Connectivity in Large-scale Cognitive Radio Ad-hoc Networks

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Overview

- Definition of connectivity
- Motivations
- Network models
- Sufficient conditions for connectivity in CRAHNs
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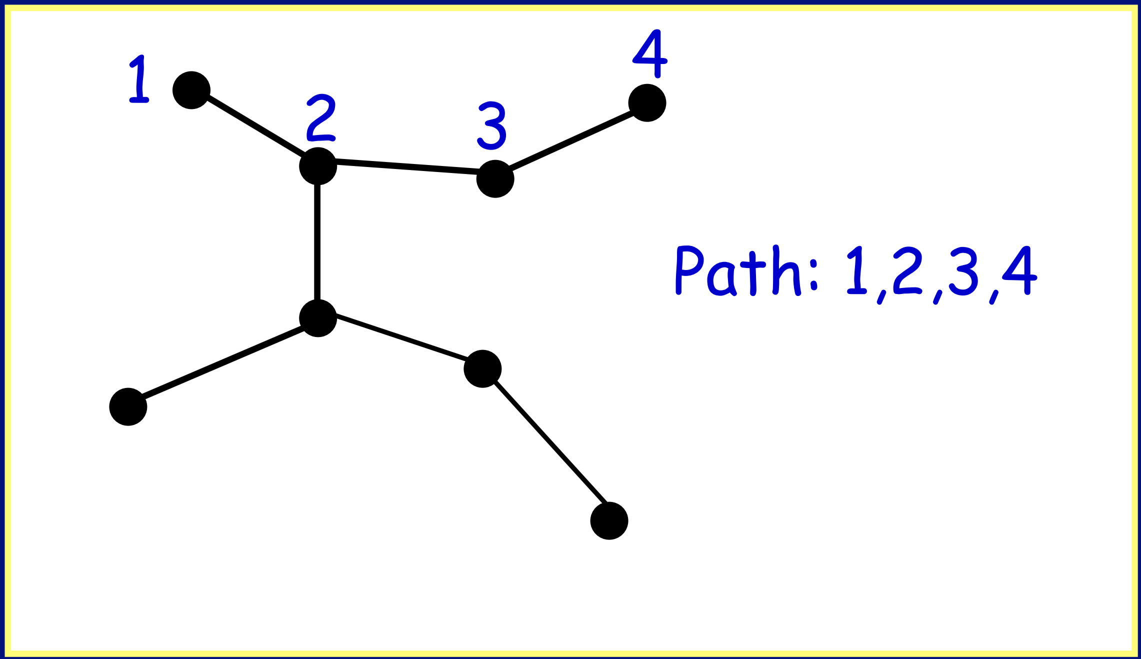
Overview

- Backgrounds
  - Conventional definition of connectivity
  - Definition of connectivity based on percolation theory
  - Keys results regarding network connectivity based percolation theory

Definition of Network Connectivity

- A network can be presented by graph G(V,E)
  - V $=\{v_1, v_2, \ldots, v_n\}$ denote nodes in the networks
  - E $=\{e_{ij}\}$ denote the link between nodes i and j if the distance between them is less than the transmission range.
  - A path is a sequence of nodes such that from each node there is link to next node in this sequence.
  - Two nodes is connected if there is at least one path between them.
  - A network is fully connected if for any two nodes in this network, there is at least one path between them

Definition of Network Connectivity



Definition of Network Connectivity

- For the finite network with a fixed topology, full connectivity can be achieved by topology control.

  Note: finite network means limited number of nodes

- The primary goal of topology control is to design power-efficient algorithms that maintain network connectivity and optimize performance metrics such as network lifetime and throughput

Definition of Network Connectivity

- For the infinite network with a random topology, full connectivity is difficult achieved.

  Note: infinite network means the network covers infinite area with a certain node density λ so that there exists a infinite number of nodes.

- To address this problem, recent research has focused on a notation of connectivity based on percolation theory.

