Competitive Pricing for Spectrum Sharing in Cognitive Radio Networks: Dynamic Game, Inefficiency of Nash Equilibrium, and Collusion

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***Abstract*—We address the problem of spectrum pricing in a cognitive radio network where multiple primary service providers compete with each other to offer spectrum access opportunities to the secondary users. By using an equilibrium pricing scheme, each of the primary service providers aims to maximize its profit under quality of service (QoS) constraint for primary users. We formulate this situation as an oligopoly market consisting of a few firms and a consumer. The QoS degradation of the primary services is considered as the cost in offering spectrum access to the secondary users. For the secondary users, we adopt a utility function to obtain the demand function. With a *Bertrand game* model, we analyze the impacts of several system parameters such as spectrum substitutability and channel quality on the Nash equilibrium (i.e., equilibrium pricing adopted by the primary services). We present distributed algorithms to obtain the solution for this dynamic game. The stability of the proposed dynamic game algorithms in terms of convergence to the Nash equilibrium is studied. However, the Nash equilibrium is not efficient in the sense that the total profit of the primary service providers is not maximized. An optimal solution to gain the highest total profit can be obtained. A collusion can be established among the primary services so that they gain higher profit than that for the Nash equilibrium. However, since one or more of the primary service providers may deviate from the optimal solution, a punishment mechanism may be applied to the deviating primary service provider. A repeated game among primary service providers is formulated to show that the collusion can be maintained if all of the primary service providers are aware of this punishment mechanism, and therefore, properly weight their profits to be obtained in the future.**

***Index Terms*—Spectrum sharing, cognitive radio, pricing scheme, game theory, Nash equilibrium, distributed adaptation, collusion.**

I. Introduction

SOFTWARE-DEFINED radio technique [1] was proposed to improve adaptability and flexibility of wireless transmission so that wireless system performance can be enhanced. Developed based on software-defined radio, "cognitive radio" has been identified as a new paradigm for designing next generation wireless networks. A cognitive radio transceiver has an ability to observe, learn, optimize, and change the transmission parameters according to the ambient radio environment [2]. With this agility of the radio transceiver, frequency spectrum can be shared among licensed (i.e., primary) and unlicensed (i.e., secondary) services to improve spectrum utilization and also to generate higher revenue to the spectrum owner. For efficient dynamic spectrum sharing, an economic model would be required for the spectrum owners and the spectrum users so that the revenue (hence profit) and the user satisfaction can be maximized.

When the allocated spectrum is not fully utilized, the spectrum owner (or primary service provider) has an opportunity to sell the spectrum opportunities to secondary service providers[1], and thereby, generate revenue. This is referred to as the spectrum trading mechanism which involves spectrum selling and buying processes. For spectrum trading, one of the challenging issues is pricing, for example, how to set the spectrum price in a competitive environment where multiple sellers (e.g., primary services) offer spectrum to the buyer (e.g., secondary service), so that the sellers are satisfied.

In this paper, we address the problem of competitive pricing in a dynamic spectrum access where a few primary services offer spectrum access opportunities to a secondary service. We formulate this as an *oligopoly market* where few firms compete with each other in terms of price to gain the highest profit. For a primary service, the cost of sharing the frequency spectrum is modeled as a function of the quality of service (QoS) degradation. For the secondary service, a spectrum demand function is established based on the utility function which depends on the channel quality. In addition, we consider *spectrum substitutability* which represents the ability of the secondary service to switch among the operating frequency spectra offered by different primary services. We use a Bertrand game model to analyze this oligopoly market situation and the Nash equilibrium is considered as the solution of this game. Also, we present distributed dynamic algorithms for price adaptation when the primary services cannot observe the profit of each other. We consider two cases, namely, the case where a primary service can observe the historical strategies adopted (i.e., prices offered) by other services, and the case where a primary service can observe only the spectrum demand from the secondary service. In the latter case, the learning rate is used to control the speed of strategy adjustment. The stability is analyzed for these strategy adaptations.

We observe that the Nash equilibrium is inefficient to maximize the total profit of the primary services. An optimal price, however, can be obtained by using a optimization formulation which is solved in a centralized manner. Since this global optimal price is not on the best response function

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[1]For brevity, from now on, we will use the terms 'primary/secondary service provider' and 'primary/secondary service' interchangeably.

of one or more of the primary services, some of the primary services may deviate from this optimal price. This deviation could provide higher profit to the deviating primary service but at the cost of lower profit for other primary services. For example, one primary service could reduce the price of the offered spectrum to generate a larger demand. In this case, the other primary services could punish (e.g., by reducing the spectrum price to generate higher demand) the deviating primary service resulting in lower than the optimal profit. We formulate a repeated game to analyze this situation. We show that if each of the primary services is aware of the punishment due to deviation, and if the profit in the future is properly weighted, a collusion among the primary services can be maintained to choose the optimal price so that the highest profit can be achieved in the long term.

The rest of this paper is organized as follows. Section II reviews the related work. Section III describes the system model and assumptions. The game formulation for competitive pricing is presented in Section IV. Section V presents the performance evaluation results. The conclusions are stated in Section VI.

II. Related Work

An introduction to cognitive radio techniques was provided in [2] where the fundamental cognitive tasks as well as the emergent behavior of cognitive radio were discussed. An overview of different spectrum sharing models, namely, open sharing, hierarchical access, and dynamic exclusive usage models, was provided in [3] and also the major issues related to primary user detection and spectrum sensing were discussed. Spectrum management is an important functionality in cognitive radio networks which involves dynamic spectrum access/sharing and pricing and it aims to satisfy the requirements of both the primary and the secondary users.

Game theory is one of the techniques that can be used for spectrum management in cognitive radio [2]. Traditionally, game theory techniques have been used for resource management (e.g., admission control, rate control [4], power control [5][6][7]) and modeling protocol misbehavior (e.g., backoff attack in CSMA/CA [8]) in wireless networks. Recently, game theoretic models have been used in the context of dynamic spectrum sharing and resource management in cognitive radio networks. In [9], a game-theoretic adaptive channel allocation scheme was proposed for cognitive radio networks. In particular, a game was formulated to capture the selfish and the cooperative behaviors of the players. The players of this game were the wireless nodes and their strategies were defined in terms of channel selection. In [10], the convergence dynamics of the different types of games in cognitive radio systems was studied (i.e., coordinated behavior, best-response, and better response for discounted repeated games, S-modular games, and potential games, respectively). Then, a game theory framework was proposed for distributed power control to achieve agility in spectrum usage in a cognitive radio network. However, the dynamic behavior of strategy adaptation was not discussed in these works. The problem of spectrum sharing was formulated as a potential game, and the Nash equilibrium of this game was obtained by a distributed sequential play [11]. However, the pricing issue was ignored.

In a wireless network, pricing and resource allocation are closely related. This is due to the fact that while a service provider wants to maximize its revenue, the user desires to maximize its satisfaction in terms of QoS performance and price. In [12], a price-based transmission rate allocation scheme was proposed for wireless ad hoc networks to achieve the highest resource utilization while maintaining fairness among end-to-end flows. For wireless networks, the problem of utility maximization with pricing was studied in [13] taking transmission rate and reliability into account. In [14], a pricing policy was introduced for voice services in a WLAN environment considering both QoS performance and users' willingness-to-pay. However, the problem of pricing in dynamic spectrum access environment was not addressed in these works.

