Optimal MIMO Antenna Geometry Analysis for Wireless Networks in Underground Tunnels

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***Abstract*—The MIMO (Multiple Input Multiple Output) systems can efficiently mitigate the severe multipath fading effects in underground tunnels. Due to the unique tunnel channel characteristics, there exist optimal antenna geometries that can maximize the MIMO channel capacity. In this paper, an optimal MIMO antenna geometry design scheme is developed based on the analysis of the channel capacity in underground tunnels. The MIMO channel capacities in both empty tunnels and tunnels with obstructions are analytically expressed to facilitate the antenna geometry design. Simulation results show that the MIMO system with designed antenna geometry has obvious improvements in channel capacity than the system with the traditional linear array antenna geometry.**

I. Introduction

In underground mines and road tunnels, robust and reliable wireless networks with high throughputs are needed to guarantee the safety, to improve the productivity and to provide convenient communications. Since the wireless signals are confined to the internal space of the underground tunnels, the multipath fading in these environments is much more severe than the terrestrial wireless channels [1].

By seeking spatial diversity, MIMO (Multiple Input Multiple Output) systems provide higher spectral efficiency and link reliability. They can greatly mitigate the impact of the multipath fading in terrestrial wireless communication systems [2], [3]. In underground tunnels, the MIMO technique also has the potential to address the multipath fading issue to improve the system robustness and reliability, as well as to increase the throughputs by multiplexing data onto multiple transmit antennas.

According to our previous analysis on the underground tunnel channel [1], the positions of the transmitter and the receiver have significant influences on the channel gain (or path loss). Hence the antenna geometry of the MIMO system may affect the MIMO channel capacity a lot. Since the electromagnetic (EM) waves have certain propagation patterns (modes) in underground tunnels, it is possible to find the optimal MIMO antenna geometry to maximize the channel capacity according to the EM field distribution of the modes.

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In [3], the effect of MIMO antenna geometry on capacity is analyzed in terrestrial wireless channel. It provides a general antenna geometry design scheme for terrestrial MIMO communications. In [4], current terrestrial MIMO techniques are evaluated in tunnel environments by simulations. However, [3] and [4] do not take into consideration the unique channel characteristics in tunnels. In [5], the MIMO channel capacity in waveguide and cavity channels are analyzed. However, the authors assume that different modes have the same attenuation rate, which is not valid in underground tunnels. In [6], the MIMO channel characteristics are analyzed based on modal theory. The separation between antenna elements are investigated. The authors assume that the tunnel is empty. However, there exist a lot of obstructions (e.g. human beings, pipes, vehicles and mining machines) in the actual underground tunnels. Currently, no analysis on MIMO channel capacity and antenna geometry in tunnels with obstructions is provided.

In this paper, we provide an optimal MIMO antenna geometry design scheme to maximize the channel capacity in underground tunnels. Specifically, we first develop a channel model in tunnels with obstructions based on our previous work [1]. Then an analytical expression of the MIMO channel capacity is provided according to the channel model. An optimal MIMO antenna geometry design scheme is developed based on the channel capacity analysis. Simulations are conducted in both empty tunnels and tunnels with obstructions. It is shown that the MIMO system with designed antenna geometry has obvious improvements in channel capacity than the system with the traditional linear array antenna geometry.

The remainder of this paper is organized as follows. In Section II, the channel model in tunnels with obstructions is developed. In Section III, the optimal MIMO antenna geometry is developed based on the analysis of the MIMO capacity in tunnels. Then, in Section IV, simulation results are provided. Finally, the paper is concluded in Section V.

II. Channel Model in Tunnels

We have developed a multi-mode operating waveguide model for empty tunnels in [1]. In this section, we first make a brief overview of this model. Then based on this model, a more general channel model is proposed, which can characterize the tunnel with obstructions of arbitrary number and shapes. Note

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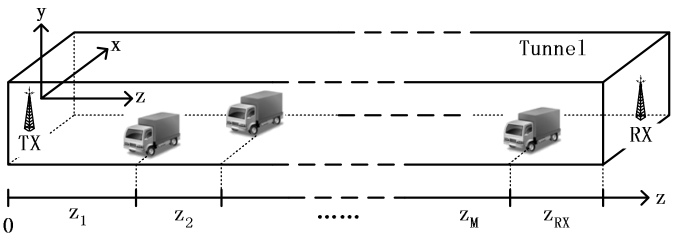

Fig. 1. Tunnel Environment

that the tunnels are assumed to be almost straight without sharp bends or corners.

