

Deployment Algorithms for Wireless Underground Sensor Networks using Magnetic Induction

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Abstract—New propagation techniques using magnetic induction (MI) waveguide solve the problems of traditional techniques in the underground soil medium. However, the deployment of the MI waveguide to connect the wireless underground sensor networks (WUSNs) is challenging due to the high deployment cost and the complex shape of the communication range of the MI waveguides. In this paper, two algorithms are proposed to deploy the MI waveguides to connect the underground sensors in the WUSNs. To minimize the number of relay coils, the *MST* algorithm based on the minimum spanning tree is developed. However, the network constructed by the *MST* algorithm is not robust to sensor failures. To enhance the network robustness with acceptable relay coil number, the *TC* algorithm based on the Voronoi diagram is developed. The effectiveness of the proposed deployment algorithms is validated by simulations.

I. INTRODUCTION

As a natural extension to the wireless sensor networks, the wireless underground sensor networks (WUSNs) [1] enable a wide variety of novel applications, including the intelligent agriculture, underground pipeline and tank leakage detection, border patrol and security monitoring, and sports-field turf management. However, the propagation medium for the WUSNs is no longer air but soil, rock and water, where the well established wireless communication techniques for terrestrial wireless sensor networks do not work well [2], [3].

Traditional wireless communication techniques using electromagnetic (EM) waves encounter two major problems in soil medium: 1) the small communication range (≤ 4 m) due to high path loss caused by the soil material absorption; and 2) the dynamic channel conditions due to the dynamic soil properties such as water content, soil makeup, and density.

To address the above problems, in [4], [5], we developed the *Magnetic Induction (MI) waveguide* technique for the wireless communications in WUSNs. The MI waveguide consists of a series of relay coils between two underground transceivers, as shown in Fig. 1. The communications are achieved by magnetic induction between two adjacent coils. These relay coils do not consume extra energy and the cost is neglectable. The MI waveguide can solve the two major problems of the EM technique in soil medium: 1) by using MI waveguides, the feasible communication range between two transceivers can achieve nearly 100 meters; and 2) the MI channel conditions remain constant, since the soil medium cause little variation in the attenuation rate of magnetic fields.

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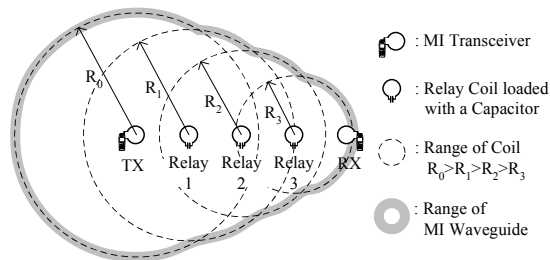


Fig. 1. The structure and the communication range of a MI waveguide.

Despite of the potential advantages, the deployment of the MI waveguides to connect the underground sensors is challenging due to the following reasons. First, on the one hand, a non-trivial number of relay coils are required to guarantee the network connectivity and robustness. On the other hand, the intensive deployment of the coils in underground soil medium cost a great amount of labor. Therefore the optimal number of relay coils needs to be found out. Second, the communication range of the MI relay coil is not the same as each other, as shown in Fig. 1. Consequently, the shape of the communication range of the MI waveguide is much more complex than the disk communication range of the traditional wireless devices. Current sensor deployment strategies [6], [7] are based on the disk communication range, and hence cannot be utilized to deploy the MI waveguides in the WUSNs.

In this paper, we analyze relay coil deployment strategies for the WUSNs using MI waveguides. In particular, we first consider the one-dimensional (1D) WUSNs. The optimal number of relay coils between two sensors are analyzed according to the required bandwidth and the distance between two sensors. Then based on the analysis of the 1D WUSNs, the optimal MI waveguide deployment strategy is developed for the two-dimensional (2D) WUSNs. To minimize the number of relay coils, we provide the *MST* algorithm, where the MI waveguides are deployed along the minimum spanning tree of the WUSN. The weight of each link of the network is the optimal relay coil number. Since the WUSN constructed by *MST* algorithm is not robust to sensor failure, we propose the *TC* algorithm. In the *TC* algorithm, the MI waveguides are deployed around the centroids of the triangle cells that are constructed by the Voronoi diagram [8]. The WUSN constructed by the *TC* algorithm is robust to sensor failure but requires more relay coils. Finally, the effectiveness of the proposed deployment strategy is analyzed by simulations.