The following works addressed the issue of pricing in dynamic spectrum access environments. In [15], an integrated pricing, allocation, and billing system for cognitive radio was proposed. In this system, the problem of pricing negotiation between the operator and the service users was formulated as a multi-unit sealed-bid auction. In [16], optimal bidding, pricing, and service differentiation mechanisms for code division multiple access (CDMA) systems were analyzed. In the considered system model, multiple CDMA operators bid for the spectrum from the spectrum manager. The objective is to maximize the revenue of the operators based on the users' willingness-to-pay. However, the issue of equilibrium among multiple operators and the stability of bidding in a competitive environment were ignored. The competition among spectrum owners was modeled as a game in [17],[18]. However, the spectrum demand function of the secondary users in presence of spectrum substitutability was not considered, and the stability of strategy adaptation was not investigated. Also, the issue of collusion among the primary services (to achieve an efficient solution maximizing total profits of the primary services) was not considered.

III. System Model and Assumptions

A. Primary and Secondary Services

We consider a wireless system with multiple primary services (total number of primary services is denoted by N) operating on the different frequency spectrum $\mathcal{F}_i$ and a secondary service which serves a group of secondary users willing to share these spectrum with the primary services (Fig. 1). In this case, primary service i serving M_i local connections wants to sell portions of the available spectrum $\mathcal{F}_i$ (e.g., time slots in a time-division multiple access (TDMA)-based wireless access system) at price p_i (per unit spectrum or bandwidth) to the secondary service. The spectrum can be shared among multiple connections (i.e., secondary users) in which the base station (BS) or access point (AP) governs the radio transmission on the allocated spectrum. The secondary users utilize adaptive modulation for transmissions on the allocated spectrum in a time-slotted manner. The spectrum demand of the secondary users depends on the transmission rate due to the adaptive modulation in the allocated frequency spectrum and the price charged by the primary services.

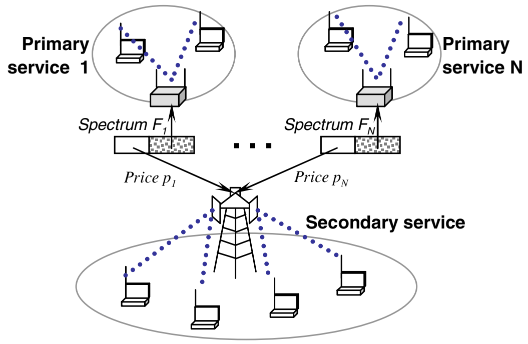

Fig. 1. System model for spectrum sharing.

B. Wireless Transmission Model

With adaptive modulation, the transmission rate can be dynamically adjusted based on the channel quality. The spectral efficiency k of transmission by a secondary user can be obtained from [19]

$$k = \log_2(1 + K\gamma), \quad \text{where} \quad K = \frac{1.5}{\ln\left(0.2/\text{BER}^{\text{tar}}\right)} \tag{1}$$

where γ is the SNR at the receiver and BER^{tar} is the target bit-error-rate (BER).

C. Oligopoly Price Competition and Bertrand Game

We model the problem of dynamic spectrum sharing under competition as an oligopoly market. In economics, oligopoly is defined as a situation where a small number of firms (i.e., oligopolists) dominate a particular market. In this market structure, the firms compete with each other noncooperatively and independently to achieve their objectives (i.e., maximize profit) by controlling the quantity or the price of the supplied product. The decision of each firm is influenced by other firms' actions and action of one firm may be observed by other firms. This Bertrand game model for price competition can be applied to analyze and obtain the equilibrium pricing scheme for a cognitive radio system consisting of multiple primary services which are willing to share the allocated spectrum with the secondary service. While the spectrum demand of the secondary service is determined from the utility of the shared frequency spectrum and the price charged by the primary service, the cost of the primary service in sharing the spectrum is computed from the degradation of the QoS for the primary service. The primary service searches for the equilibrium by adjusting the price offered to the secondary service so that the profit is maximized.

IV. Spectrum Pricing Competition and Solution

In this section, we establish a model for spectrum pricing competition based on game theory. First, we use a utility function to quantify the spectrum demand of the secondary service. Then, for a primary service, the cost for offering spectrum access to the secondary service is formulated. This cost function is based on the degradation in the QoS performance for the local connections. Next, a Bertrand game formulation is proposed. The Nash equilibrium is considered as the solution of this game.

A. Utility of Secondary Service

To quantify the spectrum demand, we consider the utility gained by the secondary service (e.g., if the spectrum creates high utility, the demand is high). In this paper, we adopt the following commonly used quadratic utility function [21]:

$$\mathscr{U}(\mathbf{b}) = \sum_{i=1}^{N} b_i k_i^{(s)} - \frac{1}{2}\left(\sum_{i=1}^{N} b_i^2 + 2\nu \sum_{i \neq j} b_i b_j\right) - \sum_{i=1}^{N} p_i b_i \tag{2}$$

where $\mathbf{b}$ is the set consisting of the size of shared spectrum from all the primary services, i.e., $\mathbf{b} = \{b_1, \ldots, b_i, \ldots, b_N\}$, p_i is the price offered by primary service i. Note that $k_i^{(s)}$ denotes the spectral efficiency of wireless transmission by a secondary user using frequency spectrum $\mathcal{F}_i$ which is owned by primary service i.

This utility function takes the spectrum substitutability into account through the parameter ν. This spectrum substitutability parameter (i.e., $\nu \in [-1.0, 1.0]$) is defined as follows. When $\nu = 0.0$, a secondary user cannot switch among the frequency spectra, while for $\nu = 1.0$ a secondary user can switch among the operating frequency spectra freely.

When $\nu < 0$, spectrum sharing by a secondary user is complementary. That is, when a secondary user wants to share one frequency spectrum, it will require to buy one or more additional spectrum simultaneously (e.g., one spectrum for uplink transmission and another for downlink transmission) from the same or different primary services.

The motivations for using the quadratic utility function in (2) can be stated as follows:

- The function is concave, and therefore, it is able to represent the saturation of user satisfaction as the transmission rate increases. Concave utility functions are widely used to quantify the satisfaction of best-effort users as a function of allocated bandwidth.
- Differentiating this quadratic utility function results in a linear bandwidth demand function, which makes the subsequent analysis tractable (e.g., standard methods can be used for stability analysis of the distributed strategy adaptation algorithms).
- The function is able to incorporate the impact of spectrum quality as well as the spectrum substitutability factor.