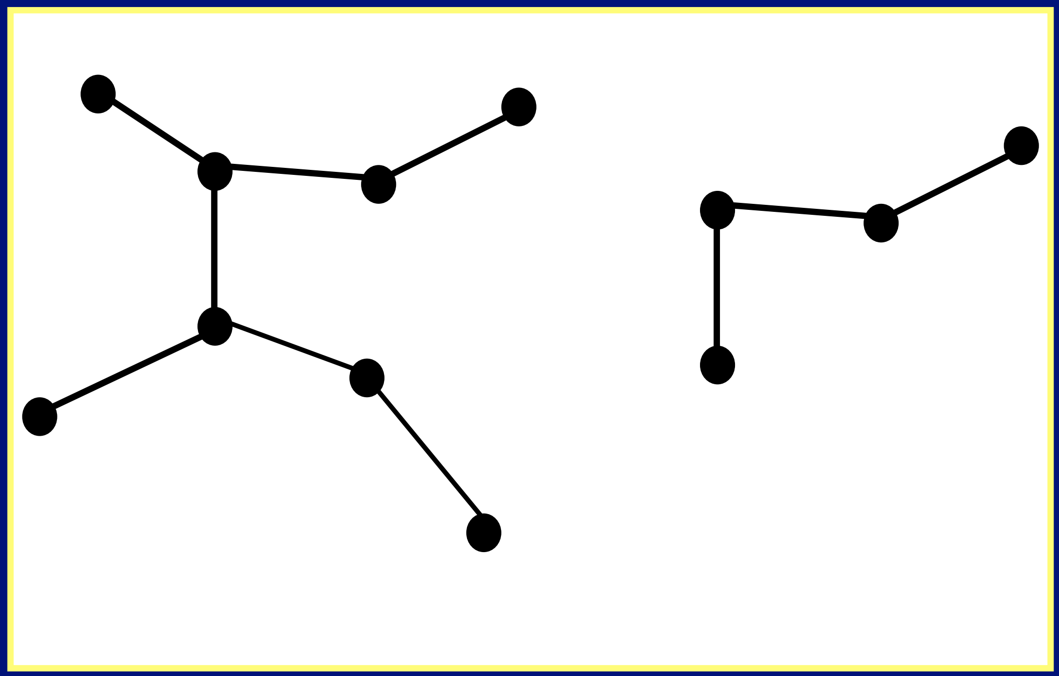
Definition of Connectivity: a notation based on percolation theory

- A infinite network is connected if there exists a infinite connected component (with high probability).

  - A connected component is a subgraph of the network in which any two nodes are connected by paths.

  - A infinite connected component is connected component consisting of a infinite number of nodes so that the nodes span almost the entire network

Definition of Network Connectivity



Keys results regarding network connectivity based percolation theory

- The fundament results of percolation theory concerns a phase transmission effect,

  i.e., there exists a critical density λ_c so that

  1. if $\lambda < \lambda_c$ (subcritical), the network only consists of small isolated components w.h.p

  2. if $\lambda > \lambda_c$ (supercritical), the network exists a unique infinite connected component w.h.p => connectivity in supercritical phase

  Note: Boolean model is used

Connectivity of SINR Graph

O. Dousse F.Baccelli and P. Thiran, "Impact of Interferences on Connectivity in Ad Hoc Networks," Proc. INFOCOM 2003
O. Dousse and P. Thiran, "Connectivity vs capacity in dense ad hoc networks," Proc. INFOCOM 2004

- Instead of assuming Boolean model, SINR model is considered, i.e., node x_i can connect with x_j, if

$$\frac{P_i L(x_i - x_j)}{N_0 + \gamma \sum_{k \neq i,j} P_k L(x_k - x_j)} \geq \beta,$$

where γ is the interference weight. L(.) is the bounded attenuation function, that is,