The remainder of this paper is organized as follows. In Section II, the optimal number of relay coils are analyzed for 1D WUSNs. Then in Section III, the deployment strategies of the MI waveguide are developed for 2D WUSNs. In Section IV, simulation studies are performed. Finally, the paper is concluded in Section V.

II. MI WAVEGUIDE DEPLOYMENT IN 1D WUSNs

In this section, the deployment of the MI waveguides in a 1D WUSN is analyzed, where the underground sensors are buried along a line or a polygonal line. This 1D network topology is applicable in the underground pipeline monitoring system. Moreover, the analysis results lay the foundation of the MI waveguide deployment strategy in 2D WUSNs.

The 1D WUSN can be divided into multiple links that starts at one sensor and ends at the next. The goal of the optimal deployment of the MI waveguide in 1D WUSNs is to use as few relay coils as possible to connect the two sensors in each link. The optimal number of relay coils for each link is determined by the length of the link and the required bandwidth, which are determined by the specific applications.

A. Path Loss of the MI Waveguide

Assuming that the length of a link is d . The required bandwidth is B . An MI waveguide with $n - 1$ relay coils is deployed along the link to connect the two sensors. Therefore the interval r between two adjacent relay coil is $r = d/n$. Assuming that the angle frequency of the transmitting signal is ω , and the center frequency of the signal is ω_0 . According to [4], the path loss of the MI waveguide can be expressed as

$$L_{MI}(d, n, \omega) \approx 6.02 + 20 \lg \zeta\left(\frac{Z}{\omega M}, n\right), \quad (1)$$

where M is the mutual induction between the adjacent coils; Z is the self impedance of one relay coil; and $\zeta\left(\frac{Z}{\omega M}, n\right)$ is the n order polynomial of $\frac{Z}{\omega M}$. The self impedance of a coil Z is designed to be resonant at the center frequency ω_0 . When $\omega = \omega_0$, Z becomes pure resistance R , which is the coil wire resistance. The polynomial $\zeta(x, n)$ can be developed as

$$\begin{aligned} \zeta(x, 1) &= x, \quad \zeta(x, 2) = x^2 + 1, \dots, \\ \zeta(x, n) &= x \cdot \zeta(x, n-1) + \zeta(x, n-2). \end{aligned} \quad (2)$$

The relay coils are placed horizontally in a planar line, as shown in Fig. 1. This MI waveguide structure guarantees the omnidirectional coverage of each relay coil, which easy the deployment of the coils. Then, the mutual induction M can be deduced by the magnetic potential of the magnetic dipole:

$$M \approx \frac{\mu\pi N^2 a^4}{4r^3} = \frac{\mu\pi N^2 a^4}{4\left(\frac{d}{n}\right)^3}, \quad (3)$$

where μ is the permeability of the soil; N is the number of turns of the wire on the coils; and a is the coil radius.

B. Optimal Number of Relay Coils

To minimize the deployment cost while maintaining the proper network functionality of the WUSNs, a MI waveguide should use the minimum number of relay coils to connect the two sensors on the link. According to (1), the path loss

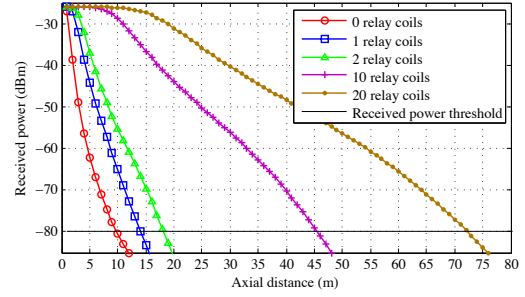


Fig. 2. Received power of MI waveguides with different relay coil numbers.