A. Channel Model for Empty Tunnels

In [1], the tunnel channel is modeled as an oversized waveguide with imperfectly lossy walls. Multiple waveguide modes with different eigenfunctions and attenuation coefficients are transmitting simultaneously in the tunnel. Since the mode's attenuation rate rises fast as the mode order increases, only a small number of modes are left after the signal propagates for a certain distance. Hence in this paper, we only consider the lowest order modes that has significant power. The tunnel cross section is treated as an equivalent rectangular with a width of $2a$ m and a height of $2b$ m. A Cartesian coordinate system is set with its origin located at the center of the rectangle tunnel, as shown in Fig. 1. Assuming that there are N significant modes in the tunnel, the complex channel gain h_{ij} between RX antenna i and TX antenna j inside the tunnel can be obtained by summing up the gain of all significant modes at the position of RX antenna, which is given by:

$$h_{ij} = \frac{\sqrt{G_t G_r}}{2k} \sum_{n=1}^{N} E^{eign}_{n,(Rx,i)} \cdot \exp(-\Gamma_n \cdot z_{tot}) \cdot C_n^j \qquad (1)$$

where G_t and G_r are the TX and RX antenna gain, respectively; k is the wave number; $E^{eign}_{n,(Rx,i)}$ is the value of the n^{th} mode's eigenfunction at the position of RX antenna i; Γ_n is the attenuation coefficient of the n^{th} mode; z_{tot} is the total distance between the transmitter and the receiver; C_n^j is the n^{th} mode's intensity induced by TX antenna j. The detailed expression of $E^{eign}_{n,(Rx,i)}$ and Γ_n can be found in [1]. C_n^j is calculated by projecting the EM field excited by TX antenna j on the eigenfunction of the n_{th} mode:

$$C_n^j = \int_{-a}^{a}\int_{-b}^{b} E^{Tx,j}_{(x,\ y)} \cdot E^{eign}_{n,(x,\ y)}\ dx\, dy \qquad (2)$$

where $E^{eign}_{n,(x,\ y)}$ is the value of the n^{th} mode's eigenfunction at the position (x, y); $E^{Tx,j}_{(x,\ y)}$ is the EM field excited by TX antenna j at the position (x, y). The $E^{Tx,j}_{(x,\ y)}$ is calculated by the Geometrical Optical (GO) model and the detailed expression is provided in [1].

B. Channel Model for Tunnels with Obstructions

In actual road tunnels or underground mine tunnels, there are always many obstructions like vehicles, road signs, mining machines, lights, as well as human beings. The obstructions have arbitrary shapes and may block part of the tunnel cross section for a certain axial distance. In this section, a more general channel model for tunnels with obstruction is developed.

In our model, there are M obstructions in the tunnel between the transmitter and the receiver, as shown in Fig. 1. It should be noted that if two or more obstructions have almost the same axial distances to the transmitter, they are considered as one obstruction. We assume that no obstructions exist in the near region of the TX antenna and the RX antenna. Then the obstructions do not affect the schemes of EM field excitation at the TX antenna and the signal receiving at the RX antenna. Moreover, since the tunnel is empty between two adjacent obstructions, the waveguide modes have the same propagation characteristics as the empty tunnel in the empty region of the tunnel. Hence, the only regions of the tunnel that the empty tunnel channel model cannot characterize are around the obstructions.

Since different waveguide modes have different EM field distributions in the tunnel cross section, the modes experience different energy loss and phase shift when propagating around the obstructions. Additionally, the obstructions also cause coupling among the propagating modes [7]. Hence, the mode intensity $C_{n,(1)}$ of the n^{th} mode after the influence of the 1^{st} obstruction should be:

$$C_{n,(1)} = \sum_{i=1}^{N} B_{ni,(1)} \cdot C_{i,(0)} \cdot \exp(-\Gamma_i \cdot z_1) \qquad (3)$$

where $B_{ni,(1)}$ is the complex coupling coefficient of the i^{th} mode on the n^{th} mode caused by the 1^{st} obstruction; $C_{i,(0)}$ is the i^{th} mode intensity excited at the TX antenna; z_1 is the distance between the 1^{st} obstruction and the TX antenna. By analogy, the mode intensity $C_{n,(1)}$ of the n^{th} mode after the influence of the v^{th} obstruction should be:

$$C_{n,(v)} = \sum_{i=1}^{N} B_{ni,(v)} \cdot C_{i,(v-1)} \cdot \exp(-\Gamma_i \cdot z_v) \qquad (4)$$

where z_v is the distance between the v^{th} obstruction and the $(v-1)^{th}$ obstruction.