To derive the demand function for spectrum $\mathcal{F}_i$ at the secondary service, we differentiate $\mathscr{U}(\mathbf{b})$ with respect to b_i as follows:

$$\frac{\partial \mathscr{U}(\mathbf{b})}{\partial b_i} = 0 = k_i^{(s)} - b_i - \nu \sum_{i \neq j} b_j - p_i. \tag{3}$$

We can obtain the spectrum demand function given the prices of all primary services by solving the set of equations in (3). The spectrum demand function can be expressed as follows:

$$\mathscr{D}_i(\mathbf{p}) = \frac{(k_i^{(s)} - p_i)(\nu(N-2)+1) - \nu \sum_{i \neq j}(k_j^{(s)} - p_j)}{(1-\nu)(\nu(N-1)+1)}. \tag{4}$$

B. Revenue and Cost Functions for Primary Service

To develop a cost function, the QoS performances of the primary users need to be considered. If a portion of the frequency spectrum (in time domain and/or frequency domain) is shared with the secondary service, degradation in the QoS performance of the primary users may occur. We assume that, for guaranteed bandwidth allocation, the connections from the primary users are charged at a flat rate. However, if the required bandwidths for the primary connections cannot be provided, the primary service provides "compensation" to the connections, and this is considered as the cost of sharing the spectrum with secondary service. For primary service i, the revenue function $\mathscr{R}_i$ and the cost function $\mathscr{C}_i$ can be defined as follows:

$$\mathscr{R}_i = c_1 M_i, \quad \mathscr{C}_i(b_i) = c_2 M_i \left(B_i^{req} - k_i^{(p)} \frac{W_i - b_i}{M_i} \right)^2 \tag{5}$$

where c_1 and c_2 denote the constant weights for the revenue and the cost functions, respectively. Here, B_i^{req} is the bandwidth requirement for a primary connection, W_i is the size of spectrum, M_i is the number of primary connections, and $k_i^{(p)}$ is the spectral efficiency of wireless transmission for primary service i.

C. Bertrand Game Model

Based on the aforementioned system model, a Bertrand game can be formulated as follows. The **players** in this game are the primary services. The **strategy** of each of the players is the price per unit of spectrum (denoted by p_i) which is non-negative. The **payoff** for each primary service i (denoted by $\mathscr{P}_i$) is the profit (i.e., revenue minus cost) due to selling spectrum to the secondary service. The *solution* of this game is Nash equilibrium.

Based on the spectrum demand, revenue, and cost functions, the profit of each primary firm can be expressed as follows: $\mathscr{P}_i(\mathbf{p}) = b_i p_i + \mathscr{R}_i - \mathscr{C}_i(b_i)$, where $\mathbf{p}$ denotes the set of prices offered by all players in the game (i.e., $\mathbf{p} = \{p_1, \ldots, p_i, \ldots, p_N\}$).

By definition, the *Nash equilibrium* of a game is a strategy profile (list of strategies, one for each player) with the property that no player can increase his payoff by choosing a different action, given other players' actions [20]. In this case, the Nash equilibrium is obtained by using the best response function which is the best strategy of one player given others' strategies. The best response function of primary service i, given a set of prices offered by the other primary services $\mathbf{p}_{-i}$ (i.e., $\mathbf{p} = \mathbf{p}_{-i} \cup \{p_i\}$), is defined as follows:

$$\mathscr{B}_i(\mathbf{p}_{-i}) = \arg\max_{p_i} \mathscr{P}_i(\mathbf{p}_{-i} \cup \{p_i\}). \tag{6}$$

The set $\mathbf{p}^* = \{p_1^*, \ldots, p_N^*\}$ denotes the Nash equilibrium of this game on competitive pricing if and only if

$$p_i^* = \mathscr{B}_i(\mathbf{p}_{-i}^*), \quad \forall i \tag{7}$$

where $\mathbf{p}_{-i}^*$ denotes the set of best responses for player j for $j \neq i$. Mathematically, to obtain the Nash equilibrium, we have to solve the following set of equations: $\frac{\partial \mathscr{P}_i(\mathbf{p})}{\partial p_i} = 0$ for all i. In this case, the size of the shared bandwidth b_i in the profit function is replaced by spectrum demand $\mathscr{D}_i(\mathbf{p})$, and the profit function can be expressed as follows:

$$\mathscr{P}_i(\mathbf{p}) = p_i \mathscr{D}_i(\mathbf{p}) + c_1 M_i - c_2 M_i \left(B_i^{req} - k_i^{(p)} \frac{W_i - \mathscr{D}_i(\mathbf{p})}{M_i} \right)^2. \tag{8}$$

The demand function in (4) can be rewritten as $\mathscr{D}_i(\mathbf{p}) = D_1(\mathbf{p}_{-i}) - D_2 p_i$, where $D_1(\mathbf{p}_{-i})$ and D_2 are constants given all p_j for $i \neq j$ and can be expressed as follows:

$$\begin{aligned} D_1(\mathbf{p}_{-i}) &= \frac{k_i^{(s)}(\nu(N-2)+1) - \nu \sum_{i \neq j}(k_j^{(s)} - p_j)}{(1-\nu)(\nu(N-1)+1)} \\ D_2 &= \frac{(\nu(N-2)+1)}{(1-\nu)(\nu(N-1)+1)}. \end{aligned}$$

Then, using $\frac{\partial \mathscr{P}_i(\mathbf{p})}{\partial p_i} = 0$, we obtain

$$\begin{aligned} 0 = {} & 2c_2 k_i^{(p)} D_2 \left(B_i^{req} - k_i^{(p)} \frac{W_i - (D_1(\mathbf{p}_{-i}) - D_2 p_i)}{M_i} \right) \\ & + D_1(\mathbf{p}_{-i}) - 2D_2 p_i. \end{aligned} \tag{9}$$

The solution p_i^*, which is a Nash equilibrium, can be obtained by solving the above set of linear equations when all the parameters in (9) are available (e.g., to a central controller). Then, given a set of prices $\mathbf{p}^*$ at the Nash equilibrium, the size of the shared spectrum can be obtained from the spectrum demand function $\mathscr{D}_i(\mathbf{p}^*)$.

For the special case of two primary services (i.e., $i = 1$ and $j = 2$), the set of equations in (9) can be expressed as in (10) and (11).

D. Dynamic Bertrand Game

In a practical cognitive radio environment, a primary service may not be able to observe the profit gained by other primary services. Also, the current strategies adopted by other primary services may be unknown. Therefore, each primary service must learn the behavior (i.e., strategy on choosing price to be offered to the secondary service) of other players from the history. Therefore, for a primary service, a distributed price adjustment algorithm is required which would gradually reach the Nash equilibrium for the pricing solution.

Let $p_i[t]$ denote the price offered by primary service i at iteration t. The sets $\mathbf{p}_{-i}[t]$ and $\mathbf{p}[t]$ are defined similarly. We first consider the case that the strategies of the primary services in the previous iteration are observable by each other (**Case I**). Given the strategies adopted by other players at time t (i.e., $\mathbf{p}_{-i}[t]$), the price offered by primary service i can be obtained iteratively from

$$p_i[t+1] = \mathscr{B}_i(\mathbf{p}_{-i}[t]), \quad \forall i. \tag{12}$$

However, in some cases, the strategies used by other primary services may not be known. Therefore, each primary service can use only local information and spectrum demand from the secondary service to adjust its strategy (**Case II**). In this case, a primary service will adjust its strategy in the direction that maximizes its profit. The relationship between the strategies in the current and the future iteration can be expressed as follows:

$$p_i[t+1] = p_i[t] + \alpha_i \left(\frac{\partial \mathscr{P}_i(\mathbf{p})}{\partial p_i} \right) \tag{13}$$

where α_i is the adjustment speed (i.e., learning rate).