increases monotonically when the signal frequency deviates from the central frequency ω_0 . Therefore, if the signal with frequency $\omega = \omega_0 + 0.5B$ is correctly received, a communication channel with bandwidth of B is established between the two sensors. Assuming that transmission power is P_t and the minimum power for correct demodulation a signal is P_{th} . Using the path loss given in (1), the received power is derived. Then the optimal number of relay coils for this link is:

$$n_{opt}(d, B) = \arg \min_n \{P_t - L_{MI}(d, n, \omega_0 + 0.5B) \geq P_{th}\}. \quad (4)$$

According to (4), the optimal number of relay coils is the function of the link length and the required bandwidth. Since the required bandwidth can be viewed as a constant, it is the link length that determines the optimal number of relay coil.

By using the parameters of the MI waveguide developed in [4], we can numerically analyze the optimal number of relay coils with different link length. In the following analysis, $P_t = 4 \text{ dBm}$ (2.5 mW) and $P_{th} = -80 \text{ dBm}$. Due to the resonant characteristics of the MI waveguide, the bandwidth of the system is much smaller than the terrestrial wireless networks. However, the small bandwidth is acceptable for WUSNs since the underground monitoring applications do not require very high data rate [1]. Therefore, the bandwidth of the MI waveguide is set to be 1 KHz. The operating frequency is 10 MHz. The relay coils have the same radius of 0.15 m and the number of turns is 20. The coil is made of copper wire with a 1.45 mm diameter. The cost and weight of coils made of this kind of wire is neglectable. The wire resistance of unit length is 0.01 Ω/m . This relatively high wire resistance also effectively mitigates the in-band signal fluctuation. The permeability of the underground soil medium is a constant and is similar to the permeability of the air, since most soil in the nature does not contain magnetite. Therefore, $\mu = 4\pi \times 10^{-7} \text{ H/m}$. The soil moisture and the soil composition do not affect the MI communication as discussed perviously.

In Fig. 2, the received power of the 10 MHz + 0.5 KHz signal using MI waveguides with different relay coil numbers is shown as the function of the link length d . The axial communication range of a MI waveguide with a certain relay coil number is shown as the intersection point of the received power and the -80 dBm threshold. Fig. 2 shows that the axial communication range increases as the relay coil number increases. However, the increment of the communication range caused by additional relay coils decreases as the relay coil number increases. For example, the axial communication range of a MI transceiver pair can be increased by 36 meters by

adding the first 10 relay coils but can be only increased by 27 meters by adding another 10 relay coils. This phenomenon is due to the fact that the coils relay the signal in a passive way and there is no extra power added at each relay coil. According to (4), the optimal relay coil number for the link with a certain length can be read from Fig. 2 by finding out the curve with the minimum relay coil number that has the axial communication range larger than the link length.

III. MI WAVEGUIDE DEPLOYMENT IN 2D WUSNs

In this section, we investigate the deployment strategies of the MI waveguides in a 2D WUSN, which is applicable for most WUSN applications. Compared with the deployment in 1D WUSNs, the deployment in 2D WUSNs is much more complicated due to the following reasons: 1) in 1D WUSNs, the route connecting the sensor nodes are determined, while in 2D WUSNs, the optimal route to connect all the sensors needs to be found out; and 2) it is possible in a 2D WUSN that some common relay coils can be shared by multiple links.

The MI waveguide deployment is also influenced by the topology of the sensors in the WUSNs, which is determined by specific applications. If full sensor coverage is required in a sensing area where underground sensors can be buried at any desired positions, the hexagonal tessellation topology is preferred due to its efficiency and simplicity. If only specific positions need to be monitored by sensors or some positions in the sensing area are not suitable to bury underground sensors, the WUSN has a random topology. In the hexagonal tessellation topology, the underground sensors of the WUSN are set in all vertexes of a hexagonal tessellation. The length of each tessellation edge is determined by the requirements of specific applications. In the random topology, the positions of the sensors can be viewed as random distributed. Note that the hexagonal tessellation topology can be viewed as a special case of the random topology. In this section, we start by analyzing the MI waveguide deployment in WUSNs with the hexagonal tessellation topology. Then we extend our research to the deployment strategy in WUSNs with random topologies.