By this iterative algorithm, the intensity of each mode at the RX antenna can be expressed in the following matrix form:

$$\begin{pmatrix} C_{1,(M)} \\ C_{2,(M)} \\ \vdots \\ C_{N,(M)} \end{pmatrix} = \mathbf{A}(z_{RX}) \cdot \prod_{v=1}^{M} \left[\mathbf{B}_v \cdot \mathbf{A}(z_v)\right] \cdot \begin{pmatrix} C_{1,(0)} \\ C_{2,(0)} \\ \vdots \\ C_{N,(0)} \end{pmatrix} \qquad (5)$$

where z_{RX} is the distance between the RX antenna and the last (M^{th}) obstruction; $\mathbf{A}(z)$ is the mode attenuation matrix for transmitting all the significant modes for z meters in an empty tunnel; $\mathbf{B}_v$ is the coupling coefficient matrix caused by the v^{th} obstructions. The detailed expressions of these two

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matrix are:

$$\mathbf{A}(z) = \begin{pmatrix} e^{-\Gamma_1 \cdot z} & 0 & \cdots & 0 \\ 0 & e^{-\Gamma_2 \cdot z} & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & e^{-\Gamma_N \cdot z} \end{pmatrix}$$

$$\mathbf{B}_v = \begin{pmatrix} B_{11,(v)} & B_{12,(v)} & \cdots & B_{1N,(v)} \\ B_{21,(v)} & B_{22,(v)} & \cdots & B_{2N,(v)} \\ \vdots & & \ddots & \vdots \\ B_{N1,(v)} & B_{N2,(v)} & \cdots & B_{NN,(v)} \end{pmatrix} \quad (6)$$

According to eq (5), we define the mode propagation matrix $\mathbf{D}$ as:

$$\mathbf{D} = \mathbf{A}(z_{RX}) \cdot \prod_{v=1}^{M} \left[\mathbf{B}_v \cdot \mathbf{A}(z_v) \right] \quad (7)$$

which is an $N \times N$ matrix and indicates the mode propagation characteristics in tunnel with obstructions. The coupling coefficient matrix $\mathbf{B}_v$ is determined by the geometry of the tunnel and the v^{th} obstruction, where the obstruction may affect the boundary conditions of the Maxwell's equations in the tunnel. Since the obstructions have arbitrary shape, size and position, it is impossible to find a universal analytical solution of the coupling coefficient matrix $\mathbf{B}_v$. However, we develop an approximate statistical solution to find the mode propagation characteristics in tunnels with obstructions.

We assume that there are a large number of random obstructions in the tunnel, and the obstructions have diverse and random shapes/positions. Hence z_v, $\mathbf{B}_v$ $(v = 1, 2, ..., M)$ and z_{RX} are all random. According to eq (6), the element d_{uv} in the u^{th} row and the v^{th} column of the mode propagation matrix $\mathbf{D}$ is expressed as:

$$d_{uv} = \sum_{i_1}^{N} \sum_{i_2}^{N} \cdots \sum_{i_{M-1}}^{N} \Bigg[B_{i_1 u,(1)} \cdot e^{-\Gamma_u \cdot z_1} \cdot \prod_{l=2}^{M-1} \left(B_{i_l i_{l-1},(l)} \cdot e^{-\Gamma_{i_l} \cdot z_l} \right) \cdot B_{v\, i_{M-1},(M)} \cdot e^{-\Gamma_v \cdot z_M} \Bigg] \cdot e^{-\Gamma_u \cdot z_{RX}} \quad (8)$$