To estimate the marginal profit, a primary service can observe the marginal spectrum demand for small variation in price ϵ (e.g., $\epsilon = 10^{-4}$) as shown in (14) and (15).

$$-\left(\frac{k_i^{(s)}-\nu k_j^{(s)}}{1-\nu^2}+\frac{2c_2k_i^{(p)}}{1-\nu^2}\left(B_i^{req}-\frac{k_i^{(p)}W_i}{M_i}+\frac{k_i^{(p)}k_i^{(s)}}{M_i(1-\nu^2)}-\frac{k^{(p)}\nu k_j^{(s)}}{M_i(1-\nu^2)}\right)\right)=$$
$$p_i\left(\frac{2c_2\left(k_i^{(p)}\right)^2}{M_i\left(1-\nu^2\right)^2}+\frac{2}{1-\nu^2}\right)+p_j\frac{\nu}{1-\nu^2}\left(1+\frac{2c_2\left(k_i^{(p)}\right)^2}{M_i(1-\nu^2)}\right) \quad (10)$$

$$-\left(\frac{k_j^{(s)}-\nu k_i^{(s)}}{1-\nu^2}+\frac{2c_2k_j^{(p)}}{1-\nu^2}\left(B_j^{req}-\frac{k_j^{(p)}W_j}{M_j}+\frac{k_j^{(p)}k_j^{(s)}}{M_j(1-\nu^2)}-\frac{k^{(p)}\nu k_i^{(s)}}{M_j(1-\nu^2)}\right)\right)=$$
$$p_j\left(\frac{2c_2\left(k_j^{(p)}\right)^2}{M_j\left(1-\nu^2\right)^2}+\frac{2}{1-\nu^2}\right)+p_i\frac{\nu}{1-\nu^2}\left(1+\frac{2c_2\left(k_j^{(p)}\right)^2}{M_j(1-\nu^2)}\right) \quad (11)$$

$$\frac{\partial \mathscr{P}_i(\mathbf{p})}{\partial p_i} \approx \frac{\mathscr{P}_i(\mathbf{p}_{-i}[t]\cup\{p_i[t]+\epsilon\})-\mathscr{P}_i(\mathbf{p}_{-i}[t]\cup\{p_i[t]-\epsilon\})}{2\epsilon} \quad (14)$$

$$\begin{aligned}\mathscr{P}_i(\mathbf{p}_{-i}[t]\cup\{p_i[t]\pm\epsilon\}) &= p_i\mathscr{D}_i(\mathbf{p}_{-i}[t]\cup\{p_i[t]\pm\epsilon\})+c_1M_i\\ &\quad -c_2M_i\left(B_i^{req}-k_i^{(p)}\frac{W_i-\mathscr{D}_i(\mathbf{p}_{-i}[t]\cup\{p_i[t]\pm\epsilon\})}{M_i}\right)^2\end{aligned} \quad (15)$$

E. Stability Analysis of the Dynamic Game

Stability is important for both the dynamic algorithms (i.e., for **Case I** and **Case II**) to ensure that the Nash equilibrium can be reached at the steady state. We analyze stability of the dynamic algorithms based on localization by considering the eigenvalues of the Jacobian matrix of the self-mapping function in (13). By definition, the self-mapping function is stable if and only if the eigenvalues λ_i are all inside the unit circle of the complex plane (i.e., $|\lambda_i| < 1$). In this case, the Jacobian matrix is defined as follows:

$$\mathbf{J}=\begin{bmatrix}\frac{\partial p_1[t+1]}{\partial p_1[t]} & \frac{\partial p_1[t+1]}{\partial p_2[t]} & \cdots & \frac{\partial p_1[t+1]}{\partial p_N[t]}\\ \frac{\partial p_2[t+1]}{\partial p_1[t]} & \frac{\partial p_2[t+1]}{\partial p_2[t]} & \cdots & \frac{\partial p_2[t+1]}{\partial p_N[t]}\\ \vdots & \vdots & & \vdots\\ \frac{\partial p_N[t+1]}{\partial p_1[t]} & \frac{\partial p_N[t+1]}{\partial p_2[t]} & \cdots & \frac{\partial p_N[t+1]}{\partial p_N[t]}\end{bmatrix}. \quad (16)$$

For the dynamic algorithm in **Case I** (i.e., for which the strategies of other primary services are observable), there is no control parameter to stabilize the price adaptation. For the special case of two primary services, the Jacobian matrix of this algorithm can be expressed as shown on the top of the following page.

Let $j_{x,y}$ denote the element at row x and column y of the Jacobian matrix. It is observed that element $j_{1,2}$ and $j_{2,1}$ are always less than one. Since $-1 \leq \nu \leq 1$, numerators of these elements are smaller than the denominators.

For the dynamic algorithm in **Case II** (i.e., for which the strategies of other primary services cannot be observed), the price adaptation depends largely on the learning rate (i.e., if the learning rate is large, the adaptation is fast). For the special case of two primary services, the corresponding Jacobian matrix can be expressed as in (17).

Since this Jacobian matrix is neither diagonal nor triangular, the characteristic equation to obtain the eigenvalues is given as follows: $(\lambda_1, \lambda_2) = \frac{(j_{1,1}+j_{2,2})\pm\sqrt{4j_{1,2}j_{2,1}+(j_{1,1}-j_{2,2})^2}}{2}$. Here, we observe that this dynamic algorithm can be either stable or unstable. The stability condition depends on the learning rate α_i, the number of local connections M_i, and the spectrum substitutability factor ν. However, the stability condition is independent of channel quality for the corresponding frequency spectrum.

Note that the stability analysis for an arbitrary number of primary services can be pursued in a similar fashion.

F. Optimal Pricing to Maximize Total Profit of Primary Services

The total profit for all the primary services is given by $\sum_{i=1}^{N}\mathscr{P}_i(\mathbf{p})$. The optimal price for all the primary services can be obtained from the following set of equations:

$$\frac{\partial \sum_{j=1}^{N}\mathscr{P}_j(\mathbf{p})}{\partial p_i}=0. \quad (18)$$

For the special case of two primary services (i.e., $i = 1$ and $j = 2$), we obtain the set of equations in (19) and (20). Clearly, these equations are not the same as those in (10) and (11) which are used to obtain the Nash equilibrium. Consequently, the optimal values of the prices p_i, which give the highest total profit, are different from those at the Nash equilibrium. Therefore, primary services may prefer to cooperate to achieve the highest profit. Note that for the case of arbitrary number of primary services, the same approach can be used to obtain the optimal solution.

G. Collusion and Repeated Game

Since the optimal prices to maximize the total profit of the primary services are different from those at the Nash equilibrium, in a noncooperative (or competitive) environment,

$$\mathbf{J} = \begin{bmatrix} 0 & \dfrac{\left(\dfrac{\nu}{1-\nu^2}+\dfrac{2c_2 M_1 (k_1^{(p)})^2 \nu}{M_1(1-\nu^2)^2}\right)}{\left(\dfrac{2}{1-\nu^2}+\dfrac{2c_2 M_1 (k_1^{(p)})^2}{M_1(1-\nu^2)^2}\right)} \\ \dfrac{\left(\dfrac{\nu}{1-\nu^2}+\dfrac{2c_2 M_2 (k_2^{(p)})^2 \nu}{M_2(1-\nu^2)^2}\right)}{\left(\dfrac{2}{1-\nu^2}+\dfrac{2c_2 M_2 (k_2^{(p)})^2}{M_2(1-\nu^2)^2}\right)} & 0 \end{bmatrix}$$

$$\mathbf{J} = \begin{bmatrix} 1-\alpha_1\left(\frac{2}{1-\nu^2}+\frac{2c_2}{M_1(1-\nu^2)^2}\right) & \alpha_1\left(\frac{\nu}{1-\nu^2}+\frac{2c_2\nu}{M_1(1-\nu^2)^2}\right) \\ \alpha_2\left(\frac{\nu}{1-\nu^2}+\frac{2c_2\nu}{M_2(1-\nu^2)^2}\right) & 1-\alpha_2\left(\frac{2}{1-\nu^2}+\frac{2c_2}{M_2(1-\nu^2)^2}\right) \end{bmatrix} \quad (17)$$