A. Deployment in Hexagonal Tessellation Topology

Hexagonal tessellations have been widely used for the wireless network topologies, such as the base station placement of the cellular networks. Due to the disk shape of the sensing range of the sensor devices, using hexagonal tessellation topology is the most efficient way to cover the whole sensing area. Different from the terrestrial wireless sensor networks, the communication range of the underground sensors is very limited. Hence, the MI waveguides are used to connect the sensors on the vertexes of the hexagonal tessellation. In the following analysis, we assume that the sensor density of the WUSN with the hexagonal tessellation topology is λ_{hex} (m^{-2}).

1) *Minimum Spanning Tree (MST) Algorithm*: If the network robustness is not considered, the optimal deployment goal is to connect all the sensors in a WUSN with minimum number of relay coils. Therefore, the minimum spanning tree can be used to find the optimal routes of MI waveguides. If the sensor number is K , the number of edges of the minimum spanning tree is $K-1$. The weight of each edge in the spanning

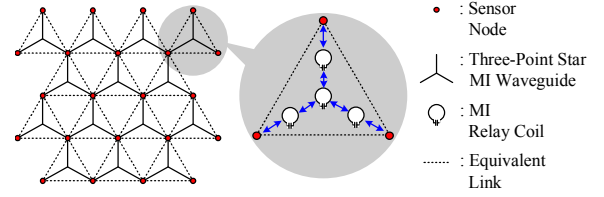


Fig. 3. The MI waveguide deployment using TC algorithm in the WUSN with hexagonal tessellation topology.

tree is the optimal number of the relay coil. As discussed in Section II, the optimal number of relay coils for a link is determined by the link length. The edges of the hexagonal tessellation have the same length e^{hex} , which is determined by the sensor density of the hexagonal tessellation λ_{hex} . Hence,

$$e^{hex} = 2 \cdot 3^{-\frac{1}{4}} \cdot \lambda_{hex}^{-\frac{1}{2}}. \quad (5)$$

Then the required number of the relay coils to connect K sensors based on the MST algorithm can be calculated as

$$N_{mst}^{hex} = (K - 1) \cdot n_{opt}(2 \cdot 3^{-\frac{1}{4}} \cdot \lambda_{hex}^{-\frac{1}{2}}, B), \quad (6)$$

where $n_{opt}(2 \cdot 3^{-\frac{1}{4}} \cdot \lambda_{hex}^{-\frac{1}{2}}, B)$ is the optimal coil number for each edge in the tessellation, which can be calculated by (4).

It should be noted that the WUSN constructed by the MST algorithm is only 1-connected. Consequently, the failure of any one sensor can disconnect the network.

2) *Triangle Centroid (TC) Algorithm*: To enhance the robustness of the network, more edges should be established. If the MI waveguides are deployed along all the edges in the hexagonal tessellation, every sensor in the WUSN is connected to all the 6 neighbors in the tessellation. Then, the network is 6-connected. We define this strategy as the *full-deployment*. However, in the full deployment strategy, the required number of relay coils for K sensors is doubled at the same time:

$$N_{full}^{hex} \simeq 2K \cdot n_{opt}(2 \cdot 3^{-\frac{1}{4}} \cdot \lambda_{hex}^{-\frac{1}{2}}, B), \quad (7)$$

To reduce the number of relay coils, we change the positions of the MI waveguides so that multiple links can share one set of the MI waveguide. In particular, the three MI waveguides along the three edges of one triangle cell can be replaced by one MI waveguide with a shape of the three-pointed star, as shown in Fig. 3. The center of the three-pointed star is located at the centroid of the triangle so that the sensors on all the three vertexes can use the same waveguide to communicate with each other directly. It can be proved that the total edge length of the three-pointed star is minimized if its center is located in the triangle centroid. Hence, the number of the relay coils to form the three-pointed star MI waveguide is minimized. To connect all the sensors in the WUSN, the three-pointed star MI waveguides are deployed in every other triangle in the tessellation, as shown in Fig. 3. The total number of triangles in the tessellation is approximately the same as the number of all sensors. Hence, the three-pointed star MI waveguides are deployed in half of the triangles. The edge length of the three-pointed star is $\sqrt{3} \cdot e_{hex}$. Then, the total required number of the relay coils to connect K sensors using TC algorithm is:

$$N_{tc}^{hex} \simeq \frac{K}{2} \cdot n_{opt}(2 \cdot 3^{\frac{1}{4}} \cdot \lambda_{hex}^{-\frac{1}{2}}, B). \quad (8)$$