Since $B_{rs,(t)}$ and Γ_n are complex, d_{uv} is also complex. Hence it can be expressed as:

$$d_{uv} = Re(d_{uv}) + j \cdot Im(d_{uv}) \quad (9)$$

According to eq (8), $Re(d_{uv})$ and $Im(d_{uv})$ are the sum of the functions of $\{B_{rs,(t)} \big| \; r, s \in [1, N], \; t \in [1, M]\}$ and $\{z_t \big| \; t \in [1, M]\}$. Since $\{B_{rs,(t)}\}$ and $\{z_t\}$ are random, their functions can be viewed as independent random variables. Moreover, the number of obstructions M is very large. According to central limit theorem, $Re(d_{uv})$ and $Im(d_{uv})$ have the normal distribution, since they are the sum of a large number of independent variables [8]. Then the norm of d_{uv} has the Rayleigh distribution. To sum up, the mode propagation matrix $\mathbf{D}$ is stochastic and the norm of each element $|d_{uv}|$ has the Rayleigh distribution.

Finally, in the tunnel with obstructions, the complex channel gain h_{ij} between RX antenna i and TX antenna j is calculated by summing up the gain of all significant modes, which is given by:

$$h_{ij} = \frac{\sqrt{G_t G_r}}{2k} \left(E^{eign}_{1,(Rx,i)}, \cdots, E^{eign}_{N,(Rx,i)} \right) \cdot \mathbf{D} \cdot \begin{pmatrix} C_1^j \\ \vdots \\ C_N^j \end{pmatrix} \quad (10)$$

where C_n^j is the mode intensity of the n^{th} mode induced by the j^{th} TX antenna, which is calculated by eq (2). As discussed in the beginning, we assume that no obstructions exist in the near region of the TX antenna and the RX antenna, which means that z_1 and z_{RX} are sufficiently long. High order modes lose most of their power in these two regions since their attenuation rates are high. Although around obstructions, some lower order modes may be coupled to higher order modes, no mode coupling happens in these two regions. Hence, the number of significant modes N is assumed to be small.

III. MIMO Antenna Geometry Optimization in Underground Tunnels

Since in underground tunnels the antenna position significantly affects the total channel gain [1], it is necessary to analyze the effect of the antenna geometry on the MIMO channel capacity in these environments. In this section, the MIMO channel capacity in tunnels is first analyzed. Then optimal MIMO antenna geometry is developed to maximize the channel capacity.

A. Analysis on MIMO Capacity in Underground Tunnels

We consider a single-user, point-to-point communication channel with p transmitting (TX) and q receiving antenna elements. The complex channel gain matrix $\mathbf{H}$ is:

$$\mathbf{H} = \begin{pmatrix} h_{11} & h_{12} & \cdots & h_{1p} \\ h_{21} & h_{22} & \cdots & h_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ h_{q1} & h_{q2} & \cdots & h_{qp} \end{pmatrix} \quad (11)$$

where h_{ij} is channel gain h_{ij} between RX antenna i and TX antenna j . Our goal is to find the optimal antenna geometry to achieve the maximum MIMO channel capacity in tunnels.

We assume that the transmitter does not have the channel knowledge. Hence, equal power is radiated from each transmitting antenna. The Ergodic MIMO capacity with equal power allocation is [2]:

$$Capacity = E\left[\log\det \left(\mathbf{I}_{q \times q} + \frac{\rho}{p} \mathbf{H}\mathbf{H}^* \right) \right] \quad (12)$$

where $\mathbf{H}^*$ denotes the conjugate transpose of the matrix $\mathbf{H}$; ρ is the signal to noise ratio (SNR), which is defined as:

$$\rho = \frac{P_{tot}}{N_0}$$

where P_{tot} is the total transmission power of all TX antenna elements; N_0 is the noise power.

