text{CR}}}(x)$ in (28), (8) is obtained.

Proof of Proposition 2: For bi-dimensional network region $\mathbf{A} = [0, a] \times [0, a]$, by using the independence and identical distribution of the random variables representing the distance in each dimension, the pdf of the Euclidean distance $S_{\text{CR}}^{\text{PU}}$ between a CR user and a mobile PU is equal to:

$$f_{S_{\text{CR}}^{\text{PU}}}^{2D}(s) = \left[f_{S_{\text{CR}}^{\text{PU}}}^{1D}(s) \otimes f_{S_{\text{CR}}^{\text{PU}}}^{1D}(s) \right] u(s) \quad (29)$$

where $f_{S_{\text{CR}}^{\text{PU}}}^{1D}(s)$ is given in (28). By substituting the expressions (8) of $f_{S_{\text{CR}}^{\text{PU}}}^{1D}(s)$ for the RWM, (10) is obtained.

Proof of Proposition 3: For one-dimensional network region, by using the same reasonings used in the proof of Proposition 1, the pdf of $S_{\text{CR}}^{\text{PU}}$ is given by (28). For the RWPM $f_{X_{\text{PU}}}(x)$ is nonuniform and, with no TTs, it is equal to [6] $f_{X_{\text{PU}}}(x) = -\frac{6}{a^3}x^2 + \frac{6}{a^2}x$, for $0 \leq x \leq a$, 0 otherwise. $f_{X_{\text{CR}}}(x)$ is uniform. By substituting the expressions of $f_{X_{\text{PU}}}(x)$ and $f_{X_{\text{CR}}}(x)$ in (28), after algebraic manipulations (12) is obtained.

Proof of Proposition 4: For bi-dimensional network region, as in the proof of Proposition 2, the pdf of $S_{\text{CR}}^{\text{PU}}$ is given by (29). By substituting (12) in (29), after some algebraic manipulations (10) is obtained.

Proof of Proposition 5: (25) and (26) are derived, by substituting in (20) the expressions of the spatial distributions for the RWM and RWPM in the one-dimensional case.

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