$L(x) = 0$ if $|x| > d$

$a < L(x) < b$ if $|x| < \delta$

Connectivity of SINR Graph

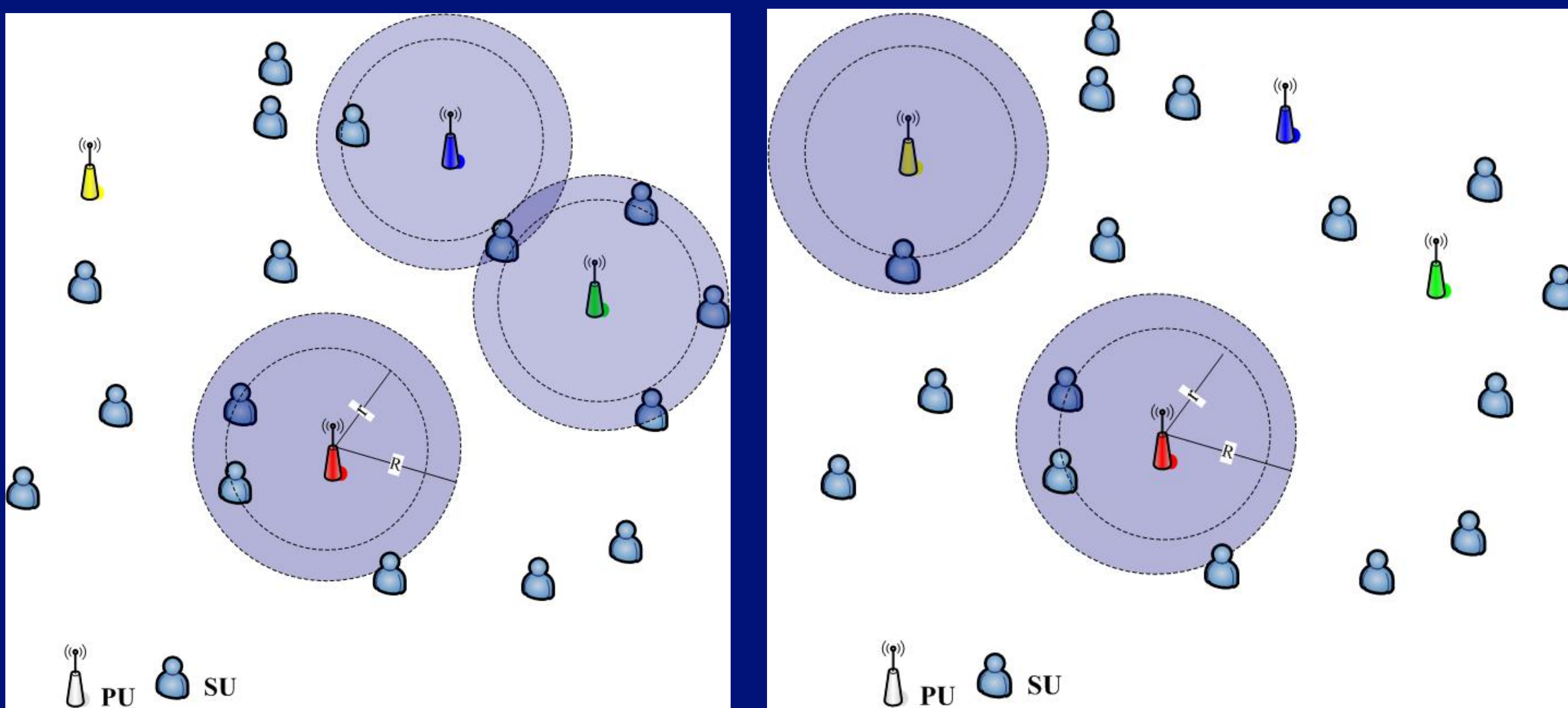
- Conclusion: if γ is small enough, there exists a critical density so that
  if $\lambda > \lambda_c$ (supercritical), the network exists a unique infinite connected component w.h.p

Connectivity of large-scale cognitive ad-hoc network

Motivation

- Connectivity of secondary network (SN) depends on behavior of primary network (PN)
- Whether two secondary users (SUs) can communicate depends on the unoccupied spectrum space in the primary networks (PNs).
- The unoccupied spectrum space, called white space, is a spatial-temporal random process determined by the topology of PN and PU activities.