The channel gain matrix $\mathbf{H}$ can be derived from eq (10):

$$\mathbf{H} = \frac{\sqrt{G_t G_r}}{2k}\ \mathbf{E}^{RX} \cdot \mathbf{D} \cdot \mathbf{C}^{TX} \tag{13}$$

where $\mathbf{D}$ is defined in eq (7); $\mathbf{E}^{RX}$ is the mode eigenfunction matrix at RX side, which is:

$$\mathbf{E}^{RX} = \begin{pmatrix} E^{eign}_{1,(Rx,1)} & E^{eign}_{2,(Rx,1)} & \cdots & E^{eign}_{N,(Rx,1)} \\ E^{eign}_{1,(Rx,2)} & E^{eign}_{2,(Rx,2)} & \cdots & E^{eign}_{N,(Rx,2)} \\ \vdots & \vdots & \ddots & \vdots \\ E^{eign}_{1,(Rx,q)} & E^{eign}_{2,(Rx,q)} & \cdots & E^{eign}_{N,(Rx,q)} \end{pmatrix} \tag{14}$$

$\mathbf{C}^{TX}$ is the mode intensity matrix at TX side, which is:

$$\mathbf{C}^{TX} = \begin{pmatrix} C_1^1 & C_1^2 & \cdots & C_1^p \\ C_2^1 & C_2^2 & \cdots & C_2^p \\ \vdots & \vdots & \ddots & \vdots \\ C_N^1 & C_N^2 & \cdots & C_N^p \end{pmatrix} \tag{15}$$

The element C_n^j in $\mathbf{C}^{TX}$ is defined in eq (2) and can be calculated by composite numerical integration. Detailed expression of C_n^j can be found in [1]. The number of TX antenna elements p and RX antenna elements q should be larger than the number of significant modes N (i.e. $p \geq N$ and $q \geq N$) to guarantee the matrix $\mathbf{C}^{TX}$ and $\mathbf{E}^{RX}$ are full-rank. Intuitively, it suggests that each significant mode needs at least one TX antenna element and one RX antenna element to be efficiently excited and received.

Substituting (13) into (12), the MIMO channel capacity is converted to:

$$\begin{aligned} Capacity = E\Bigg[& \log\det\Big(\mathbf{I}_{q\times q} \\ & + \frac{G_t G_r \rho}{2kp}\mathbf{E}^{RX}\cdot\mathbf{D}\cdot\mathbf{C}^{TX}\cdot\mathbf{C}^{TX*}\cdot\mathbf{D}^*\cdot\mathbf{E}^{RX*}\Big)\Bigg] \end{aligned} \tag{16}$$

Since $\det(\mathbf{I}+\mathbf{AB}) = \det(\mathbf{I}+\mathbf{BA})$ [9], the channel capacity becomes:

$$\begin{aligned} Capacity = E\Bigg\{ & \log\det\Big[\mathbf{I}_{N\times N} \\ & + \frac{G_t G_r \rho}{2kp}(\mathbf{E}^{RX*}\mathbf{E}^{RX})\cdot\mathbf{D}\cdot(\mathbf{C}^{TX}\mathbf{C}^{TX*})\cdot\mathbf{D}^*\Big]\Bigg\} \end{aligned} \tag{17}$$

Note that $\mathbf{E}^{RX}$, $\mathbf{D}$ and $\mathbf{C}^{TX}$ are all full-rank matrix. If the system SNR is high, then Ergodic channel capacity of the MIMO system in tunnels becomes:

$$\begin{aligned} Capacity \approx E\Bigg\{ & \log\det\Big[\frac{G_t G_r \rho}{2k}(\mathbf{D}\mathbf{D}^*)\Big]\Bigg\} \\ & + \log\det\Big(\mathbf{E}^{RX*}\mathbf{E}^{RX}\Big) + \log\det\Big(\frac{1}{p}\mathbf{C}^{TX}\mathbf{C}^{TX*}\Big) \end{aligned} \tag{18}$$

The first term in eq (18) is determined by the mode propagation matrix $\mathbf{D}$, which is governed by the tunnel environment and the obstructions. As discussed in section II, if there are a large number of random obstructions in the tunnel, the norm of the elements in the mode propagation matrix $\mathbf{D}$ have the Rayleigh distribution. The TX or RX antenna geometries cannot influence the mode propagation in tunnels. The second term and the third term in eq (18) are determined by the geometry of the RX antennas and the TX antennas, respectively. Hence, the design of TX and RX antenna geometry can be conducted separately.