$$-\left(\frac{k_i^{(s)}-\nu k_j^{(s)}}{1-\nu^2}+\frac{2c_2 k_i^{(p)}}{1-\nu^2}\left(B_i^{req}-\frac{k_i^{(p)}W_i}{M_i}+\frac{k_i^{(p)}k_i^{(s)}}{M_i(1-\nu^2)}-\frac{k^{(p)}\nu k_j^{(s)}}{M_i(1-\nu^2)}\right)\right.$$
$$\left.-\frac{2c_2\nu k_j^{(p)}}{1-\nu^2}\left(B_j^{req}-\frac{k_j^{(p)}W_j}{M_j}+\frac{k_j^{(p)}k_j^{(s)}}{M_j(1-\nu^2)}-\frac{k_j^{(p)}\nu k_i^{(s)}}{M_j(1-\nu^2)}\right)\right)=$$
$$p_i\left(\frac{2c_2\left(k_i^{(p)}\right)^2}{M_i\left(1-\nu^2\right)^2}+\frac{2}{1-\nu^2}-\frac{2c_2\left(k_j^{(p)}\right)^2}{M_j\left(1-\nu^2\right)^2}\right)+p_j\frac{\alpha_i\nu}{1-\nu^2}\left(1+\frac{2c_2\left(k_i^{(p)}\right)^2}{M_i(1-\nu^2)}+\frac{2c_2\left(k_j^{(p)}\right)^2}{M_j(1-\nu^2)}\right) \quad (19)$$

$$-\left(\frac{k_j^{(s)}-\nu k_i^{(s)}}{1-\nu^2}+\frac{2c_2 k_j^{(p)}}{1-\nu^2}\left(B_j^{req}-\frac{k_j^{(p)}W_j}{M_j}+\frac{k_j^{(p)}k_j^{(s)}}{M_j(1-\nu^2)}-\frac{k^{(p)}\nu k_i^{(s)}}{M_j(1-\nu^2)}\right)\right.$$
$$\left.-\frac{2c_2\nu k_i^{(p)}}{1-\nu^2}\left(B_i^{req}-\frac{k_i^{(p)}W_i}{M_i}+\frac{k_i^{(p)}k_i^{(s)}}{M_i(1-\nu^2)}-\frac{k_i^{(p)}\nu k_j^{(s)}}{M_i(1-\nu^2)}\right)\right)=$$
$$p_j\left(\frac{2c_2\left(k_j^{(p)}\right)^2}{M_j\left(1-\nu^2\right)^2}+\frac{2}{1-\nu^2}-\frac{2c_2\left(k_i^{(p)}\right)^2}{M_i\left(1-\nu^2\right)^2}\right)+p_i\frac{\alpha_j\nu}{1-\nu^2}\left(1+\frac{2c_2\left(k_j^{(p)}\right)^2}{M_j(1-\nu^2)}+\frac{2c_2\left(k_i^{(p)}\right)^2}{M_i(1-\nu^2)}\right) \quad (20)$$

some primary services could unilaterally deviate from optimal price, especially when the game is played only once (i.e., one-shot game). In other words, the optimal pricing does not give a stable equilibrium. However, if the game is repeated, it is desirable for the players in the game to achieve an efficient solution to earn a high long-term profit. This can be achieved if all of the primary services establish a collusion. To model this, we formulate a repeated game which captures the behavior of the primary services when the pricing game is infinitely repeated. Since the game is repeated, a punishment mechanism can be applied to deter any primary service from deviating from the optimal price.

In a repeated game, the players play the game multiple times, and the outcome of the previous play can be observed. As a result, the players can learn and coordinate their actions so that the desired result is achieved. If the game is repeated, each step in playing the game can be defined as a stage. In general, one stage lasts for the period starting at the time when the players choose their actions and ending at the time when the outcome is observed. In a dynamic Bertrand game, since the price adaptation can be in transient state before it reaches the steady state, the outcome of the game cannot be instantaneously observed. In this case, the transient state exists for a time period during which the difference between prices in two consecutive iterations is larger than the threshold ϵ (otherwise, the price adaptation is in steady state). This dynamic Bertrand game can be defined as a repeated game for which each stage is defined from the time that the players change the parameters of price adaptation to the time that the steady state is reached. The stages of a repeated dynamic game can be shown as in Fig. 2. In this repeated dynamic game of spectrum price competition, the set of actions consists of *maintaining collusion*, *deviating from collusion*, and *punishment actions*. When the punishment action is used, all of the primary services choose the Nash equilibrium strategy from which none of the primary services wants to deviate. Also, the total profit at the Nash equilibrium is less than or equal to that due to the optimal pricing.

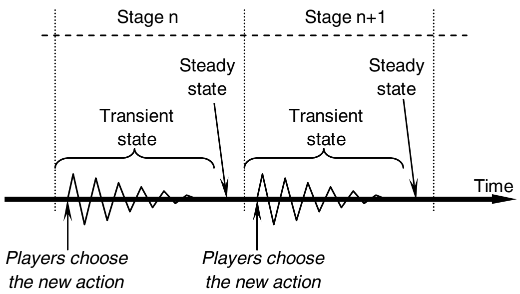

Fig. 2. Repeated game.

We consider a *trigger strategy* for this repeated game. With this *trigger strategy*, any primary service will maintain collusion as long as other services agree to do so. However, if a primary service deviates from the collusion (i.e., triggers), then all of the other primary services will use the punishment action permanently. In this case, the primary service will consider the long-term profit. However, since part of the long-term profit will be gained in the future stages, a primary service usually gives smaller weight to the profits in the future stages than that to the current profit. In particular, if the current profit is $\mathscr{P}_i$, the profit in the next stage is worth of $\delta_i \mathscr{P}_i$, where δ_i is the weight ($0 \leq \delta_i \leq 1$).

Note that the parameter δ can be considered as a discount factor. The discount factor is able to consider the fact that the profit that a primary service receives in the current stage is more worthy than the profit received in a future stage due to the interest and inflation. For example, if the inflation rate is high, the value of the current profit is higher than the value of the profit in the future. In such a case, the discount factor is small since the primary service will be interested in the current profit more than the future profit. Similarly, if the interest rate is high, the current profit can be invested to earn more benefit. Therefore, the current value of the profit is higher, and the discount factor is small.