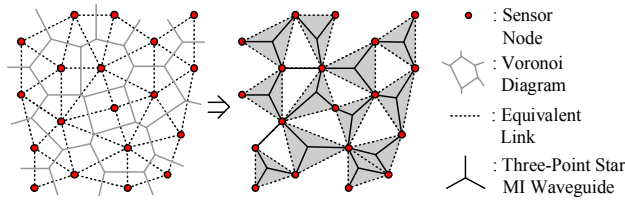


Fig. 4. The MI waveguide deployment using TC algorithm in the WUSN with random topology.

The WUSN constructed by the TC algorithm is 6-connected, the same as the full-deployment strategy. By comparing (7) with (8), we can find that the required relay coil number of the TC algorithm is much smaller than that of the full deployment. Detailed numerical analysis is given in Section IV.

B. Deployment in WUSNs with Random Topology

Based on the analysis on the WUSNs with the hexagonal tessellation topology, we investigate the deployment algorithms for WUSNs with random topology in this section. Assuming that the underground sensors are uniformly distributed with the spatial density λ_{rand} (m^{-2}). Similar to the strategy in hexagonal tessellation, the MST algorithm are provided to achieve the minimum relay coil number, while the TC algorithm are implemented to provide the robustness to sensor failure with acceptable relay coil number.

1) *MST Algorithm*: The MST algorithm for WUSN with random topology is similar with that in hexagonal tessellations. First, the edge lengths between any two underground sensor nodes are calculated. Second, the optimal number of relay coils for each edge is calculated by (4), which is the weight of each edge. Third, the minimum spanning tree of the WUSN is found out. Finally, the MI waveguides with the optimal coil number are deployed along each edge of the minimum spanning tree.

2) *TC Algorithm*: As discussed previously, the TC algorithm needs to locate the centroid in each triangle cell of the network. In the hexagonal tessellation topology, the network is well partitioned into equilateral triangle cells. Therefore the centroid in each triangle cell is easy to be located. However, in random topologies, TC algorithm encounters two problems: 1) how to partition the random network into non-overlapped triangle cells; and 2) how to deploy the three-pointed star MI waveguide in those randomly distributed triangle cells.

To solve the above problems, we introduce the Voronoi diagram [6]. As shown in the left of Fig. 4, the Voronoi diagram of the sensors partitions the whole area into polygons (Voronoi cells). Each Voronoi cell contains only one sensor. All the points in one Voronoi cell are closer to the sensor in this Voronoi cell than to any other sensors. By connecting the sensors that are in the adjacent Voronoi cells, the sensing area can be partitioned into non-overlapped triangle cells. Then in every other triangle cell, the MI waveguide is deployed along the three lines connecting the triangle vertexes and the centroid, which forms the three-pointed star MI waveguide, as shown in the right of Fig. 4. The detailed procedure of the TC algorithm for random topology is described in Algorithm 1.