As a special case, if the tunnel is empty, there is no mode coupling in the tunnel. Hence all the mode coupling coefficient matrix $\mathbf{B}_v$ defined in eq (6) are $\mathbf{I}_{N\times N}$. Then the mode propagation matrix $\mathbf{D}$ in empty tunnel becomes deterministic and $\mathbf{D} = \mathbf{A}(z_{tot})$, where z_{tot} is the total distance between the transmitter and the receiver. Consequently the Ergodic channel capacity of the MIMO system in empty tunnels is converted to:

$$\begin{aligned} Capacity \approx & \sum_u^N \log\det\Big(\frac{G_t G_r \rho}{2k}\cdot\big|e^{-\Gamma_u\cdot z_{tot}}\big|^2\Big) \\ & + \log\det\Big(\mathbf{E}^{RX*}\mathbf{E}^{RX}\Big) + \log\det\Big(\frac{1}{p}\mathbf{C}^{TX}\mathbf{C}^{TX*}\Big) \end{aligned} \tag{19}$$

Eq (19) shows that the channel in an empty tunnel can be divided into N independent sub-channels. Each sub-channel corresponds to a significant mode in the tunnel. The geometry of the RX antennas and the TX antennas may influence the efficiency of exciting and receiving of those modes.

B. Optimal Antenna Geometry Optimization Design Scheme

According to eq (18) and (19), to maximize the MIMO channel capacity in tunnels, it is equal to maximize $\det(\mathbf{E}^{RX*}\mathbf{E}^{RX})$ by selecting optimal RX antenna geometry, and to maximize $\det(\frac{1}{p}\mathbf{C}^{TX}\mathbf{C}^{TX*})$ by selecting optimal TX antenna geometry.

1) Optimal RX Antenna Geometry: According to [1], [6], the eigenfunctions of different modes in a rectangle tunnel are approximately orthogonal to each other, i.e.

$$\int_{-a}^{a}\int_{-b}^{b} E^{eign}_{i,(x,y)} \cdot E^{eign*}_{j,(x,y)}\ dx\,dy \simeq \begin{cases} \xi, & \text{if } i = j \\ 0, & \text{otherwise} \end{cases} \tag{20}$$

The matrix $\mathbf{E}^{RX*}\mathbf{E}^{RX}$ is in fact the covariance matrix of the eigenfunctions of all significant modes at the RX elements' positions. Due to the mode orthogonality, $\det(\mathbf{E}^{RX*}\mathbf{E}^{RX})$ is maximized if $\mathbf{E}^{RX*}\mathbf{E}^{RX}$ is diagonal. If the RX antenna elements are placed at all the positions where the eigenfunctions of significant modes have extrema values, the matrix $\mathbf{E}^{RX}\mathbf{E}^{RX*}$ can be approximately diagonalized. Therefore, the optimal RX antenna geometry in tunnels is described as follows: the number and geometry of the antenna elements depend on which modes have significant power in the tunnel. For mode EM_{mn}, $q = m \times n$ antenna elements are needed. Their positions $\{(x_u, y_v)\}$ should be

$$\begin{cases} x_u = -a + (u - \frac{1}{2})\frac{2a}{m}, & u \in [1, m] \\ y_v = -b + (v - \frac{1}{2})\frac{2b}{n}, & v \in [1, n] \end{cases} \tag{21}$$

TABLE I
OPTIMAL TX ANTENNA GEOMETRY

Significant modes	Optimal TX antenna positions
EM_{11}	(0, 0)
EM_{11}, EM_{21}	(1.148, 0), (-0.001, 0.128)
EM_{11}, EM_{21}, EM_{12}	(-1.183, 0.058), (0.302, 0.007), (-0.004, 1.025)
EM_{11}, EM_{21}, EM_{12}, EM_{31}	(1.496, 0.452), (-0.001, 0.157), (-1.160, 0.040), (0.129, -0.929)

For example, if mode EM_{11} and EM_{21} have significant power in the tunnel, the number of RX antenna is $q = 1 \times 1 + 1 \times 2 = 3$. Their positions should be $(0, 0)$, $(-a/2, 0)$ and $(a/2, 0)$.