For primary service i, let $\mathscr{P}_i^o$, $\mathscr{P}_i^n$, and $\mathscr{P}_i^d$ denote the profits due to the optimal price, the price at the Nash equilibrium, and the price due to deviation, respectively. Then, for the case that the collusion is maintained forever, the long-term profit of primary service i can be expressed as follows:

$$\mathscr{P}_i^o + \delta_i \mathscr{P}_i^o + \delta_i^2 \mathscr{P}_i^o + \delta_i^3 \mathscr{P}_i^o + \ldots = \frac{1}{1-\delta_i} \mathscr{P}_i^o. \quad (21)$$

In other words, the long-term profit is the weighted-sum of current profit and the profit in all the future stages. On the other hand, if one primary service deviates from the optimal price, that service will gain deviated profit in the first stage, while during the rest of the stages, this primary service will experience the profit at the Nash equilibrium. Therefore, the long-term profit can be expressed as follows:

$$\mathscr{P}_i^d + \delta_i \mathscr{P}_i^n + \delta_i^2 \mathscr{P}_i^n + \delta_i^3 \mathscr{P}_i^n + \ldots = \mathscr{P}_i^d + \frac{\delta_i}{1-\delta_i} \mathscr{P}_i^n. \quad (22)$$

The collusion will be maintained if the long-term profit due to adopting collusion is higher than that due to deviation, i.e.,

$$\frac{1}{1-\delta_i} \mathscr{P}_i^o \geq \mathscr{P}_i^d + \frac{\delta_i}{1-\delta_i} \mathscr{P}_i^n. \quad (23)$$

Therefore, the lower bound for the value of δ_i is obtained as follows:

$$\delta_i \geq \frac{\mathscr{P}_i^d - \mathscr{P}_i^o}{\mathscr{P}_i^d - \mathscr{P}_i^n}. \quad (24)$$

For the values of δ_i obtained as above, a collusion will be maintained in which the optimal price will be used to gain the highest profit in each state, and consequently, the highest long-term profit.

V. Performance Evaluation

A. Parameter Setting

We consider a cognitive radio environment with two primary services (primary service *one* and *two*) and a secondary service (Fig. 1). The total frequency spectrum available to each primary service is 20 MHz (i.e., $W_i = 20$). The number of local connections at each primary service is set as $M_1 = M_2 = 10$. The target BER for the secondary service is $\text{BER}^{\text{tar}} = 10^{-4}$. The bandwidth requirement of the connections at each primary service is 2 Mbps (i.e., $B^{req} = 2$), and $c_1 = c_2 = 2$. The channel quality for the secondary service varies between 9 to 22 dB. The spectrum substitutability factor lies between 0.1 to 0.6. For the dynamic price adaptation algorithms, the initial prices are set as follows: $p_1[0] = p_2[0] = 1$. Note that some of these parameters will be varied according to the evaluation scenarios.

B. Numerical Results

1) Variation of Primary Services' Profit with Offered Price: Fig. 3 shows the profit of primary service *one* as a function of offered price. When the offered price increases, the profit increases since more revenue is generated due to higher price. However, after a certain point, since demand from the secondary service decreases, this profit decreases. The price which results in the highest profit is the best response. That is, given the prices of other primary services, the best response for a particular primary service is the price for which the profit is maximized. Also, we observe that a higher price offered by primary service *two* results in a larger value of the best response (i.e., offered price) for primary service *one*. This is due to the fact that when the price offered by service *two* increases, the demand for spectrum from service *one* increases. As a result, primary service *one* can offer a higher price to gain a higher profit.

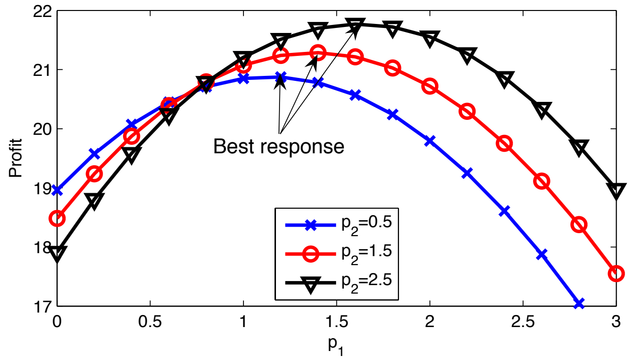

Fig. 3. Profit of primary service *one* as a function of offered price.

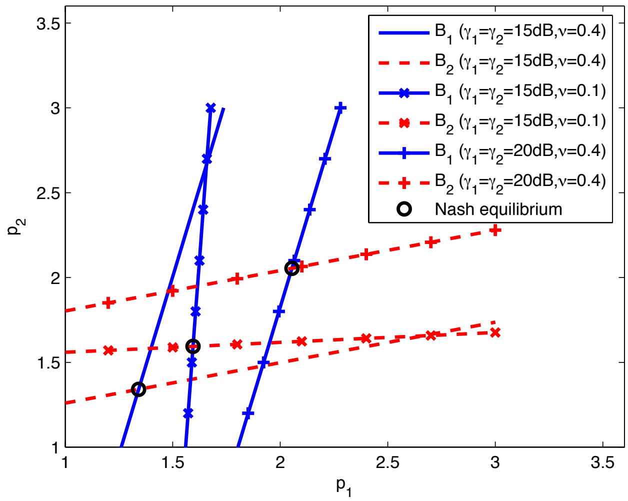

Fig. 4. Best response functions and Nash equilibrium.

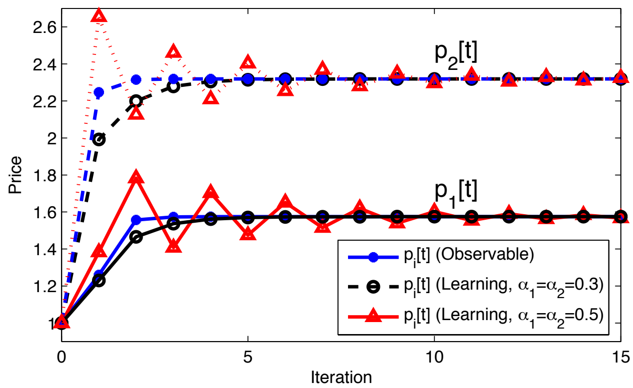

Fig. 5. Convergence to the Nash equilibrium.

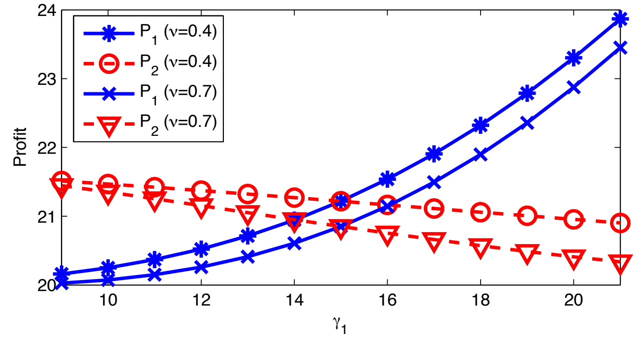

Fig. 6. Profit of each primary service at equilibrium under different channel qualities corresponding to the frequency spectrum offered by primary service *one*.

2) Best Response Functions and Nash Equilibrium: The best response functions for both the primary services are shown in Fig. 4 under different channel quality (γ_1, γ_2) for the secondary service and under different spectrum substitutability factor ν. The Nash equilibrium is located at the point where the best response functions of the primary services intersect. When the channel quality becomes better, since a secondary user can transmit at a higher rate due to adaptive modulation, the demand for spectrum opportunities increases. As a result, the primary service can offer a higher price.

The spectrum substitutability factor also has a significant impact on the Nash equilibrium. Specifically, when the secondary service can switch freely among the frequency spectra offered by the primary services (i.e., ν is high), the Nash equilibrium results in a smaller price. Since the secondary service can switch to the cheaper spectrum easily, the degree of competition among the primary services becomes higher. Therefore, each primary service offers a lower price to attract the secondary service. Note that while the channel quality impacts the location of the best response functions, this spectrum substitutability factor impacts their slopes. Also, both of these parameters impact the location of the Nash equilibrium.