Snapshots of PN and SN



Motivation

- Now, we have two questions:

  1) Under what conditions (densities) is the secondary network (SN) connected *at all times?*

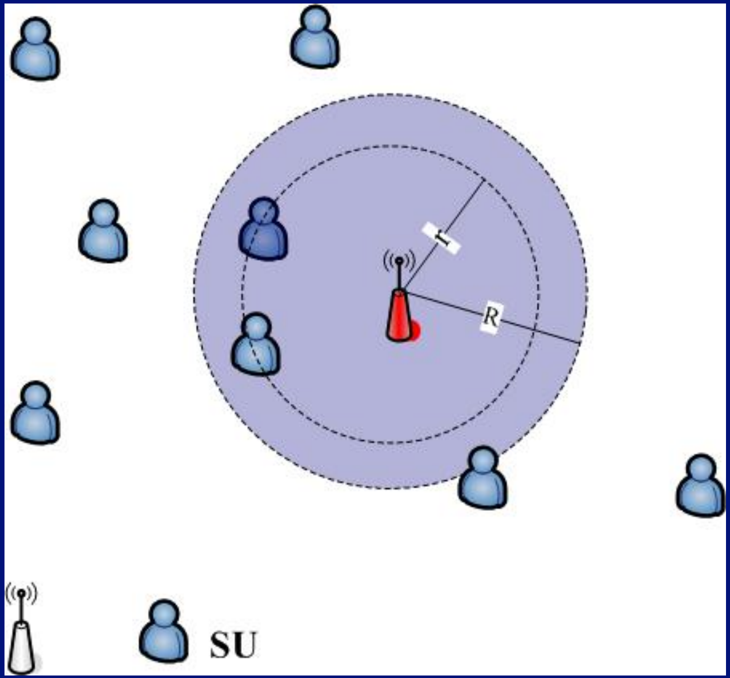
  2) what are the bounds of the transmission latency in the SN ?

Network models

- Both PN and SN are ad-hoc networks.
- The PN follows Poisson point processes denoted by $G(H_{\lambda}^{p}, r)$ with density λ^{p}. We associate each PU with a receiver which is at distance d from PU. d follows uniform distribution.
- The SN follows Poisson point processes denoted by $G(H_{\lambda}^{s}, r)$ with density λ^{p} and λ^{s}.
- Both PU and SU has transmission range r

Interference model of PU

- Disc inference model of each PU and SU



PU activity/MAC model

- Random access/contention-based MAC for PU
- Assume pure-Aloha MAC or slotted Aloha due to its simplicity and traceability.
  - each node independently transmits its own data without considering other nodes' activities.

PU activity/MAC model

- PU activity is modeled by stationary ON/OFF process, denoted by $W_i(t)$. $W_i(t) = 1$ if PU_i is transmitting at t and $W_i(t) = 0$ if PU_i is inactive at t.
- Assume $W_i(t)$ is probabilistically identical for each PU_i and we use $W(t)$ instead of $W_i(t)$.

PU activity/MAC model

- Under above assumptions, the stationary distribution of W(t) is given by

$$\Pi_1 = \Pr\{W(t) = 1\} = \frac{E[\tau_{ON}]}{E[\tau_{ON}] + E[\tau_{OFF}]}$$

$$\Pi_2 = \Pr\{W(t) = 0\} = \frac{E[\tau_{OFF}]}{E[\tau_{ON}] + E[\tau_{OFF}]}$$

where $E(\tau_{ON})$ ($E(\tau_{OFF})$) is the expectation of the duration of ON (OFF) period.

Sufficient conditions of connectivity in secondary network: dynamic percolation

- Definition 1: let $G_1 = G(H_\lambda^p, 1, W(t))$ denote the sampled graph of PN at time t consisting of all the PUs that are active at time t, along with their associated links.