2) Optimal TX Antenna Geometry: To find the optimal TX antenna geometry, it is equal to an optimization problem, which is:

$$\begin{aligned} Given: &\quad \text{Tunnel size and existing significant modes} \\ Find: &\quad \text{Positions of } p \text{ TX antenna elements} \\ Maximize: &\quad \det(\frac{1}{p}\mathbf{C}^{TX}\mathbf{C}^{TX*}) \end{aligned}$$

This optimization problem can be solved by the Matlab Optimization Toolbox. The optimization results in a tunnel with a height of 3.6 m and a width of 6 m are shown in Table I, where only low order modes are considered because the lower order modes have the lowest attenuation rates and are more efficient to excite [1]. According to Table I, every optimal position is near an extrema value of a certain mode's eigenfunction. This phenomenon can be understood intuitively: if an TX antenna is placed at the position where a certain mode has the maximum EM field, this mode can be efficiently excited. If all the significant modes are efficiently excited, maximum channel capacity is possible to be achieved. Hence, an approximate but more simple optimal TX antenna geometry design scheme is to: 1) find out what are the significant modes in the tunnel; 2) for each mode, place one TX antenna at the position where the mode's eigenfunction achieves the extrema value.

To sum up, in both empty tunnels and tunnels with obstructions, the optimal MIMO antenna geometry design scheme can be described as follows: 1) Estimating which modes have significant power in the tunnel (usually the lowest order modes). 2) Placing RX antenna elements at all positions where the eigenvalue of a significant mode reaches its extrema value. For mode EM_{mn}, $m \times n$ RX antenna elements are needed. 3) For each significant mode, placing one TX antenna element at one of the positions where the mode's eigenvalue reaches its extrema value. Each significant mode needs one TX antenna element.

IV. SIMULATION RESULTS

In this section, the effects of antenna geometry on MIMO channel capacity in tunnels are analyzed by simulations. Multiple RX and TX antenna geometries are employed in the simulation for both empty tunnels and tunnels with obstructions. For empty tunnels, the channel model developed in [1] is used. For tunnels with obstructions, the channel model described in section II is utilized.

The environment parameters in simulations are set as follows: The tunnel cross section shape is a rectangle with a height of 3.6 m and a width of 6 m. The tunnel walls are made of concrete with the dielectric constant $\varepsilon = 10\varepsilon_0$ and the conductivity $\sigma = 0.01\ S/m$. The tunnel interior is filled with air ($\varepsilon = \varepsilon_0$, $\sigma = 0\ S/m$). The signal operating frequency is 900 MHz. The total transmitting power of all antenna elements P_{tot} is set to 10 dBm. We assume the underground mine tunnel or road tunnel environments have high noise power level N_0, which is set to -60 dBm.

According to the design scheme developed in section III-B, we first estimate which modes are significant in the tunnel. If we estimate that only mode EM_{11} is significant, the optimal SISO (1 TX and 1 RX) antenna geometry is derived; If we assume mode EM_{11} and EM_{21} are significant, the optimal MIMO (2 TX and 3 RX) antenna geometry is derived, so on and so forth. The more modes are considered, the more RX and TX antenna elements are needed. We also compare our results with the channel capacity of a MIMO (3 TX and 5 RX) antenna with traditional linear array geometry. The space between each array element is 0.33 m (the wavelength of the 900 MHz signal). The linear array is placed at the center of the tunnel and is parallel to the floor.

A. Empty Tunnel

Fig. 2 shows the Ergodic MIMO channel capacity with different antenna geometries in empty tunnels as a function of axial distance. It indicates that, the channel capacity with optimal antenna geometry is much higher than that of the undesigned geometry. Even though the undesigned linear array MIMO has more antenna elements, and we place it at the tunnel center (one of the optimal positions), its capacity is only slightly higher than the optimal positioned SISO system. In the near region to the transmitter, since more modes exist, the MIMO with more antenna elements has higher capacity. It is because that more modes are considered when designing the antenna geometry. As the distance between transceivers increases, fewer modes are left. The advantages of more antenna elements become trivial. After a sufficient long distance, all systems with different antenna geometries have similar channel capacities. It is because that only the lowest order mode EM_{11} is left and all antenna geometries have one RX and TX elements placed in the tunnel center, which is the optimal position to excite and receive EM_{11} mode. Notice that the optimal MIMO (4 TX and 7 RX) only performs better than the optimal MIMO (3 TX and 5 RX) in the very near region. In the far region, it has even worse performance. It is because that one TX antenna element corresponds to one mode. If the corresponding mode attenuates very fast and disappears at the RX side, the energy is wasted at the corresponding TX