3) Dynamic Behavior and Convergence to Nash Equilibrium: Fig. 5 shows the convergence of price adaptation by the primary services for $M_1 = 10$ and $M_2 = 14$. When the strategies of the primary services are observable by each other, a fast convergence to the equilibrium price is expected (i.e., requires only a few iterations). However, if a primary service can observe only the spectrum demand from the secondary service (i.e., **Case II**), and the price is adjusted based on this information, the speed of convergence depends largely on the learning rate. If this learning rate is properly set (e.g., $\alpha_1 = \alpha_2 = 0.3$), the algorithm converges to the equilibrium price as fast as that for the case when the strategies of the other players are observable (i.e., **Case I**). However, if the learning rate is large (e.g., $\alpha_1 = \alpha_2 = 0.5$), it causes fluctuations in the price adaptation, and the algorithm may require a larger number of iterations to reach the equilibrium. Note that a larger number of local connections at primary service *two* results in a higher cost of sharing the spectrum. As a result, the price offered by this primary service becomes higher.

4) Price and Profit under Different Channel Qualities and Spectrum Substitutability Factors: Figs. 7-6 show the price and the profit of both the primary services at the Nash equilibrium. When the channel quality (i.e., SNR at the secondary receiver) corresponding to the spectrum offered by primary service *one* becomes better, the spectrum demand becomes higher. Therefore, primary service *one* can increase the price as well as the size of the offered spectrum share to gain a higher revenue. While primary service *one* gains a higher profit due to larger demand, primary service *two* gains a lower profit due to smaller demand.

The spectrum substitutability factor ν also impacts the rates of increase and decrease in prices due to the different channel qualities. While the price offered by primary service *one* is only sightly affected by a larger value of ν, the price offered by primary service *two* decreases at a higher rate for a larger value of ν. Since with a smaller value of ν the price offered by primary service *one* is lower, the rate of decrease in the price offered by service *two* has to be higher to attract the secondary service. This is required to achieve the highest profit given the channel qualities corresponding to the spectrum offered by primary service *one*.

5) Stability Region and Bifurcation Analysis: Fig. 8 shows the stability region of the learning rates for the dynamic algorithm in the case that the strategies of other primary services cannot be observed. When the learning rates α_1 and α_2 are set with values taken from the stability region, the distributed dynamic algorithm successfully converges to the

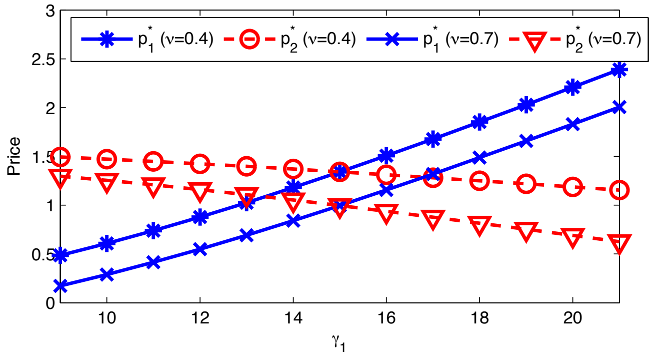

Fig. 7. Equilibrium price under different channel qualities corresponding to the frequency spectrum offered by primary service *one*.

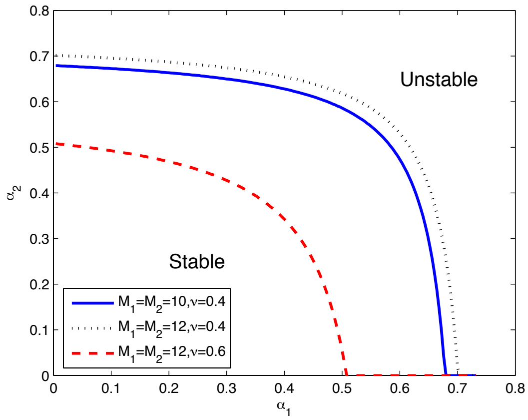

Fig. 8. Stability region under different learning rates.

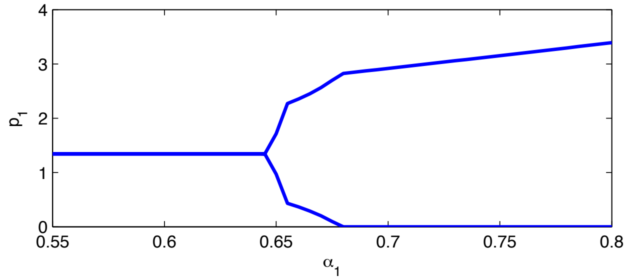

Fig. 9. Bifurcation diagram under different learning rates.

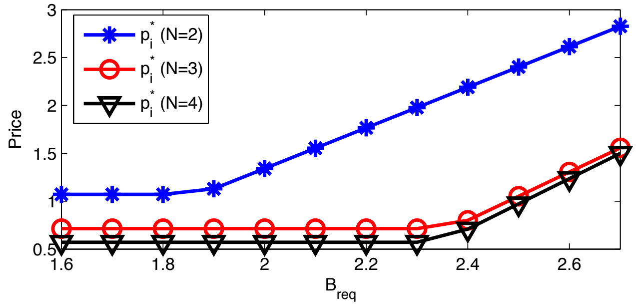

Fig. 10. Price adaptation under different number of primary services.

Nash equilibrium. Here, we observe that a smaller number of connections at the primary service causes a destabilization effect. This is due to the smaller price at the equilibrium. Larger values of learning rates will create high fluctuations in price adaptation which diverges from the equilibrium when the price is small. This destabilization effect is also observed when the substitutability factor becomes large (i.e., for a small equilibrium price).

By using bifurcation diagram we investigate the system behavior when the price adaptation is unstable (Fig. 9). In this case, α_2 is fixed at 0.3 while α_1 (i.e., learning rate of primary service *one*) is varied. From the bifurcation diagram, we observe that when the learning rate is small (i.e., $\alpha_1 < 0.645$), the price adaptation is stable and at the steady state a single solution, i.e., Nash equilibrium, is reached. However, when $\alpha_1 > 0.645$, the adaption is unstable and it swings between two values. When the learning rate increases, the gap between these two values becomes larger due to the fluctuation. Note that this strategy adaptation can result in *pitchfork bifurcation* [22] since at a particular value of the learning rate (i.e., the control parameter), the strategy at the steady state changes from single to multiple values. This observation on strategy adaptation would be useful for adjusting the learning rate so that the Nash equilibrium can be achieved.

6) Impact of Number of Primary Services: Next, we investigate the effect of the number of primary services N on the prices at the equilibrium (Fig. 10). In this case, the size of total spectrum is 60 MHz and the total number of primary connections served by all primary services is 24. For the case of $N = 2$, the size of available spectrum is $W_i = 30$ MHz and the number of local connections is $M_i = 12$. For $N = 3$, we assume $W_i = 20$ MHz and $M_i = 8$. For $N = 4$, we assume $W_i = 15$ MHz and $M_i = 6$. Here, we observe that a higher number of primary services results in a lower offered price. Since the level of competition becomes higher, the primary services must decrease their offered price to attract the secondary service to gain the highest revenue and profit. Note that when the bandwidth requirement of each of primary connections increases, the offered price increases due to the QoS performance degradation. However, when the bandwidth requirement is small, the price remains constant. The offered price starts increasing when the bandwidth requirement cannot be satisfied and the primary service must charge higher price to the secondary service to compensate this cost.