For the random topology, the WUSN constructed by the MST algorithm is only 1-connected. Meanwhile, the network created by the TC algorithm in random topology is k -

Algorithm 1 TC Algorithm for Random Topology

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Create the Voronoi diagram of the  $K$  sensors, and derive  $K$  Voronoi cells  $\mathbf{VC} = \{VC_1, VC_2, \dots, VC_K\}$ .
Keep a subset  $\mathbf{G}$  of  $\mathbf{VC}$ ;  $\mathbf{G}$  initially contains  $VC_1$ .
while (Not all Voronoi cells are in  $\mathbf{G}$ ) do
    Find a Voronoi cell  $VC_x$  in  $\mathbf{G}$  that has the neighbor Voronoi cells  $\{VC_x^1, VC_x^2, \dots, VC_x^j\}$  which are not in  $\mathbf{G}$ .
    Connect the adjacent sensors in  $\{VC_x^1, VC_x^2, \dots, VC_x^j\}$  and  $VC_x$ , and derive the non-overlapped triangle cells  $\{Tr_1, Tr_2, \dots, Tr_{j-1}\}$ . The IDs of triangle cells are assigned in a clockwise manner.
    if ( $j$  is odd) then
        In  $Tr_1, Tr_2, Tr_4, \dots, Tr_{j-1}$ , deploy the MI waveguide along the the three lines connecting the vertexes and the centroid.
    else
        In  $Tr_1, Tr_3, Tr_5, \dots, Tr_{j-1}$ , deploy the MI waveguide along the the three lines connecting the vertexes and the centroid.
    end if
    Add  $\{VC_x^1, VC_x^2, \dots, VC_x^j\}$  to  $\mathbf{G}$ .
end while
    
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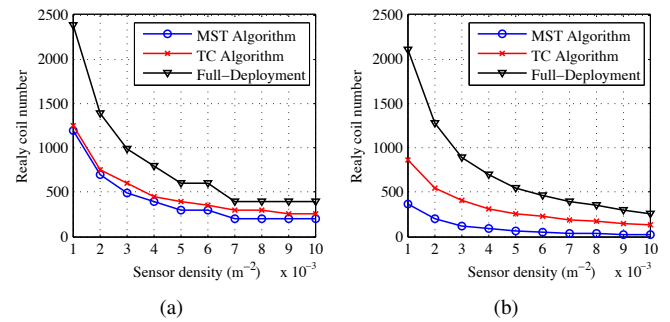


Fig. 5. The number of relay coils to connect 100 sensors in WUSNs with (a) hexagonal tessellation topology and (b) random topology.

connected, where $k \geq 3$. The required number of relay coils of the MST algorithm as well as the TC algorithm in the WUSN with random topology cannot be accurately estimated since the positions of the sensors are highly random.

IV. PERFORMANCE EVALUATION

In this section, we numerically evaluate the required relay coil number and the network robustness of the MST algorithm and the TC algorithm in WUSNs with hexagonal tessellation topology or random topology. The performance of the full-deployment strategy is also given as a reference. In the simulations, 100 underground sensors are deployed in a square area. The size of the square area is determined by the sensor density. The MI waveguide parameters used in the simulations are the same as those used in Section II.

A. Hexagonal Tessellation Topology

In Fig. 5(a), the required relay coil numbers of the deployment algorithms are given as a function of the sensor density in the WUSN with hexagonal tessellation topology. It shows that the relay coil number required by the TC algorithm is slightly larger than the number required by the MST algorithm but much smaller than the number required by the full-deployment strategy. Meanwhile, the network constructed by the TC algorithm is 6-connected, the same as the full-deployment strategy and far more robust than the 1-connected network constructed by the MST algorithm. Therefore, for