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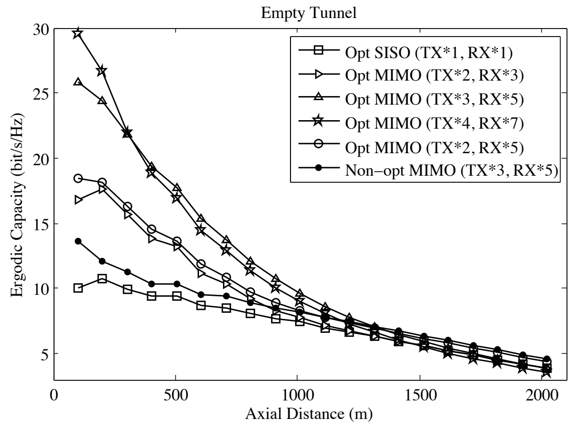

Fig. 2. Ergodic MIMO channel capacity with different antenna geometries in empty tunnels

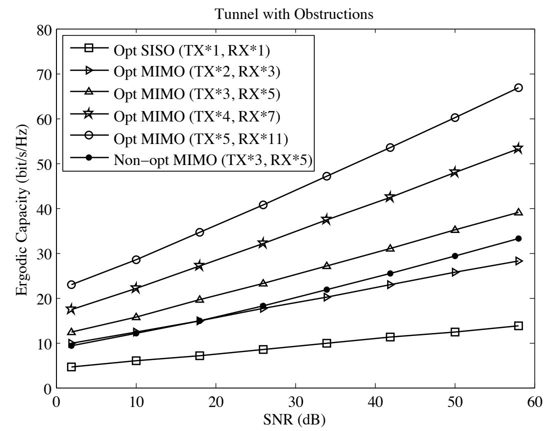

Fig. 3. Ergodic MIMO channel capacity with different antenna geometries in tunnels with obstructions

element. Meanwhile, the optimal MIMO (2 TX and 5 RX) has only a little higher channel capacity than the optimal MIMO (2 TX and 3 RX) even though it has two more RX elements, which indicates that more RX elements can enlarge the channel capacity but it is not efficient.

B. Tunnels with Obstructions

In Fig. 3, the Ergodic MIMO channel capacity with different antenna geometries in tunnels with obstruction as a function of SNR is shown. The channel model developed in section II is adopted, where we assume that the first five lowest order modes (EM_{11}, EM_{21}, EM_{12}, EM_{22} and EM_{31}) have significant power. The tunnel is filled with a large number of random obstructions. The norms of elements d_{uv} in mode propagation matrix $\mathbf{D}$ have independent Rayleigh distributions. The Ergodic channel capacity in Fig. 3 is the mean value of the results of 1000 iterations. In each iteration, every $Re(d_{uv})$ and $Im(d_{uv})$ are generated by two Gaussian random variables with zero mean and unit variance. Fig. 3 shows that, given the same number of antenna elements, the capacity of MIMO with optimal antenna geometry is obviously higher than the traditional linear array MIMO antenna. If all the five significant modes are considered when designing the antenna geometry, highest channel capacity is achieved (optimal MIMO with 5 TX and 11 RX elements). If fewer modes are considered when designing the antenna geometry, there is an obvious capacity decrease since all the modes have comparable power at the RX side. With less antenna elements, there will be at least one significant mode cannot be efficiently excited or received. That is the reason why the advantages of MIMO with optimal antenna geometry are not as significant as the empty tunnel case.

V. Conclusion

In this paper, an optimal antenna geometry design scheme for MIMO systems in underground tunnels is developed based on the analysis of channel characteristics. It can be concluded that: in tunnels with a large number of random obstructions, the channel is stochastic where the modes are coupling with each other. The norm of the coupling coefficients have the Rayleigh distribution. In empty tunnels, the channel is deterministic and can be divided into N independent sub-channels. Each sub-channel corresponds to a significant mode in the tunnel. To maximize the MIMO channel capacity, the antennas with optimal geometry should be able to efficiently excite and receive those modes that have significant power. Hence the antenna elements are placed at the positions where the eigenfunctions of the significant modes reach their extrema values. In empty tunnels, less antenna elements are needed since the number of significant modes become less and less as the transmission distance increases. In tunnels with obstructions, more antenna elements are needed since obstructions can couple the lower order modes to the higher order modes.

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