- Definition 2: let $G_2 = G(H_\lambda^s, 1, W(t))$ denote the sampled graph of SN at time t consisting of all the SUs residing in the white space at time t, along with their associated links

Sufficient conditions of connectivity in secondary network

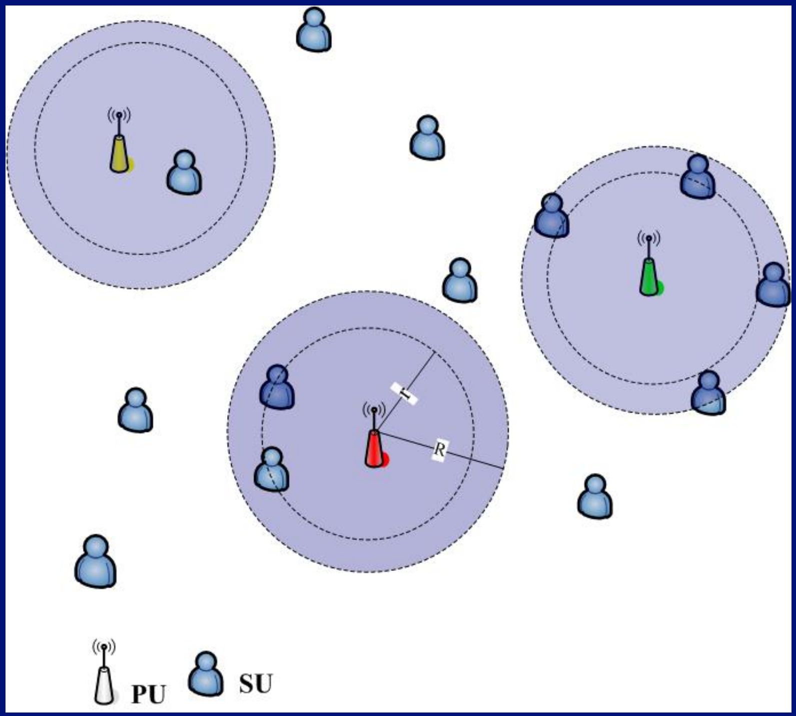
- Theorem 1: Given a SN modeled by $G(H_\lambda^s, r)$ and a PN modeled by $G(H_\lambda^p, r)$, if

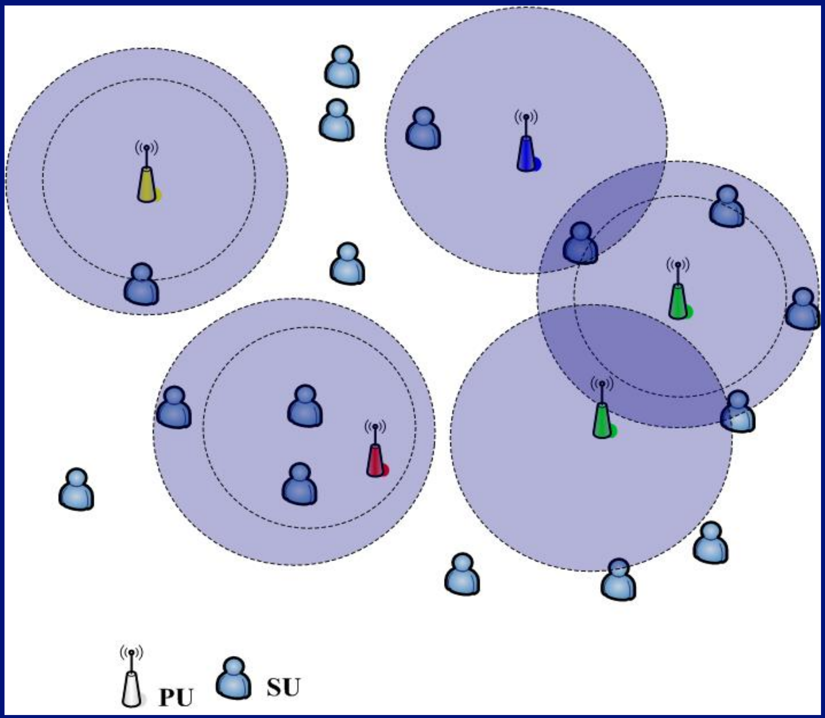
$$\lambda^s > \max\{\lambda_c^{\ s}, \frac{5}{r^2}\ln\frac{1}{1-\sqrt{1-p}\exp[2\Pi_1\lambda^p(\pi R^2+4\sqrt{1/5}R)]}\}$$