7) Inefficiency of Nash Equilibrium: Fig. 11 shows the variations in profit corresponding to Pareto optimality (i.e., when one player cannot increases its payoff without decreasing other players' payoffs). It is clear that Nash equilibrium is not Pareto optimal while the optimal solution is. This shows that Nash equilibrium is inefficient (i.e., the total profit is not maximized). Even though the profit at the Nash equilibrium is not the highest, the Nash equilibrium can provide a "stable" solution from which none of the primary services wants to deviate. However, if a collusion among the primary services can be established at the optimal price, each of the primary services can achieve a profit higher than that at the Nash equilibrium. Note that the collusion is naturally "unstable" since the optimal price does not lie on the best response function. That is, it represents a feasible strategy for which the profit of one or more primary services can increase.

8) Collusion among Primary Services: Fig. 12 shows the prices at the Nash equilibrium, and the optimal prices which result in the highest profit. The optimal price is higher than that at the Nash equilibrium. If both of the primary services make an agreement to establish a collusion, the optimal price will be offered to the secondary service. However, one primary

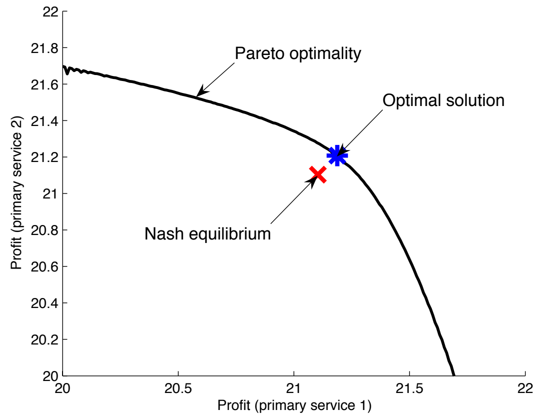

Fig. 11. Pareto optimality, Nash equilibrium, and optimal solution.

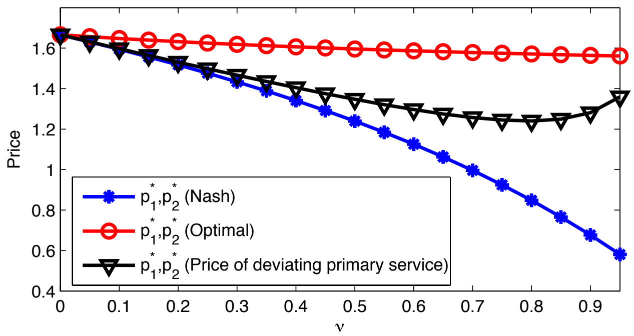

Fig. 12. Price at the Nash equilibrium, the optimal price, and the price of deviating primary service.

service can unilaterally deviate to achieve a higher profit than that it can gain at the optimal price. This price due to deviation, which is observed to be always lower than the optimal price and higher than that at the Nash equilibrium, is also shown in Fig. 12.

The above observation can be explained as follows. If primary service *one* believes that service *two* will offer the optimal price, primary service *one* can decrease the offered price to increase the spectrum demand from secondary service. This will result in higher revenue and profit. However, primary service *two* will experience lower profit than that when both the primary services offer the optimal price. In this case, primary service *two* will punish service *one* by offering the price at the Nash equilibrium which is lower than the optimal price. Consequently, primary service *one* must offer the price at the Nash equilibrium as well to maintain its highest profit. Note that as the spectrum substitutability factor increases, both the optimal price and the price at the Nash equilibrium decrease due to a higher level of competition.

In order to the maintain the collusion, both of the primary services must consider the profit in the future when the punishment will be enforced, if one of the primary services deviates from the optimal price. In this case, to maintain a collusion, the required smallest value of weight δ_i can be obtained for the profit in the future.

Fig. 13 shows variations in the lower bound of δ_i under different channel qualities. In this case, when the channel quality corresponding to the spectrum offered by primary service *two* becomes higher, the corresponding weight δ_i becomes smaller while that of primary service *one* becomes

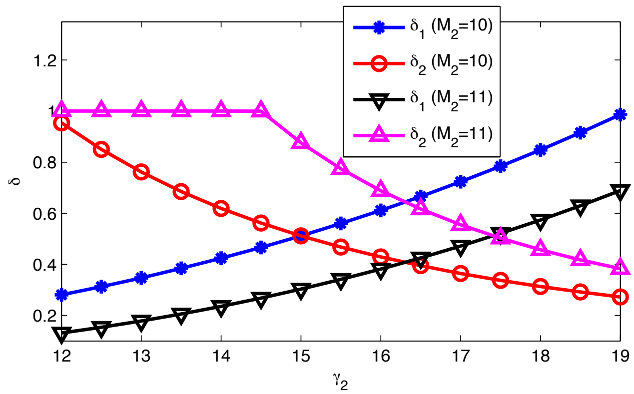

Fig. 13. The smallest value of the weight to maintain collusion.

larger. Since a better channel quality results in smaller benefit (i.e., profit) obtained due to deviation from the optimal price, primary service *two* has less motivation to deviate. As a result, the value of δ_i can be reduced to maintain collusion. On the other hand, for primary service *one*, the profit from the optimal price decreases. In particular, the punishment becomes less severe, and thus it has more motivation to deviate. Consequently, a larger value of δ_i is required for primary service *one* to maintain the collusion. When the number of primary connections served by primary service *two* increases, the price increases while the profit decreases, especially, at small average SNR. In this case, the profit obtained due to collusion is smaller than that due to Nash equilibrium. Therefore, primary service *two* will always deviate (i.e., value of δ_i to maintain the collusion is one).

VI. Conclusion

We have presented a game-theoretic model to obtain the optimal pricing for dynamic spectrum sharing in cognitive radio networks. We have considered an environment in which multiple primary services compete with each other to offer spectrum access opportunities to the secondary service. We have formulated this competition as an oligopoly market in which the firms adjust their prices dynamically to gain the highest profit. For the primary services, the cost of sharing the spectrum with the secondary service has been calculated as a function of the QoS performance degradation of the primary connections. We have used a utility function for the secondary service to obtain the spectrum demand function. The channel quality and the spectrum substitutability of the secondary service have been taken into account in the presented model.

We have analyzed the problem as a Bertrand game and obtained the Nash equilibrium which provides the optimal pricing (i.e., maximizes the total profit of all the primary services). We have also considered a distributed environment in which for any primary service, the profit functions of the other primary services are not available; however, the primary services can observe only the strategies (i.e., offered prices) of each other. This information is used by each primary service to dynamically adjust its strategy to achieve the Nash equilibrium. We have investigated further the case that the primary services cannot observe the strategies of each other. In the case, only the spectrum demand information from the secondary service is used by each primary service. The strategy adaptation of each primary service can be controlled by the learning rate. The stability condition of this algorithm has been studied.

To this end, we have shown that the Nash equilibrium is inefficient to achieve the highest total profit for all of the primary services. The optimal price to gain the highest profit can be obtained if all of the primary services can make an agreement to establish a collusion. Unlike a Nash equilibrium, this optimal price does not lie on the best response function; therefore, any primary service can deviate to gain higher profit. However, this will decrease the profit of other primary services. This situation has been studied by using a repeated game. We have shown that if all of the primary services are aware of the punishment due to the deviation, by properly weighting the profit in the future, a collusion can be maintained in the long-term so that all of the primary services gain higher profit compared with that at the Nash equilibrium.

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