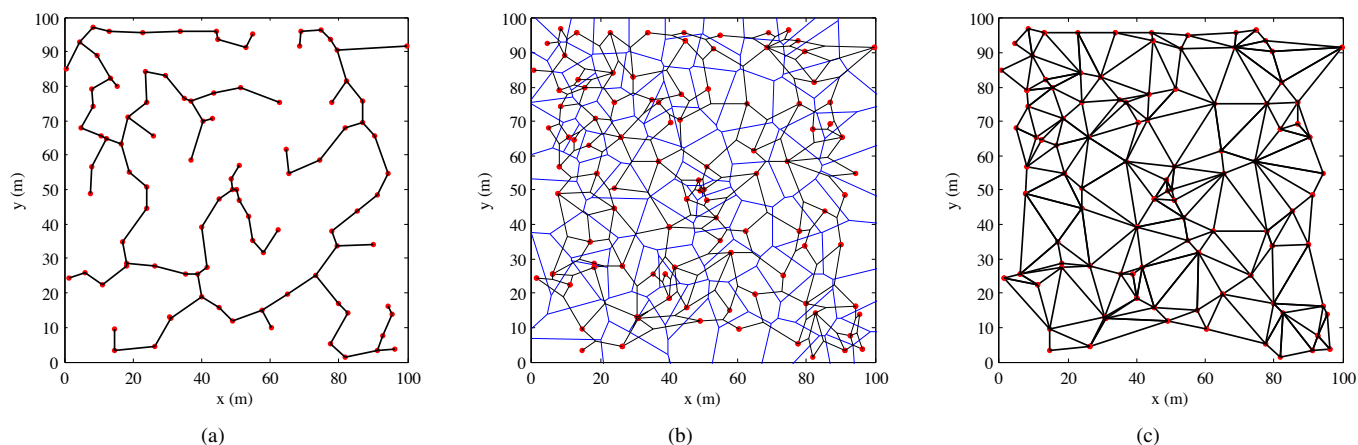


Fig. 6. The deployment results of (a) the MST algorithm, (b) the TC algorithm, and (c) the full-deployment strategy. (The red dots are the sensors; the black lines are the MI waveguides; and the blue cells are the Voronoi diagrams. 100 sensors are uniformly distributed with a spatial intensity $\lambda_{rand} = 0.01 \text{ m}^{-2}$.)

the hexagonal tessellation topology, the TC algorithm achieves both small relay coil number and high network robustness.

B. Random Topology

Fig. 6 shows the deployment results of the MST algorithm, the TC algorithm and the full-deployment strategy. On the one hand, the network constructed by the MST algorithm is only 1-connected. Consequently, the failure of any one sensor can disconnect the network. On the other hand, the networks constructed by the TC algorithm and the full-deployment strategy have the same network topology, since the three-pointed star MI waveguide in a triangle cell is equivalent to the three MI waveguides on the edges of the triangle cell. Except the sensors on the border, the network constructed by the TC algorithm or the full-deployment strategy is k -connected. k is determined by the sensor topology and $k \geq 3$. Therefore, the TC algorithm and the full-deployment strategy are more robust to sensor failures.

In Fig. 5(b), the required relay coil numbers of the deployment algorithms are given as a function of the sensor density in WUSNs with random topologies. It indicates that the relay coil number required by the TC algorithm is obviously larger than the number required by the MST algorithm. As the sensor density increases, the differences in terms of the coil number between the deployment algorithms become smaller. Compared with the hexagonal tessellation topology, the advantages of the MST algorithm in terms of the relay coil number is much more obvious in the random topology.

Therefore, in the WUSNs with random topology, the relay coils number required by the MST algorithm is significantly smaller than other deployment algorithms. However, the MST algorithm is not robust to sensor failures. Although the TC algorithm requires more relay coils than the MST algorithm, it can construct a robust k -connected WUSN. Moreover, the required coil number of the TC algorithm is much smaller than the number required by the full-deployment strategy.

V. CONCLUSION

The MI waveguides solve the propagation problems encountered by the traditional EM wave techniques in WUSNs.

This paper investigate the deployment of those MI waveguides to connect the underground sensors in the WUSNs. In 1D WUSNs, the optimal number of relay coils between two adjacent sensors is analyzed. Based on the results in 1D WUSNs, two solutions are developed to deploy the MI waveguides in 2D WUSNs. To minimize the relay coil number, the MST algorithm is provided. The MST algorithm uses the minimum spanning tree to connect all the sensors with the optimal relay coil number. However, the WUSN constructed by the MST algorithm is 1-connected hence is not robust to sensor failure. To enhance the network robustness while not increasing the relay coil number too much, the TC algorithm is proposed. The TC algorithm first use the Voronoi diagram to partition the whole network into non-overlapping triangle cells. Then the MI waveguides with the shape of the three-pointed star is deployed in every other triangle cells. The network constructed by the TC algorithm is k -connected ($k \geq 3$). Hence, the TC algorithm is robust to sensor failures.

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