$$\lambda^p < \min\{\frac{r^2}{4R^2\Pi_1}\lambda_c^{\ p}, \frac{\ln 1/\sqrt{1-p}}{\Pi_1(\pi R^2+4\sqrt{1/5}R)}\}$$

then the SN $G(H_\lambda^s, 1, W(t))$ is percolated (or connected) at all t.
where $p = (11-2*10^{1/2})/27$. r is trans. Range of PU. R is the interference range of PU. λ_c^p and λ_c^s are critical densities of PN and SN

=> connectivity in supercritical phase





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Sufficient conditions of connectivity in secondary network

- Theorem 2 (latency): If a SN $G(H_{\lambda s}, r, w(t))$ is not connected at any time t (subcritical), then the message from any node in $G(H\lambda s, r)$ can eventually reach any destination after a certain T.

  => connectivity in subcritical phase

Connectivity in cognitive ad-hoc networks with static spectrum pool

W.Ren, Q. Zhao, and A. Swam, "Connectivity of Heterogeneous Wireless networks," submitted to IEEE Trans. Information Theory, Aug. 2009

- Assume that primary users keep transmitting data all the time.

  Conclusion: the SN is connected if

$$\lambda^s > \lambda_c^{\,s}$$

$$\lambda^p < \min\{\frac{\lambda_c(1)}{4R^2 - r^2}, \frac{1}{2\pi R^2 - I(R,r)} \ln \frac{1-\exp(-\frac{\lambda^s r^2}{8})}{1-(1/3)^{(2k+1)^2}}\}$$

where λc is critical density when r = 1. I(R,r) is a certain function of R and r. k is some constant.

-The availability of spectrum is not changing over time and only depends on the locations of PUs and SUs. Thus, connectivity is not related to PU activities

-If network is not connected (subcritical), it is impossible for any node to transmit data to the nodes far away. Therefore, there is no latency issue.

Basic proof steps for connectivity in Cognitive radio

- Principle: to prove there exists a infinite connected component w.h.p.

  => To prove the origin belongs to infinite connected component with a positive probability. (Kolmogorov's zero-one law)

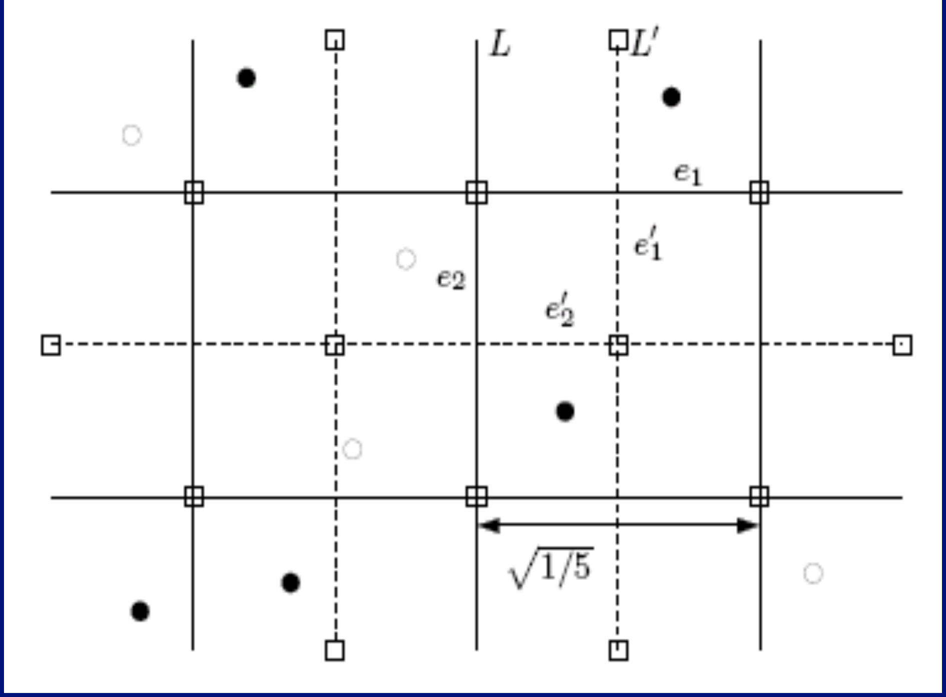
- Steps:

  First, map a network to a lattice

  Second, show there is a closed circuit around with probability less than 1

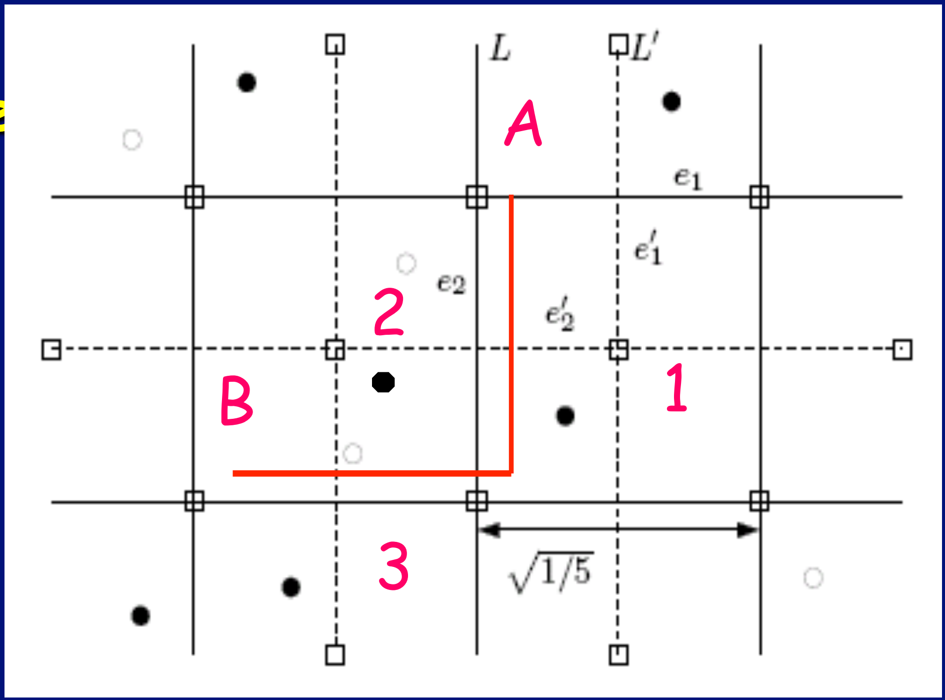
Basic proof steps

- L : original lattice
- L': dual lattice of L'
- e : edge (bond) of L
- e': edge (bond) of L' (perpendicular to e)
- e is open : there exists at least one node in each adjacent cell
- e' is close when e is close



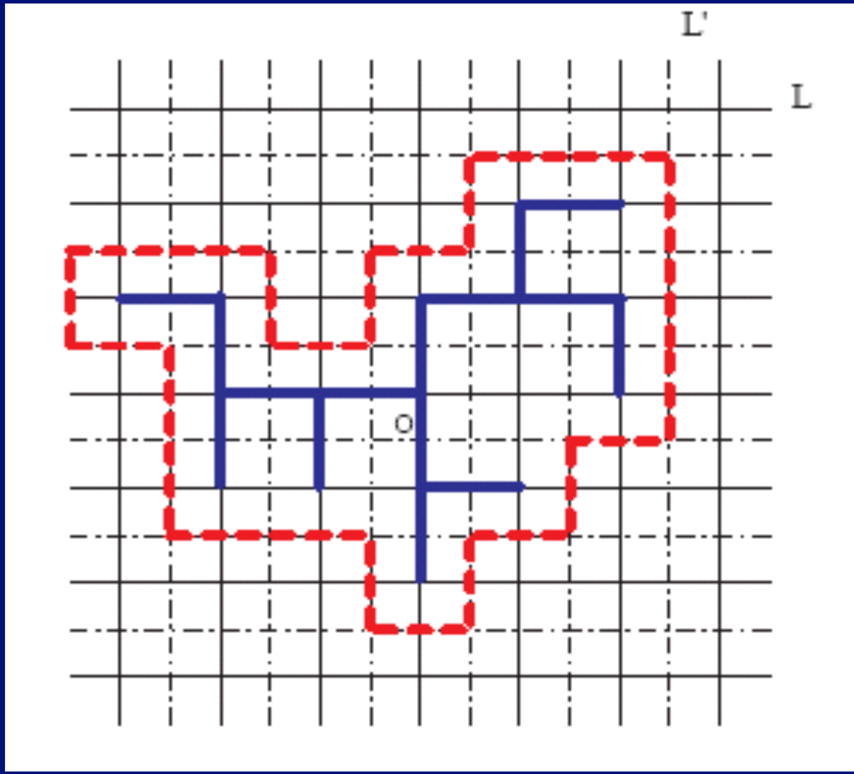
Basic proof strategy

- Open edge in L => two nodes in adjacent cells are connected d = $(1/5)^{0.5}$
- Open path (edges are open) in L=> Connected component
- Open path with unlimited length in L => infinite connected component



Basic proof strategy

- There is infinite connected component with positive probability => closed circuits exist with probability < 1
- A circuit is closed => all edges are closed
- Close probability depends on interested event



Basic proof strategy

- A circuit is closed => all edges are closed
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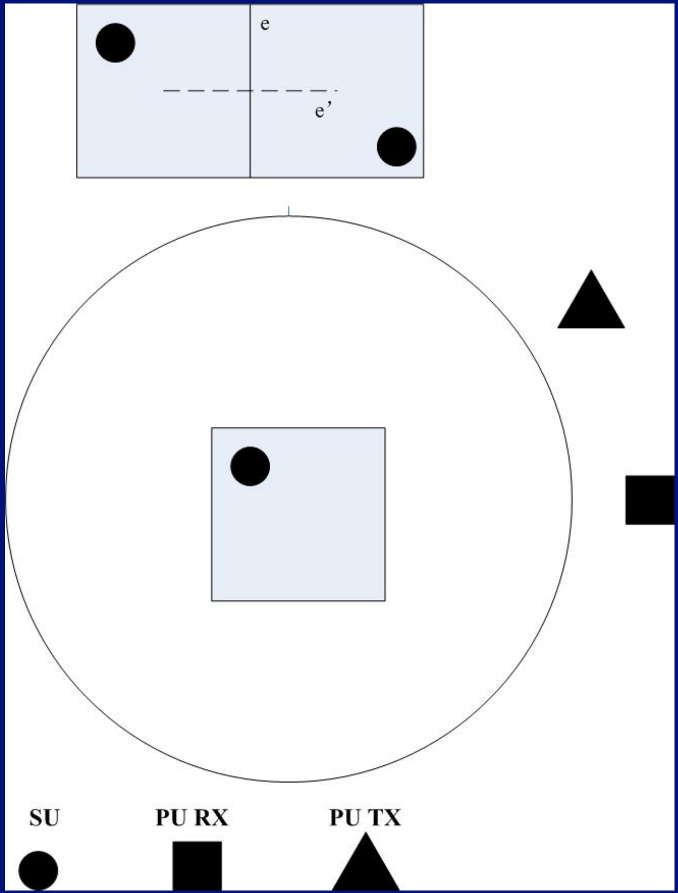

Next step:

latency

First Passage Percolation

- FPP is time dependent model for the flow of liquid through a porous body

- Suppose these exists a two dimensional lattice where each edge *e* is associated with a random variable T(e), called time coordinate.

- For any path *L*, the passage time is the sum of time coordinates of all edges in *L*

First Passage Percolation

- The first passage time (FPT) is defined as the infimum of the passages times of all paths between source and destination.

- In our case, we need to reveal the relationship between FPT and network settings, e.g., ON and OFF statistics and Euclidean distance

Q&A

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